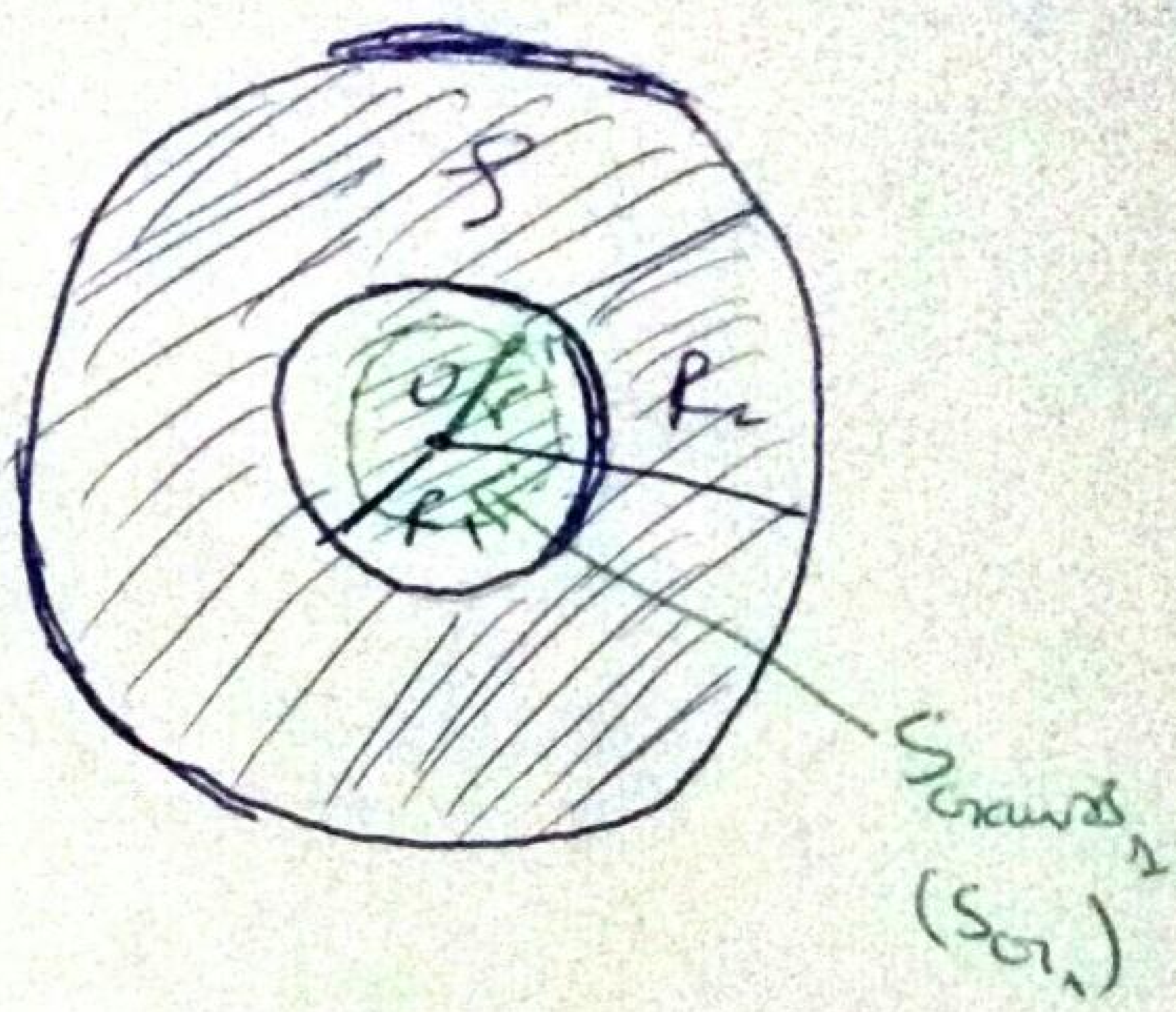
$\vec{E}_M(r) = ?$ en utilisant le théorème de Gauss



