Chapitre V : Travail, Puissance & énergie
Chapter V : Power, Work and Energy

1. Généralités — General concepts
2. Travail d'une force — Work of a force
3. Puissance d'une force — Power of a force
4. Energie — Energy

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Chapitre V Travail, Puissance & énergie
Chapter V : Power, Work and Energy

- Energy is a fundamental quantity in physics that allows solving certain problems in point mechanics through a scalar equation that could also be solved using the vector form of the fundamental principle of dynamics.
- Energy is the ability to do work. There are various forms of energy in physics, including kinetic energy (energy of motion), potential energy (energy due to position), and many others.
- In summary, work is the energy transfer that occurs when a force acts on an object, energy is the capacity to do work, and power is the rate at which work is done or energy is transferred. These concepts are fundamental in understanding how objects interact in the physical world.

Travail d'une force
Work of a force

- A force that alters the motion of an object that was initially at rest or causes its deformation does work. Therefore, the work of a force expresses the effort required to move an object. It is denoted as 'W' from the English word work.
- The elementary work of the force $\vec{F}$ during the time dt is defined as the dot product of this force with the infinitesimal displacement vector $\overrightarrow{dl}$ also denoted as $\overrightarrow{dOM}$

$$\delta w = \vec{F} \cdot \overrightarrow{dl}$$

δw represents the infinitesimal amount of work done.

We can write:

$$\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{\overrightarrow{dl}}{dt} \qquad \text{donc} \qquad \overrightarrow{dl} = \vec{V}\,dt$$

$$\delta w = \vec{F}\,.\,\overrightarrow{dl} = \vec{F}.\,\vec{V}\,.dt$$

The work done by the force vector $\vec{F}$ along a path AB (or curve C) is equal to the sum of the elementary works. To find the total work over a certain path, you would integrate *dW* over that path. The integration involves summing up all these infinitesimally small contributions along the entire path.

$$W(\vec{F})_{A\_B} = \sum \delta w = \int_A^B \delta w$$

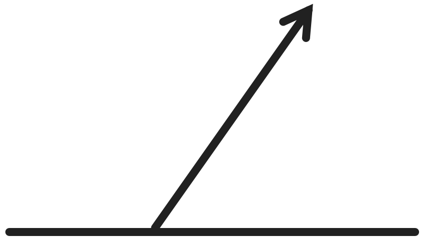
$$W(\vec{F})_{A\_B} = \int_A^B \vec{F}\,.\,\overrightarrow{dl}$$

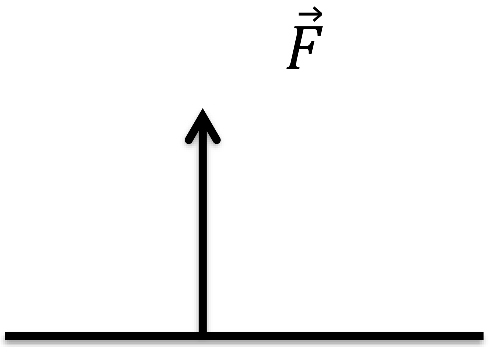
Travail d’une force constante sur un déplacement rectiligne/Work of a constant force on a rectilinear displacement.

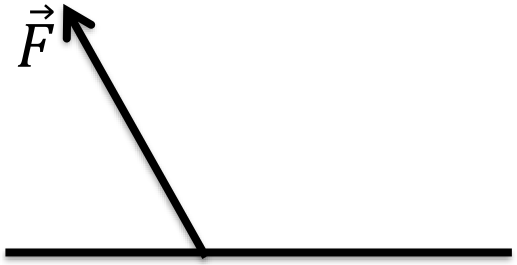
The work of the force on the displacement AB is given by the dot product of this force and the displacement AB . The Unit is : N.m , 1N.m = 1 joule

$$W(\vec{F})_{A\_B} = \int_A^B \vec{F} \,.\, \overrightarrow{dl} = \vec{F} \int_A^B \overrightarrow{dl}$$

$$W(\vec{F})_{A\_B} = \vec{F}.\overrightarrow{AB} = F \,.\, AB \,.\, \cos\alpha$$





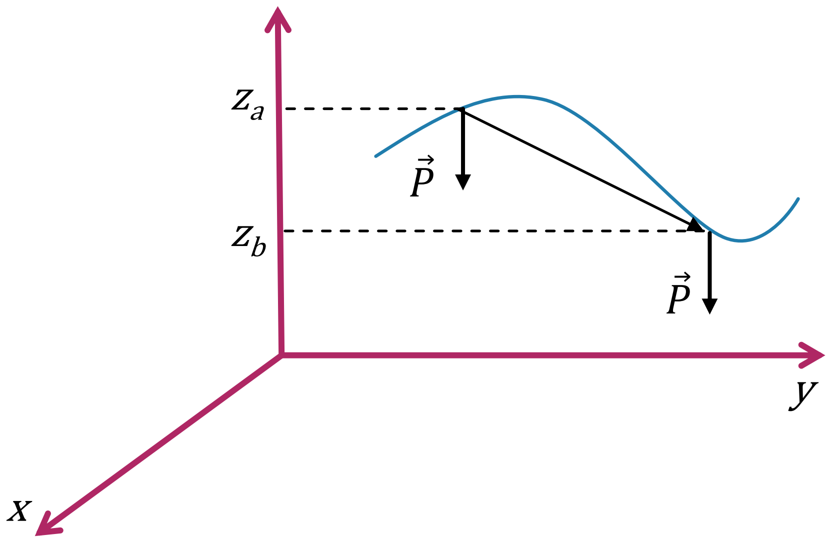


- The work is a driving force (or The work is motive) — The work is zero — The work is resistant
- Le travail est moteur — le travail est nul — le travail est résistant

