Coordinate systems

EXERCISE 1: We give the time equations of a point M in the form:

- $\begin{cases} x = 2t+3 \\ y = 4t+2 \end{cases}$ - $\begin{cases} x = t+1 \\ y = t^2+2t \end{cases}$ - $\begin{cases} x = 2\cos(t)+2 \\ y = 2\sin(t)-1 \end{cases}$

- Determine for the three cases the equation of the trajectory described by the point M.
- Deduce for each case the components of the velocity and acceleration of the point M.

EXERCISE 2 : We consider a material point M moving in a frame of reference $\Re\,(O,xyz)$ provided with the Cartesian base $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$. The coordinates of point M in the frame $\Re$ are given by:

x = t+1 , y = t^2+1 et z = 0

1. Write the expression for the position vector $\overrightarrow{OM}$
2. Give the equation of the trajectory of M in $\Re$. Deduce its nature.
3. Give the expression of the velocity and acceleration of point M.

EXERCISE 3 : A material point M describes the plane curve whose equation in the polar basis $(\vec{u}_\rho, \vec{u}_\theta)$ is $\rho = b\,(1+\cos\theta)$ with b a given constant.

We consider that the angle θ varies over time according to the time law $\theta = \omega t$ with ω a constant.

1. Give the expression for the Cartesian coordinates of the mobile.
2. Give in Cartesian coordinates the expression for the velocity of M.
3. Give the expression for the velocity and acceleration in polar coordinates.

EXERCISE 4 : A material point M identified by its Cartesian coordinates (x, y, z), has a movement of time equations:

$$x = R\sin\omega t \quad , \quad y = R\,(1-\cos\omega t) \quad , \quad z = kt$$

With R, ω and k positive constants

1. Determine the time equations ($\rho(t)$, $\theta(t)$, $z(t)$) in cylindrical coordinates.
2. Express the velocity and acceleration in the Cartesian and cylindrical base.

We recall: $1-\cos(\omega t) = 2\sin^2\left(\frac{\omega t}{2}\right)$ et $\sin(\omega t) = 2\sin\left(\frac{\omega t}{2}\right)\cos\left(\frac{\omega t}{2}\right)$

EXERCISE 5: Knowing that for w and R two positive constants, the speed of a particle M is given by:

$$V = 4\,w\,R\cos wt\ \vec{u}_x + 4\,w\,R\sin wt\ \vec{u}_y + 3\,w\,R\ \vec{u}_z$$

And that at the initial instant t =0, x (0) =0, y (0) = 0 and z (0) =0

1. Give the expression for the position vector $\overrightarrow{OM}$.
2. Give the expression for the acceleration vector as well as its two normal and tangential components.
3. What is the radius of curvature of the trajectory at any instant t?