$$\Phi = \oiint \vec{E}.\vec{ds} = \oiint E.ds \cos\alpha$$

$\vec{E}$ et $\vec{ds}$ sont colinéaire, alors $\cos\alpha = 1$

$$\oiint \vec{E}.\vec{ds} = \oiint E.ds = \frac{Q_{int}}{\varepsilon_0}$$

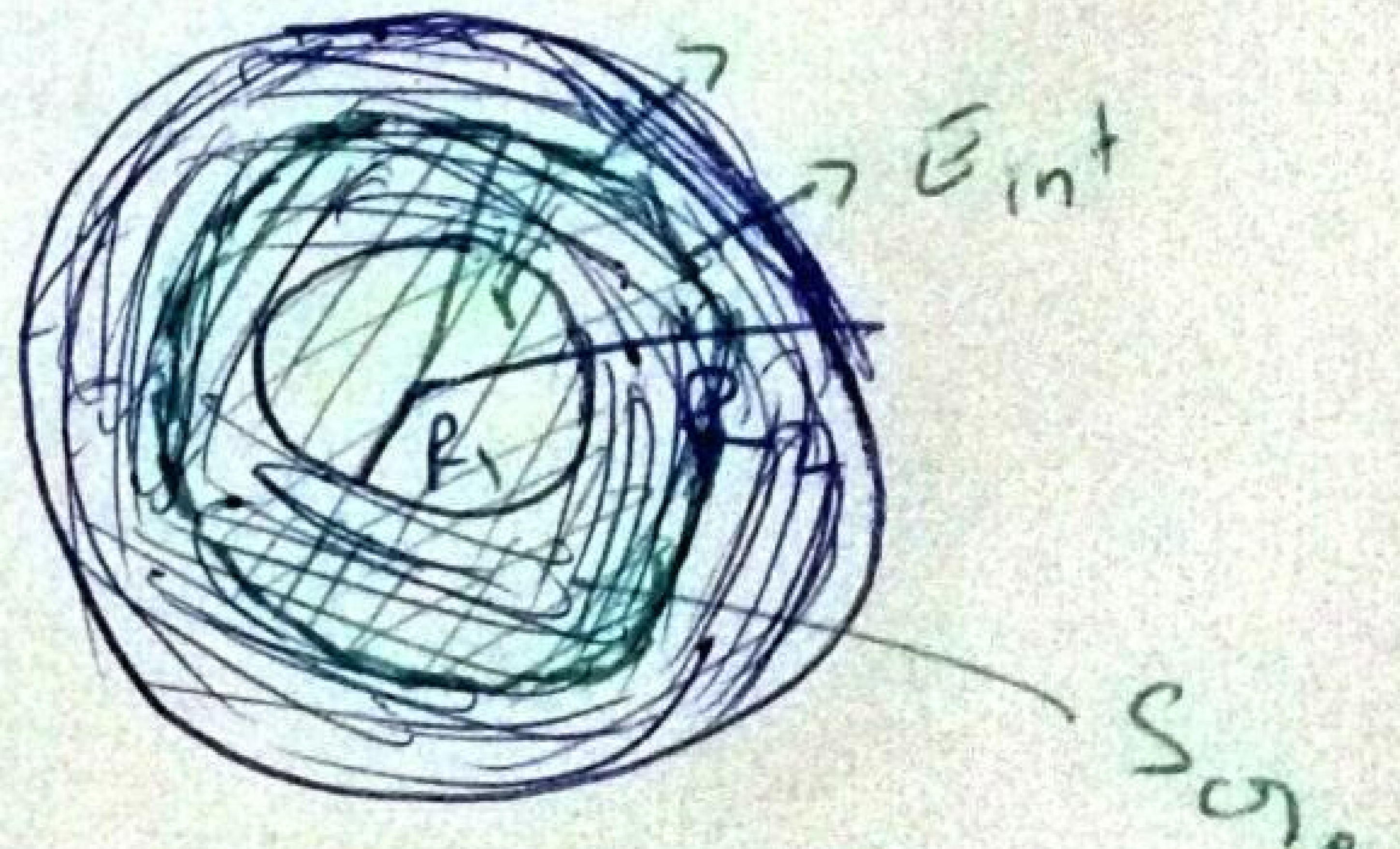
1) 1ier cas $r < R_1$

$Q_{int} = 0$ (y'a pas de charge électrique)

$$\Rightarrow \oiint E_{int}.ds = E_{int} \oiint ds = 0 \Rightarrow E_{int} = 0$$

2) 2ième cas $R_1 < r < R_2$

dans ce cas la charge électrique dans la surface de gauss est entre ... la sphère de rayon R_1 et la sphère de Gauss S_{G_2}



$$\Rightarrow Q_{int} = \rho \iiint V = \rho V = \rho \Big(\underbrace{\tfrac{4}{3}\pi (r^3)}_{\text{Volume de la sphère de Gauss}} - \underbrace{\tfrac{4}{3}\pi R_1^3}_{\text{Volume de la sphère de rayon } R_1} \Big)$$

$$\boxed{Q_{int} = \rho \frac{4}{3}\pi (r^3 - R_1^3)}$$

alors $\oiint \vec{E}_{int} \cdot \vec{ds} = \oiint E_{int}\, ds_{\sigma_2} = E_{int} S_{\sigma_2} = \frac{Q_{int}}{\varepsilon_0}$

