University year : 2023-2024
Department :MI
Module : Algebra 1

Tutorial Series(1)

Exercise 1 *Let p, q and r three propositions. Show, using the truth table, the equivalence of the following propositions :*

1. $\overline{p \wedge q} \Leftrightarrow \overline{p} \vee \overline{q}$.
2. $\overline{p \vee q} \Leftrightarrow \overline{p} \wedge \overline{q}$.
3. $p \Rightarrow q \Leftrightarrow \overline{p} \vee q$
4. $[(p \wedge q) \Rightarrow r] \Leftrightarrow [(p \Rightarrow r) \vee (q \Rightarrow r)]$.
5. $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$.
6. $[p \Rightarrow (q \vee r)] \Leftrightarrow [(p \Rightarrow q) \vee (p \Rightarrow r)]$.

Exercise 2 *Let p and q two propositions. Simplify the following expression :*

$$(\overline{p} \wedge q) \vee (\overline{p} \wedge \overline{q}) \vee (p \wedge q).$$

(using the truth table).

Exercise 3 *Let the following propositions :*

a. *248 is a multiple of 6 and 3 divides 48.*

b. *248 is a multiple of 6 or 3 divides 48.*

c. $(\exists x \in \mathbb{R},\ x + 5 = 0)$ *or* $(\exists x \in \mathbb{R},\ x + 9 = 0)$.

d. $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R};\ x + y < 0$.

e. $\exists x \in \mathbb{R},\ \forall y \in \mathbb{R};\ x + y < 0$.

f. $\forall x \in \mathbb{R},\ \forall y \in \mathbb{R};\ x + y < 0$.

g. $\exists x \in \mathbb{R},\ \exists y \in \mathbb{R};\ x + y < 0$.

1. *Noting the order of the quantifiers, indicate which of these propositions are true and which are false (Justify your answer).*
2. *Give the negation of each of them.*

Exercise 4 *Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function. deny the following propositions :*

1. $\forall x \in \mathbb{R},\ f(x) \neq 0$.
2. $\forall M > 0,\ \exists A > 0,\ \forall x \geq A,\ f(x) > M$.

3. $\forall x \in \mathbb{R},\ f(x) > 0 \Longrightarrow x \leq 0$.
4. $\forall \varepsilon > 0,\ \exists \eta > 0,\ \forall (x, y) \in I^2,\ (|x - y| \leq \eta \Rightarrow |f(x) - f(y)| \leq \varepsilon)$.

Exercise 5 *Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function. Express the following propositions using quantifiers :*

1. *f est constant.*
2. *f is not constant.*
3. *f cancels.*
4. *f is increased.*
5. *f is bounded.*
6. *f is even.*
7. *f is odd.*
8. *f is periodic.*
9. *f reaches all values of $\mathbb{N}$.*
10. *f increasing.*
11. *f strictly decreasing.*
12. *f less than g.*
13. *f is not less than g.*

Exercise 6 *The following questions are independent.*

1. *Show by direct reasoning that : $a, b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.*
2. *Show, using case by case reasoning that : $\forall n \in \mathbb{N},\ n(n+1)$is divisible by 2.*
3. *Show, using contrapositive reasoning that : $\forall n \in \mathbb{N}$, n^2 is even $\Rightarrow$ n is even.*
4. *Show by the absurd that : $\forall a, b \geq 0,\ (\frac{a}{1+b} = \frac{b}{1+a}) \Rightarrow (a = b)$.*
5. *Show by counter example that : $\forall x \in \mathbb{R},\ x < 2 \Rightarrow x^2 < 4$.*
6. *Show by recurrence that : $\sum_{k=0}^{n} k^3 = \frac{n^2}{4}(n+1)^2$.*