$W > 0 \quad (\alpha < \frac{\pi}{2})$ — $W = 0 \ (\alpha = \frac{\pi}{2})$ — $W < 0 \ (\alpha > \frac{\pi}{2})$

Travail d'une force constante sur un déplacement quelconque/Work of a constant force on any displacement.

- Let's take, as an example of the work of a constant force, the work done by gravity.
- Consider a material point M with mass m; it is therefore subjected to its weight $\vec{P}$ which is a constant force over time.
- The work done by the weight during a displacement AB is therefore the dot product of the weight $\vec{P}$ = m$\vec{g}$ with the displacement vector $\overrightarrow{AB}$.



$$W(\vec{P})_{A\_B} = \int_A^B \vec{P}\,.\,\overrightarrow{dl} = \vec{P}\int_A^B \overrightarrow{dl} = \vec{P}.\,\overrightarrow{AB} = P.\,AB\,.\cos\alpha$$

$$\vec{P}.\,\overrightarrow{AB} = (\,o\,\vec{\imath} + o\,\vec{\jmath} - mg\,\vec{k}\,)\,.\,(x_b - x_a)\,\vec{\imath} + (y_b - y_a)\,\vec{\jmath} + (z_b - z_a)\,\vec{k}$$

$$\vec{P}.\,\overrightarrow{AB} = -mg\,(z_b - z_a) = -m\,g\,\Delta h$$

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- The difference in altitude between points A and B is given by:
- $\Delta h = z_b - z_a$ With $AB \cdot \cos\alpha = z_a - z_b$
- $\vec{P} \cdot \overrightarrow{AB} = -mg\,(z_b - z_a) = -m\,g\,\Delta h = mg \cdot AB \cdot \cos\alpha$

We observe that the work done by gravity does not depend on the path taken but only on the difference in altitude between the starting point A and the ending point B. This property leads to the classification of weight as a conservative force.

If point M ascends ($\Delta h>0$), the work is negative, and it is termed resistant.

If point M descends ($\Delta h<0$), the work is positive, and it is termed motive.

Travail d'une force variable sur un déplacement quelconque/The work of a variable force on an arbitrary displacement.

Let's take the example of a variable force, the elastic force $\vec{T}$, or the tension in a spring that varies with the extension state x : $\vec{T}$ = - K x $\vec{i}$

where K is the spring constant, and x is the displacement

The work done by the spring tension from position A to position B is given by:

$$W(\vec{T})_{A\_B} = \int_A^B \vec{T}\,.\,\overrightarrow{dl} = \int_A^B \vec{T}\,.\,\overrightarrow{dx} = \int_A^B -Kx.dx = -K\int_{Xa}^{Xb} x.dx$$

$$W(\vec{T})_{A\_B} = \frac{1}{2}Kx_a^2 - \frac{1}{2}Kx_b^2$$

Forces conservatives $\vec{F}_C$
Conservative forces

- Conservative forces are those for which the work done by the force in moving an object from one point to another is independent of the path taken. In other words, the total work done by a conservative force is determined only by the initial and final positions of the object and is not affected by the specific path the object follows.
- A key characteristic of conservative forces is that they are associated with a potential energy function. The gravitational force and the elastic force in a spring are common examples of conservative forces. The work done by these forces can be expressed as a change in potential energy.

- The gravitational force.

- The elastic force.

- Work of a constant force in magnitude and direction.

Forces non conservatives $\vec{F}_{NC}$
Non-conservative forces

- Non-conservative forces are forces for which the work done in moving an object from one point to another depends on the specific path taken. The total work done by a non-conservative force is not solely determined by the initial and final positions of the object but also by the particular route or trajectory the object follows.
- Examples of non-conservative forces include friction, air resistance, and applied forces that are not conservative. These forces dissipate mechanical energy and typically convert it into other forms such as heat or sound.

Puissance d'une force/ The power of a force

The power of a force is the rate at which work is done or energy is transferred by that force. Mathematically, power (P) is defined as the work done (W) per unit of time (t), and it is given by the formula:

$$P_m = \frac{W}{\Delta t}$$

The unit of power is the watt, which corresponds to 1 joule of work done in 1 second.