$S_{\sigma_2} = 4\pi r^2$

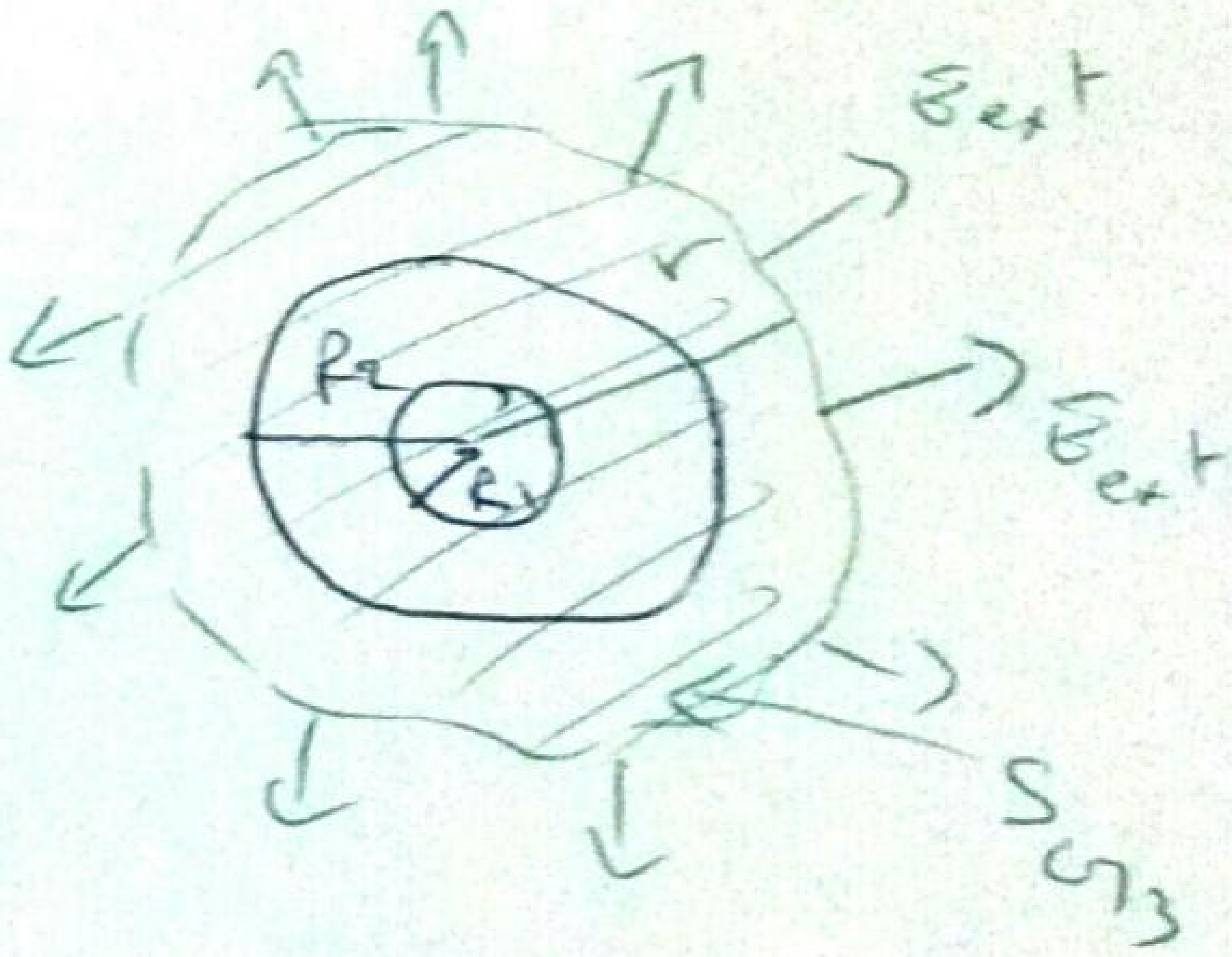
$E_{int} \cancel{4}\pi r^2 = \frac{\rho}{\varepsilon_0} \frac{\cancel{4}}{3} \pi (r^3 - R_1^3)$

$\Rightarrow \boxed{E_{int} = \frac{\rho}{3\varepsilon_0} \frac{(r^3 - R_1^3)}{r^2}}$ et $\boxed{\vec{E}_{int} = \frac{\rho}{3\varepsilon_0} \frac{(r^3 - R_1^3)}{r^2} \vec{e_r}}$

3ième cas $\quad r > R_2$

$\oiint E \cdot ds_{\sigma_3} = E_{ext} S_{\sigma_3} = \frac{Q_{int}}{\varepsilon_0}$

$E_{ext} S_{\sigma_3} = E_{ext} 4\pi r^2$



$Q_{int} = \rho V = \rho \left(\frac{4}{3}\pi R_2 - \frac{4}{3}\pi R_1^3\right)$

↑ la charge électrique dans la surface de Gauss est entre la sphère de rayon R_1 et celle de rayon R_2

$Q_{int} = \rho \frac{4}{3} \pi (R_2^3 - R_1^3)$

$\Rightarrow E_{ext} \cancel{4\pi} r^2 = \frac{\rho}{\varepsilon_0} \frac{\cancel{4\pi}}{3} (R_2^3 - R_1^3)$

$\Rightarrow E_{ext} = \frac{\rho}{3\varepsilon_0} \frac{R_2^3 - R_1^3}{r^2}$

$$\boxed{\vec{E}_{ext} = \frac{\rho}{3\varepsilon_0} \frac{R_2^3 - R_1^3}{r^2} \vec{e_r}}$$

