TD N°2

Exercise 1 :

1) Calculate the complexity of the following unary addition algorithm:
We define two basic operations: ++ and −−. If a is a string of ones, then a++ is formed from a by appending a '1' to a, while a−− is formed from a by deleting a '1' from the end of a.

Algorithm *Unary integer addition.*

Input: integers $a \geq b \geq 0$ encoded in unary.
Output: $a + b$ in unary.
Algorithm:

```
while b ≠ 0
    a ← a++
    b ← b−−
end-while
output a
```

2) Calculate the complexity of the following binary addition algorithm:

$$\text{sum} : \{0, 1\} \times \{0, 1\} \times \{0, 1\} \to \{0, 1, 2, 3\}, \quad \text{sum}(a, b, c) = a + b + c.$$

Algorithm 2.3 *Binary integer addition.*

Input: integers $a \geq b \geq 0$ encoded in binary as $a_n \cdots a_1$ and $b_n \cdots b_1$.
Output: $a + b$ in binary.
Algorithm:

```
c ← 0
for i = 1 to n
    if sum(a_i, b_i, c) equals 1 or 3
        then d_i ← 1
        else d_i ← 0
    if sum(a_i, b_i, c) ≥ 2
        then c ← 1
        else c ← 0
next i
if c = 1
    then output 1 d_n d_{n−1} ··· d_1
    else output d_n d_{n−1} ··· d_1.
```

3) Is the binary addition algorithm more efficient than the unary one?

Exercise 2:

Consider the following primality test algorithm:

Algorithm *Naive Primality Testing.*

Input: an integer $N \geq 2$.
Output: true if N is prime and false otherwise.
Algorithm:
$D \leftarrow 2$
$P \leftarrow$ true
while P is true and $D \leq \sqrt{N}$
    if D divides N exactly
        then $P \leftarrow$ false
        else $D \leftarrow D + 1$
end-while
output P

1) Calculate this complexity.
2) Is it effective? justify by giving an example.

Exercise 3:
1) Give the DTM machine of the “unary addition” algorithm
2) Test for 5+2 in binary
3) Calculate the time (number of steps) of DTM
4) Give the DTM machine of the “binary addition” algorithm
5) Test for 5+2 in binary
6) Calculate the time (number of steps) of DTM
7) What is the most efficient machine?

Exercise 4:

Let the graph CLIQUE be a decision problem.

CLIQUE
Input: a graph G of order n and an integer $2 \leq k \leq n$.
Question: Does G contain a clique of order k?

1) Does CLIQUE ∈ P?

Exercise 5:

1) Prove that 2-SAT ∈ P?

Exercise 6:

1) Consider the multiplication function $multi : \mathbb{Z}^+ \to \mathbb{Z}^+$ where $multi(a,b) = ab$
   Show that *multi* ∈ FP?

2) Consider the divisor function $div : \mathbb{Z}^+ \times \quad \to \mathbb{Z}^+$ where $div(a,b) = \lfloor a/b \rfloor$
   Show that div ∈ FP?

3) Consider the exponentiation function $\exp(a,b,c) : \mathbb{Z}^+ \times \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}_c$ where
   $\exp(a,b,c) = a^b \bmod c$
   Show that exp ∈ FP?

4) 4) Let the function be the greatest common divisor $pgdc : \quad N \times N \to N$ where
   $pgcd(a,b) = \max\{\text{d } \geq 1 \mid \text{d divise a et d divise b}\}$
   Show that *pgcd* ∈ FP?

Exercise 7:
Prove that P ⊆ PSPACE ⊆ EXP?