Batna2 University **Academic Year**

Common Core Science and Technology **2023/2024**

Tutorials(2) P1 Engineer (Kinematics of Material Point)

Exercise 1/

Let C be the trajectory defined by : $\vec{r} = 3\cos 2t\,\vec{\imath} + 3\sin 2t\,\vec{\jmath} + (8t-4)\vec{k}$
Find a unit vector tangent to the curve

Exercise 2/

The motion of a material point M moving in the (xOy) plane is described by the position vector :

$$\overrightarrow{OM} = 3\,\cos 2t\,\vec{\imath} + 3\,\sin 2t\,\vec{\jmath}$$

1- Determine the equation of the trajectory, what is its nature ?
2- Express the velocity vector and its magnitude (modulus), deduce the nature of the motion.
3- Determine the acceleration vector and its magnitude (modulus)
4- Give the value of the angular velocity $\omega(t)$ and the angulare position $\theta(t)$ of the motion for $\theta_0 = 0$
5- Give the position, velocity and acceleration vectors in polar coordinates
6- Demonstrate that the velocity vector is perpendicular to the acceleration vector $\vec{v} \perp \vec{a}$.

Exercise 3/

Consider a mobile M treated as a material point moving in the XOY plane. It is identified by its polar coordinates : $r(t) = t^2 / 4$; $\theta(t) = \frac{\pi}{4}\,t$ (t in s, r in m et θ in rd).
1/ Express the position, velocity and acceleration vectors in polar coordinates.
2/ Calculate the magnitude (modulus) of the velocity vector and acceleration vector at t=6s.
3/ Give the Cartesian coordinates of point M.
4/ Deduce the expression of the velocity vector in Cartesian coordinates.

Exercise 4/

In a Cartesian coordinate system (O, x, y) , equipped with the basis vectors $(\vec{\imath}, \vec{\jmath})$ a moving point M has parametric equations : $X = 2\cos(3t + 2)$ et $Y = 2\sin(3t+2)$

1. Give the equation of the trajectory, what is its nature ?
2. Express the velocity vector $\vec{V}$, and determine its magnitude (modulus).
3. Give the acceleration vector $\vec{a}$, and determine its magnitude (modulus).
4. Give the Polar coordinates of point M.
5. Give the position, velocity and acceleration vectors in polar coordinates.

Exercise 5/

In a Cartesian coordinate system (O, x, y), equipped with the basis vectors $(O, \vec{\imath}, \vec{\jmath})$, a material point M is in motion such that :

$$\overrightarrow{OM} = \cos t\,\vec{\imath} + \sin t\,\vec{\jmath}$$

1. Determine the nature of the trajectory of M.

2. Express the velocity vector $\vec{V}$ in Cartesian coordinates and determine its magnitude (modulus)
3. Deduce the nature of the motion and determine the angular velocity ω.
4. Express the acceleration vector $\vec{a}$ in Cartesian coordinates and determine its magnitude (modulus). What does this acceleration represent in the Frenet frame and why?
5. Determine the angle α between acceleration and velocity.
6. Express the velocity and acceleration vectors in polar coordinates

Exercise 6/

A mobile point M follows a plane trajectory given by the equations in polar coordinates $(O, \vec{e_r}, \vec{e_\theta})$

$$\begin{cases} r(t) = e^t \\ \theta(t) = t \end{cases} \qquad (\text{t in s, r in m et } \theta \text{ in rad})$$

1. Express $(\vec{e_r}, \vec{e_\theta})$ in terms of fixed basis $(\vec{i}, \vec{j})$.
2. Express the position vector $\overrightarrow{OM}$ in polar coordinates.
3. Calculate the velocity vector $\vec{V}$, and determine its magnitude (modulus).
4. Calculate the acceleration vector $\vec{a}$, and determine its magnitude (modulus).
5. Deduce the position vector $\overrightarrow{OM}$ in Cartesian coordinates

Exercise 7/

The motion of a material point M moving in the (xOy) plane initially located at point (0,3) is defined by its velocity as a function of time: $\vec{V} = 2\vec{i} + 2t\vec{j}$

1. Give the magnitude of the velocity.
2. Determine the acceleration vector and its magnitude (modulus).
3. Derermine the position vector.
4. Determine the equation of the trajectory.
5. Give the tangential and normal components of the acceleration vector, and deduce the radius of curvature for t=1s.