EXO 2

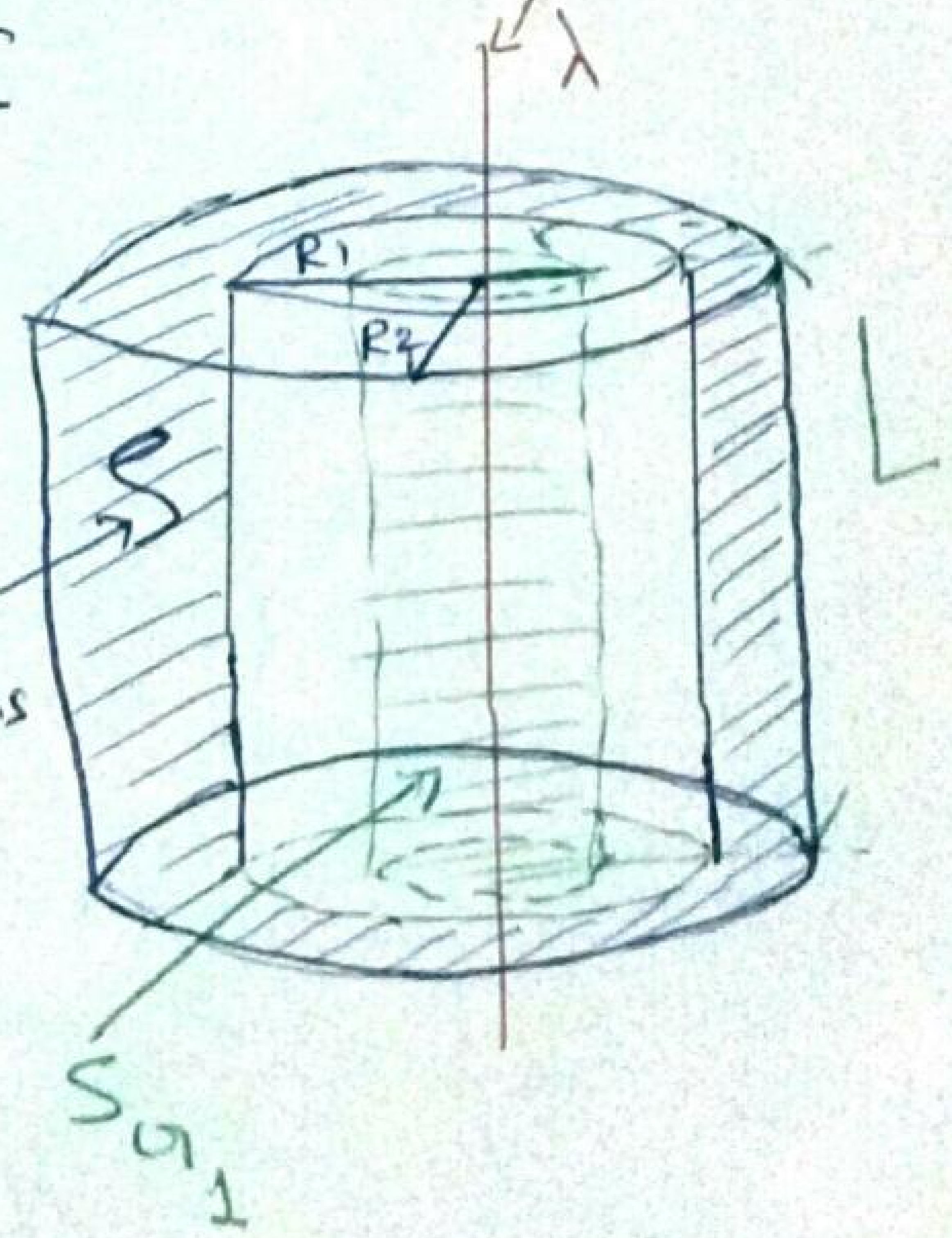
Trouvons l'expression du champ électrostatique $\vec{E}_M(r)$ en tout point dans l'espace

Rq: en tout point dans l'espace
C.a.d on choit la surface de Gauss de rayon r à l'intérieur du cylindre de rayon R_1 puis entre le cylindre de rayon R_1 et R_2 ensuite à l'extérieur du cylindre de rayon R_2

1ier cas $r < R_1$

$$\phi = \oiint \vec{E}_{int} \, \vec{dS}_{\sigma} = E_{int} \, S_{\sigma_1} = \frac{Q_{int}}{\varepsilon_0}$$

$S_{\sigma_1} = 2\pi r L$



Q_{int} = la charge intérieur de la surface latérale de Gauss, dans ce cas, elle est uniquement sur le fil conducteur

