

Exercise 1

Let X and Y be discrete random variables, with joint probability function given by

$$P_{XY} = \begin{cases} 1/2 & x = 3, y = 5 \\ 1/6 & x = 3, y = 9 \\ 1/6 & x = 6, y = 5 \\ 1/6 & x = 6, y = 9 \\ 0 & \text{otherwise} \end{cases}$$

1. Compute $E(X)$, $E(Y)$ and $E(XY)$.
2. What do you conclude ?
3. Prove that if X and Y are independent then $E(XY) = E(X)E(Y)$.

Exercise 2

Compute C and $E(X)$ when the density function of X is given by each of the following.

(a)

$$f_X(x) = \begin{cases} C(x+1) & 6 \leq x \leq 8 \\ 0 & \text{Otherwise} \end{cases}$$

(b)

$$f_X(x) = \begin{cases} Cx^4 & -5 \leq x \leq -2 \\ 0 & \text{Otherwise} \end{cases}$$

(c)

$$f_X(x) = \begin{cases} \frac{1}{x^2} & x \leq 1 \\ \frac{C}{x^2} & x > 1 \end{cases}$$

Exercise 3

Prove the monotonicity and the linearity of the expectation of a discrete random variable

Exercise 4

(a). Compute the expectation of the following distributions:

1. Geometric distribution
2. Poisson distribution
3. Standard normal distribution $N(0, 1)$

(b) Using the properties of the expected value, conclude the expectation of the normal distribution $N(\mu, \sigma^2)$

Exercise 5

Suppose the joint probability function of X and Y is given by

$$P(X = x, Y = y) = \begin{cases} 1/7 & x = 5, y = 0 \\ 1/7 & x = 5, y = 3 \\ 1/7 & x = 5, y = 4 \\ 3/7 & x = 8, y = 0 \\ 1/7 & x = 8, y = 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Compute $E(X)$ and $E(Y)$.
- (b) Compute $Var(X)$ and $Var(Y)$.
- (c) Compute $Cov(X, Y)$.
- (d) Compute $\rho(X, Y)$.

Exercise 6

Let X and Y have joint density

$$f(x, y) = \begin{cases} 2x^3y + 3y^3 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $\rho(X, Y)$.

Exercise 7

Suppose you roll two fair six sided dice. Let X be the number showing on the first die, and let Z be the sum of the two numbers showing.

- (a) Compute $E(X)$.
- (b) Compute $E(Z|X = 1)$.
- (c) Compute $E(Z|X = 6)$.
- (d) Compute $E(X|Z = 2)$.
- (e) Compute $E(X|Z = 4)$.
- (f) Compute $E(X|Z = 6)$.
- (g) Compute $E(X|Z = 7)$.

Exercise 8

Let X and Y have joint density

$$f(x, y) = \begin{cases} 8x^3y & 0 \leq x, y \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

- (a) Compute $E(X|Y)$.
- (b) Compute $E(Y|X)$.
- (c) what do you conclude?