University of Mostefa Ben Boulaïd - Batna 2
Faculty of Mathematics and computer science
Departement of Mathematics

Tutorial 01

Exercise 1 *: $E = \mathbb{R}^3$ and $\mathbb{K} = \mathbb{R}$.*

1. *Determine the linear forms f, g and h on $\mathbb{R}^3 \longrightarrow \mathbb{R}$ such as :*

   - ▷ $f(1,1,1) = 0$; $f(2,0,1) = 1$; $f(1,2,3) = 4$
   - ▷ $g(-1,0,1) = 1$; $g(2,0,1) = 0$; $g(2,2,3) = 2$
   - ▷ $h(1,3,0) = 0$; $h(2,0,-1) = -1$; $h(1,0,2) = -2$

2. *Provide the basis for the ker of f, g and h.*

Exercise 2 *: $E = \mathbb{R}^4$.*

Let $\mathcal{B} = \{e_1, e_2, e_3, e_4\}$ be the canonical basis of E. Consider application :

$$\begin{array}{rccl} f: & E & \longrightarrow & \mathbb{R} \\ & X & \longmapsto & f(X) = x_1 + x_2 - x_3 - x_4 \end{array}$$

1. *Show that f is a linear form.*
2. *Let F be a subset of E such that : $F = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 - x_3 - x_4 = 0\}$.*
   Justify that F is a hyperplane of E, deduce its dimension, then give it a base, all its equations and all its supplementary in E.
3. *Let $m \in \mathbb{R}$ and $v = (m, m+1, 2m, m-2)$: for which values of m are the subspaces F and $\triangle = vect(v)$ supplementary in E ?*

Exercise 3 *: Let f_1, f_2 be the two elements of $\mathcal{L}(\mathbb{R}^2, \mathbb{R})$ defined by :*

$$f_1(x,y) = x + y \text{ and } f_2(x,y) = x - y.$$

1. *Show that (f_1, f_2) forms a base of $(\mathbb{R}^2)^*$.*
2. *Express the following linear forms g, h and k in the base (f_1, f_2) :*

$$g(x,y) = x, \qquad h(x,y) = 2x - 6y \qquad \text{and} \quad k(x,y) = -y.$$

Exercise 4 *: Let $E = \mathbb{R}^5$ with the canonical basis $\mathcal{B} = \{e_1, e_2, e_3, e_4, e_5\}$.*

We consider the set F of vectors (x, y, z, s, t) of E such as

$$\begin{cases} x + 2y + 3z + 4s + 5t = 0 \\ x + y + z + s + t = 0 \\ 5x + 4y + 3z + 2s + t = 0 \end{cases}$$

1. *Justify that F is a E e.v and $2 \leq dim(F) \leq 4$.*
2. *Does the vectors $u = (6, -9, 1, 1, 1)$ and $v = (2, -1, 1, -2, 1)$ belong to F ?*
3. *Determine a base of F, deduce its dimension.*
4. *Determine a suplementary G of F in E, give a system of equations.*
5. *We put $H = Vect(e_1, e_2)$. Detrmine $F \cap H$.*

Exercise 5 *: Let $E = \mathbb{R}^4$ with the canonical basis $\mathcal{B} = \{e_1, e_2, e_3, e_4\}$*

We set $F = \{(x, y, z, t)E | x + y - z - t = 0\}$.

1. *Justify that F is a hyperplane.*
2. *Deduce its dimension.*
3. *Give all equations of F.*
4. *Give a base of F.*
5. *Give all his supplementary.*

Exercise 6 *: Let $E = \mathbb{R}_2[X]$ with the canonical base $\mathcal{B} = \{1, X, X^2\}$.*

We pose $F = \{P \in E | P(1) + P^{'}(0) = 0\}$.

1. *Justify that F is a hyperplane.*
2. *Deduce its dimension.*
3. *Give all equations of F.*
4. *Give a base of F.*
5. *Give all his supplementary.*