Academic year : 2023-2024
Department :MI
Module : Algebra1

Tutorial Series(2)

Exercise 1 *Let A, B and C be three parts of a non empty set E.*
Show that :

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Exercise 2 *Let A and B be two parts of a set E.*
Show that :

1. $A \subset B \Leftrightarrow A \cup B = B$.
2. $A \subset B \Leftrightarrow A \cap \overline{B} = \emptyset$.

Exercise 3 *Let A and B be two parts of a non empty set E.*
We call the symmetric difference of A and B denoted by $A \vartriangle B$, the set defined by

$$A \vartriangle B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

where $\overline{A}$ the complement of A in E and $\overline{B}$ the complement of B in E.

1. *Determine the sets $A \vartriangle E$, $A \vartriangle \emptyset$ and $A \vartriangle A$.*
2. *Show that the operator $\vartriangle$ is commutative.*

Exercise 4 *Let $f : E \longrightarrow F$ be an application and let A_1 , A_2 be two subsets of E.*
Prove that :

1. $A_1 \subset A_2 \Rightarrow f(A_1) \subset f(A_2)$.
2. $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$.
   - *Show by a counterexample that :*$f(A_1) \cap f(A_2) \not\subset f(A_1 \cap A_2)$.
3. $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$.

Exercise 5 *Let $f : E \longrightarrow F$ be an application and let B_1 , B_2 be two subsets of F.*
Prove that :

1. $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$.
2. $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$.
3. $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$.

Exercise 6 *Consider three sets E, F and G and two applications* $f : E \longrightarrow F$ *and* $g : F \longrightarrow G$. *Show that :*

1. $g \circ f$ *is injective* $\Longrightarrow f$ *is injective.*
2. $g \circ f$ *is surjective* $\Longrightarrow f$ *is surjective.*
3. $g \circ f$ *is injective and* f *is surjective* $\Longleftrightarrow g$ *is injective.*
4. $g \circ f$ *is injective et* g *is injective* $\Longleftrightarrow f$ *is surjective.*

Exercise 7 *Using the definitions of surjection, injection, and bijection of an application, determine whether the following applications are surjective, injective, or bijective.*

1. $\begin{array}{rcl} f : [-1, 1] & \longrightarrow & \mathbb{R} \\ x & \longmapsto & f(x) = \sqrt{1 - x^2} \end{array}$

2. $\begin{array}{rcl} f : \mathbb{R} & \longrightarrow & [-3, +\infty[\\ x & \longmapsto & f(x) = x^2 + 4x + 1 \end{array}$

3. $\begin{array}{rcl} f : [0, +\infty[& \longrightarrow & [1, +\infty[\\ x & \longmapsto & f(x) = 3x^2 + 4x + 1 \end{array}$