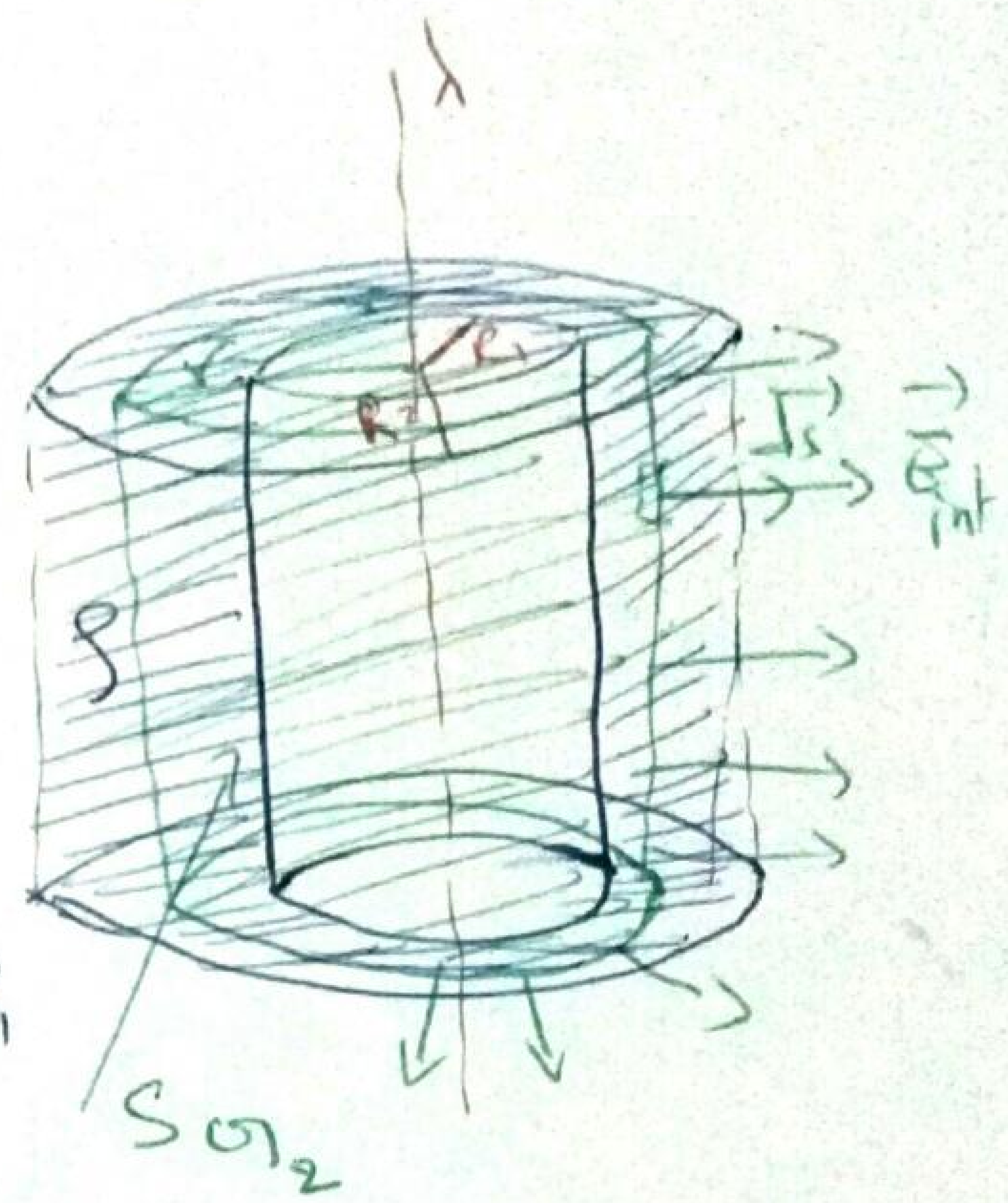
$$Q_{int} = \lambda \int_0^L dl = \lambda L$$

$$\Rightarrow E_{int} \, 2\pi r \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0} \Rightarrow \boxed{E_{int} = \frac{\lambda}{2\pi r \varepsilon_0}}$$

Rq: S_σ = est la surface de Gauss, la surface latérale du Cylindre de rayon r et de hauteur L $S_\sigma = 2\pi r L$

$2^{ième}$ cas : $R_1 < r < R_2$

$$E_{int} S_{\sigma_2} = \frac{Q_{int}}{\varepsilon_0}$$



$$S_{\sigma_2} = 2\pi r L$$

Q_{int} = charge du fil + charge entre le cylindre de rayon r et de celui de rayon R_1

