

Course : Algebra 3
Chapter 3 : Endomorphisms

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Tutorial series 3

Exercise 0.1 Let D be similar to A where A and D are square matrices. Show that

1. $\det D = \det A$.
2. In the case where $P^{-1}X$ represents an eigenvector of D , X represents an eigenvector of A .

Exercise 0.2 Let A and D be similar such that A and D are square matrices. Prove that

$$\det(D - \lambda I) = \det(A - \lambda I).$$

Exercise 0.3 Let A be a square matrix. Prove that

$$\det(A - \lambda I) = \det(A^T - \lambda I).$$

Exercise 0.4 Let A be a matrix

$$A = \begin{pmatrix} 4 & 1 \\ 9 & 4 \end{pmatrix}.$$

1. Find the characteristic polynomial of A .
2. Determine the eigenvalues and eigenvectors of A .
3. Is the matrix A diagonalizable.
4. In the case where A is diagonalizable, diagonalize it.

Exercise 0.5 Let A be a matrix where

$$A = \begin{pmatrix} 1 & 2 & -3 \\ 1 & 1 & 2 \\ 1 & 0 & 3 \end{pmatrix}.$$

Is the matrix A diagonalizable.

Exercise 0.6 Let A be a square matrix defined by

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 0 & 3 & 4 \\ -9 & 4 & -3 \end{pmatrix}.$$

Diagonalize A .

Exercise 0.7 Let $X' = AX$ be a system of differential equations where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 4 & 2 \\ 26 & 24 & 6 \end{pmatrix}.$$

Solve the given system.