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Tutorial 01

Exercise 1

We consider in $\mathcal{N}$ the following subsets :
$A = \{1; 2; 3; 4; 5; 6; 7\}$, $B = \{1; 3; 5; 7\}$,$C = \{2; 4; 6\}$,$D = \{3; 6\}$.

1. *Determine $B \cap D$, $C \cap D$, Is one of these meetings disjunct ?.*
2. *Determine $B \cup C$, $C \cup D$.*
3. *Determine $C \triangle D$.*
4. *Determine the complementary in A of B, C and D.*

Exercise 2

Let E be a set, A, B, C and D be subsets of E. Show that :

1. $A \cap B = \emptyset \Leftrightarrow A \subset C_E B$.
2. $A \subset B \Leftrightarrow A \cap C_E B = \emptyset$.
3. $A \subset B \Leftrightarrow C_E B \subset C_E A$.
4. $(A \setminus B) \cap (A \cap B) = \emptyset$.
5. $(A \setminus B) \cup (A \cap B) = A$.

Exercise 3

Let $f : E \longrightarrow F$ be an application. Let A_1 and A_2 two parts of E. Show that :

1°) $f(A_1 \cup A_2) = f(A_1) \cup f(A_2)$

2°) $f(A_1 \cap A_2) \subset f(A_1) \cap f(A_2)$

3°) Si $A_1 \subset A_2$ on a $f(A_1) \subset f(A_2)$

4°) $A \subset f^{-1}(f(A))$

Exercise 4

Let $f : E \longrightarrow F$ be an application. Let B_1 and B_2 two parts of F. Show that :

1°) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

2°) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

3°) Si $B_1 \subset B_2$ on a $f^{-1}(B_1) \subset f^{-1}(B_2)$

4°) $f(f^{-1}(B)) \subset B$.

Exercise 5

$I-$ Let f be an application defined by :

$$g : \begin{array}{ccc} \mathbb{Z} & \longrightarrow & \mathbb{Z} \\ n & \longmapsto & g(n) = n + 2. \end{array}$$

Show that g is bijective and give g^{-1}.

$II-$ Let f be an application defined by :

$$f: \quad \begin{array}{ccc} \mathbb{R}-\{1\} & \longrightarrow & \mathbb{R}-\{2\} \\ x & \longmapsto & f(x)=\frac{2x+5}{x-1}. \end{array}$$

1. *Show that f is bijective.*
2. *Let f_1, f_1 and f_3 be the following applications :*

$$f_1: \quad \begin{array}{ccc} \mathbb{R}-\{1\} & \longrightarrow & \mathbb{R}-\{0\} \\ x & \longmapsto & {}_1(x)=x-1. \end{array} \qquad f_2: \quad \begin{array}{ccc} \mathbb{R}-\{0\} & \longrightarrow & \mathbb{R}-\{0\} \\ x & \longmapsto & f_2(x)=\frac{7}{x}. \end{array}$$

$$f_3: \quad \begin{array}{ccc} \mathbb{R}-\{0\} & \longrightarrow & \mathbb{R}-\{2\} \\ x & \longmapsto & f_3(x)=x+2. \end{array}$$

   (a) *Show that f_1, f_2 and f_3 are bijective.*
   (b) *Show that : $f = f_3 \circ f_2 \circ f_1$.*
3. *Determine the reciprocal application f^{-1} with the same way as f.*

Exercise 6

Let $\mathcal{S}$ be the binary relationship defined on $\mathbb{R}$ by :

$$\forall x, y \in \mathbb{R}, \quad x\mathcal{S}y \Leftrightarrow (|x| \geq |y| \textbf{ and } xy \geq 0)$$

1. *Show that $\mathcal{S}$ is a relation of order.*
2. *Is the order total ?*

Exercise 7

Let $E = \mathbb{N} \times \mathbb{N}$, $\mathcal{R}$ is defined in E as :

$$(x,y)\mathcal{R}(x^{'},y^{'}) \Leftrightarrow x + y^{'} = y + x^{'}$$

Show that $\mathcal{R}$ is an equivalence relation on E.

Exercise 8

1. *Let $\mathcal{R}$ be a binary relation on $\mathbb{R}^2$ defined by :*

$$(x,y)\mathcal{R}(a,b) \Leftrightarrow |x-a| \leq b-y$$

   (a) *Show that it is a relation of order ; is it total ?*
   (b) *Represent the majors of $(1;1)$.*
2. *Let $\mathcal{T}$ be a binary relation on $\mathbb{R}^2$ defined by :*

$$(x,y)\mathcal{T}(a,b) \Leftrightarrow \Big((x+y<a+b) \text{ or } (x+y=a+b \text{ and } x \leq a)\Big)$$

   (a) *verify that it is a relation of order ; is it total ?*
   (b) *Represent the majors of $(1;1)$.*