$\Rightarrow$

$$Q_{int} = \lambda \int_0^L dl + \rho \iiint dV$$

$$= \lambda L + \rho V = \lambda L + \rho(\pi r^2 L - \pi R_1^2 L)$$

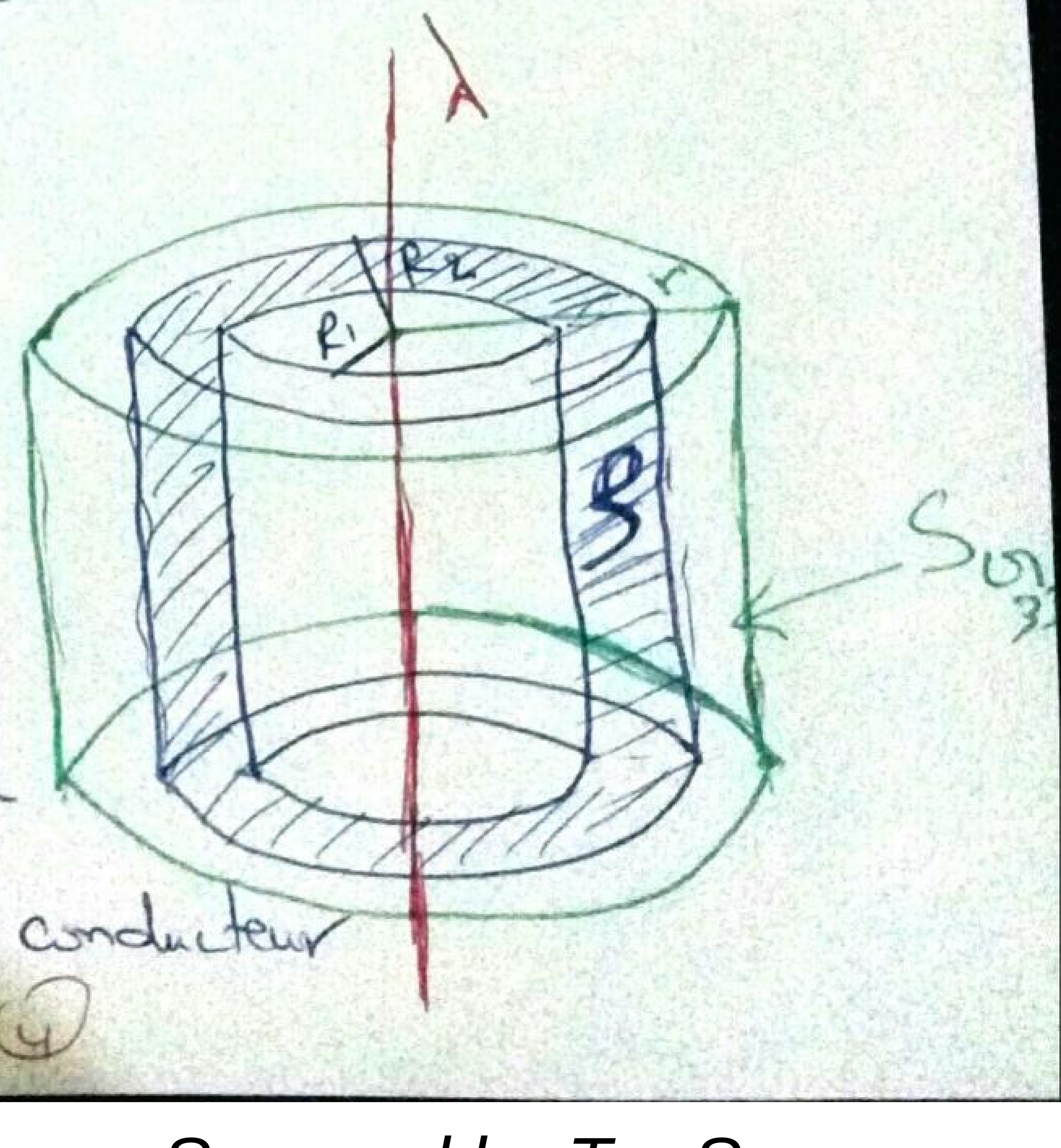
$$Q_{int} = \lambda L + \rho \pi L (r^2 - R_1^2)$$

alors $E_{int}\, 2\pi r \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0} + \frac{\rho \pi \cancel{L}(r^2 - R_1^2)}{\varepsilon_0}$

$$E_{int} = \frac{\lambda}{\varepsilon_0 2\pi r} + \frac{\rho}{2\varepsilon_0 r}(r^2 - R_1^2)$$

$3^{ième}$ cas : $r > R_2$

$$E_{ext} S_{\sigma_3} = \frac{Q_{int}}{\varepsilon_0}$$



$$S_{\sigma_3} = 2\pi r L$$

Q_{int} : la charge à l'intérieur de la surface de Gauss est entre le cylindre de rayon R_1 et celui de rayon R_2 + charge du fil conducteur

$$\Rightarrow Q_{int} = \lambda L + \rho V = \lambda L + \rho(\pi R_2^2 L - \pi R_1^2 L)$$
$$= \lambda L + \rho \pi L (R_2^2 - R_1^2)$$

