Academic year : 2023-2024
Department :MI
Module : Algebra1

Tutorial Series(3)

Exercise 1 *In the set of integers $\mathbb{Z}$, we define the binary relation $\mathcal{R}$ by :*

$$x\mathcal{R}y \Leftrightarrow \exists k \in \mathbb{Z} : x - y = 3k$$

1. *Show that $\mathcal{R}$ is an equivalence relation .*
2. *Determine equivalence classes of zero, one and two.*
3. *Find the quotient set $\mathbb{Z}/\mathcal{R}$.*

Exercise 2 *Let E be a nonempty set, and let $\mathcal{R}$ be a reflexive relation in E such that :*

$$\forall x, y, z \in E, (x\mathcal{R}y \text{ et } y\mathcal{R}z) \Rightarrow (z\mathcal{R}x)$$

— *Show that $\mathcal{R}$ is an equivalence relation .*

Exercise 3 *On $\mathbb{N}^*$, we define the relation $\mathcal{R}$ as :*

$$a\mathcal{R}b \Leftrightarrow \exists q \in \mathbb{N}^* : b = q.a$$

1. *Demonstrate that $\mathcal{R}$ is an order relation .*
2. *Is the order total ?*

Exercise 4 *Over the power set of E, we define the relation $\mathcal{R}$ as :*

$$A\mathcal{R}B \Leftrightarrow A \subset B$$

1. *Demonstrate that $\mathcal{R}$ is an order relation .*
2. *Is the order total ?*