- Instantaneous power corresponds to the work done by the force during the infinitesimal time interval dt

$$P(t) = \frac{\delta W}{dt} = \frac{\vec{F}\,.\,\vec{dl}}{dt} = \vec{F}.\ \vec{V}$$

- A relationship can be established between the work and the power of a force:

$$W(\vec{F})_{A\_B} = \sum \delta w = \int_A^B \delta w$$

$$W(\vec{F})_{A\_B} = \int_A^B \vec{F}\ .\ \vec{dl}$$

$$W(\vec{F})_{A\_B} = \int_{ta}^{tb} \vec{F}.\ \vec{V}\ .\ dt = \int_{ta}^{tb} P\ dt$$

Energie/Energy

Energy is a fundamental concept in physics that describes the ability of a system to perform work. Energy is a scalar quantity that enables the resolution of numerous problems in dynamics. There are various forms of energy

- **Energie cinétique/ Kinetic Energy :**

This is the energy an object possesses due to its motion, so it depends of the velocity of this object $\vec{V}$:

$$E_c = \frac{1}{2} m V^2$$

The kinetic energy theorem is given by :

$$\frac{1}{2} m V_b^2 - \frac{1}{2} m V_a^2 = E_c(B) - E_c(A) = \Delta E_c = \sum W A\_B\ (\vec{F}_{ext})$$

Le théorème de l’énergie cinétique
The kinetic energy theorem

The change in kinetic energy of a particle, subjected to a set of external forces between positions A and B, is equal to the sum of the works done by all these forces (both conservative and non-conservative).

Demonstration :

- $W(\vec{F})_{A\_B} = \int_A^B \vec{F} \,.\, \overrightarrow{dl}$

- $\sum\ W(\vec{F})_{A\_B} = \sum \int_A^B \vec{F} \,.\, \overrightarrow{dl} = \int_A^B \sum \vec{F}.\overrightarrow{dl}$

- $\sum \vec{F}_{ext} = m\, \vec{a} = m \frac{d\vec{V}}{dt}$

- et $\vec{V} = \frac{d\overrightarrow{OM}}{dt} = \frac{d\vec{l}}{dt}$ donc $\vec{dl} = \vec{V}\, dt$

- $\sum\ W(\vec{F})_{A\_B} = \int_A^B m \frac{d\vec{V}}{dt} \,.\, \overrightarrow{dl} = \int_A^B m \frac{d\vec{l}}{dt} \,.\, \overrightarrow{dV} = \int_A^B \vec{V}.\, d\vec{v}$

- $\sum\ W(\vec{F})_{A\_B} = \int_A^B \vec{V}.\, d\vec{v} = \frac{1}{2} m\, V_b^2 - \frac{1}{2} m\, V_a^2 = E_c(B) - E_c(A) = \Delta E_c$

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Energie potentielle/Potential Energy

- This is the energy an object possesses due to its position .
- The work done by a conservative force does not depend on the path taken but only on the initial (A) and final (B) states. This work can be expressed using a potential energy function, denoted as Ep.
- The potential energy theorem is given by:

$$E_p(B) - E_p(A) = \Delta E_p = -\sum W_{A\_B}(\vec{F}_c)$$

- The change in potential energy between two points A and B is equal to the negative of the work done by conservative forces between these two points.
- **Gravitational Potential Energy** : $E_{pp} = mgh = - W_{A\text{-}B}(\vec{P})$
- **Elastic Potential Energy**: $E_{pe} = \frac{1}{2} K\Delta l = - W_{A\text{-}B}(\vec{T})$

Energie mécanique

- The mechanical energy of a system is equal to the sum of the kinetic and potential energies of that system.

$$E_{mec} = E_c + E_p$$

- The theorem of mechanical energy is given by:

$$\Delta E = E_{mec}(B) - E_{mec}(A) = \sum W_{A\_B}\,(\vec{F}_{NC})$$

- The change in mechanical energy between two points A and B is equal to the sum of the works done by non-conservative forces between these two points.

Demonstration :

The kinetic energy theorem gives : $\Delta E_c = \sum W_{A\_B}\,(\vec{F}_{ext})$

$$\Delta E_c = \sum W_{A\_B}(\vec{F}_C) + \sum W_{A\_B}(\vec{F}_{NC}) = E_c(B) - E_c(A)$$

$$E_p(B) - E_p(A) = \Delta E_p = -\sum W_{A\_B}(\vec{F}_c)$$

$$E_p(A) - E_p(B) = \sum W_{A\_B}(\vec{F}_c)$$

$$\Delta E_c = E_c(B) - E_c(A) = E_p(A) - E_p(B) + \sum W_{A\_B}(\vec{F}_{NC})$$

$$\sum W_{A\_B}(\vec{F}_{NC}) = E_c(B) - E_c(A) + E_p(B) - E_p(A)$$

$$\sum W_{A\_B}(\vec{F}_{NC}) = \Delta E_c + \Delta E_p = \Delta E$$

- If the system is conservative or isolated, then there is conservation of mechanical energy: $\boldsymbol{\Delta E_c + \Delta E_p = \Delta E = 0}$

So we can say that if a system is conservative, meaning it is subjected only to conservative forces or forces that do no work, then the change in mechanical energy of this system is zero.