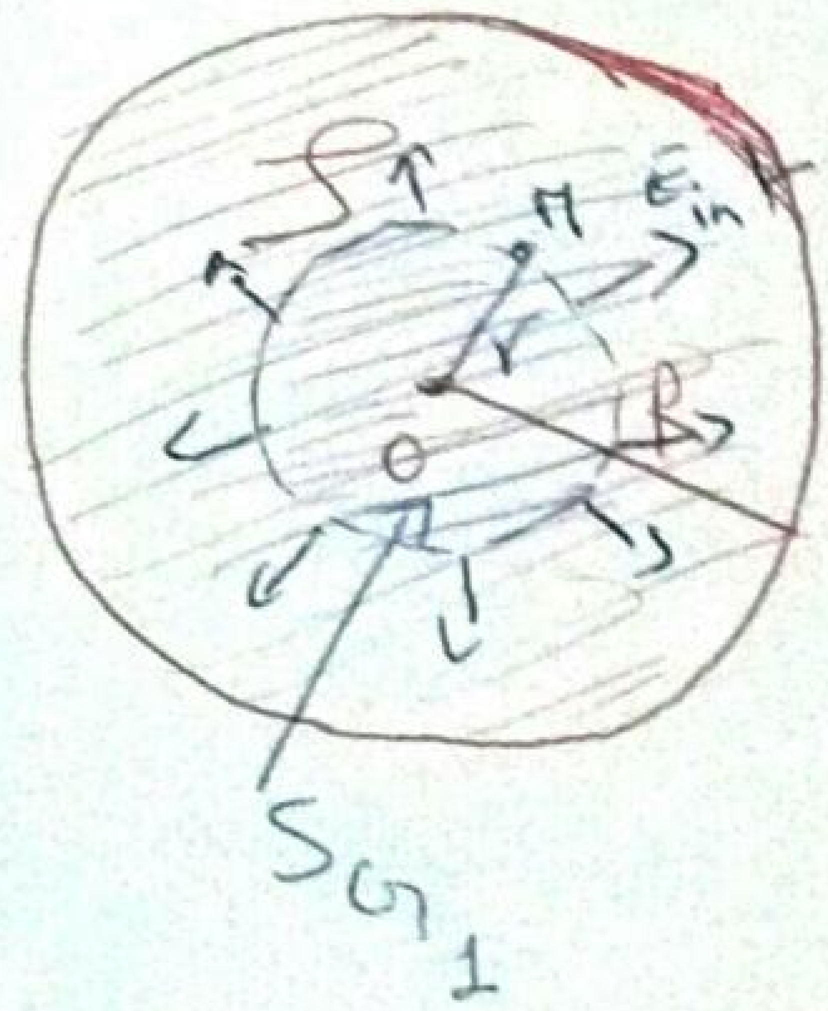
$$\Rightarrow E_{ext} S_{G_3} = E_{ext}\, 2\pi r L = \frac{Q_{int}}{\varepsilon_0} = \frac{\lambda L + \rho\pi L(R_2^2 - R_1^2)}{\varepsilon_0}$$

$$\Rightarrow \boxed{E_{ext} = \frac{\lambda + \rho\pi(R_2^2 - R_1^2)}{2\varepsilon_0 r}}$$

Exo 4

1) Détermination de $\vec{E}_M(r)$

1^{er} cas : $r < R$ (M à l'intérieur de la sphère)



$$\phi = \oiint E_{int}\, dS_{G_1} = \frac{Q_{int}}{\varepsilon_0}$$

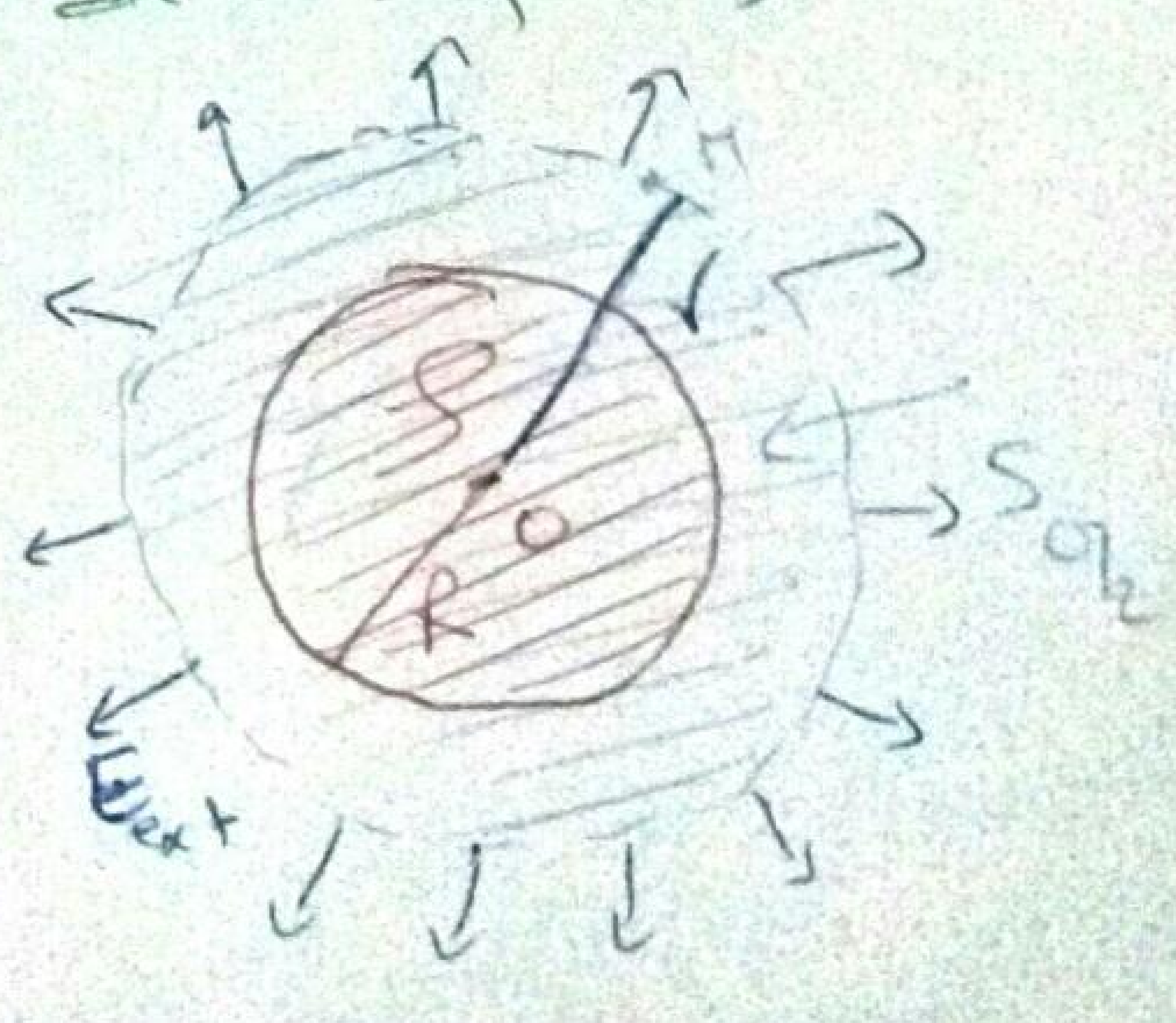
$$E_{int} S_{G_1} = E_{int}\, 4\pi r^2$$

$$Q_{int} = \rho V = \rho \frac{4}{3}\pi r^3$$

$$\Rightarrow E_{int} S_{G_1} = E_{int}\, \cancel{4\pi} \cancel{r^2} = \frac{\rho}{\varepsilon_0}\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}}$$

alors $\boxed{E_{int} = \frac{\rho}{3\varepsilon_0} r}$ et $\boxed{\vec{E}_{int} = \frac{\rho}{3\varepsilon_0} r\, \vec{e}_r}$

$2^{ième}$ cas : $r > R$ (M à l'extérieur de la sphère)



$$E_{ext} S_{G_2} = \frac{Q_{int}}{\varepsilon_0} = E_{ext}\, 4\pi r^2$$

$$Q_{int} = \rho V = \rho \frac{4}{3}\pi R^3$$

$$\Rightarrow E_{ext}\, \cancel{4\pi} r^2 = \frac{\rho}{\varepsilon_0}\frac{\cancel{4}}{3}\cancel{\pi} R^3$$

alors $\boxed{E_{ext} = \frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2}}$

$$\boxed{\vec{E}_{ext} = \frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2} \vec{e_r}}$$

2) Déduction de $V(r)$

$\underline{1^{er} \text{ cas}}$: $r < R$

$\vec{E} = -\overrightarrow{grad}\, V = -\frac{dV}{dl} \Rightarrow dV = -\vec{E} \cdot \vec{dl}$

$\Rightarrow V = -\int \vec{E} \cdot \vec{dl} = -\int E \cdot dr = -\int E_{int}\, dr = \int \frac{-\rho}{3\varepsilon_0} r\, dr$

$\Rightarrow V_{int} = -\frac{\rho}{3\varepsilon_0} \int r\, dr = \frac{-\rho}{3\varepsilon_0} \frac{1}{2} r^2 + C_1$

$\underline{2^{ième} \text{ cas}}$: $r > R$

$V_{ext} = -\int E_{ext}\, dr = -\int \frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2} dr = -\frac{\rho R^3}{3\varepsilon_0} \int r^{-2} dr$

avec $\int r^{-2} dr = \int u^n du = \frac{u^{n+1}}{n+1}$

$V(r)_{ext} = -\frac{\rho}{3\varepsilon_0} R^3 \frac{r^{-1}}{-1} + C_2 = +\frac{\rho}{3\varepsilon_0} R^3 \frac{1}{r} + C_2$

pour $r \to \infty$, $V_{ext\,(r \to \infty)} = 0 \Rightarrow C_2 = 0$ $\Rightarrow V(r) = \frac{\rho}{3\varepsilon_0} R^3 \frac{1}{r}$

pour $r = R$, $V_{ext}(r=R) = \frac{\rho}{6\varepsilon_0} R$

Pour trouver C_1 on a $V_{int}(r=R) = -\frac{\rho}{3\varepsilon_0} \frac{1}{2} R^2 + C_1 = \frac{\rho}{3\varepsilon_0} R^2 = V_{ext}(r=R)$

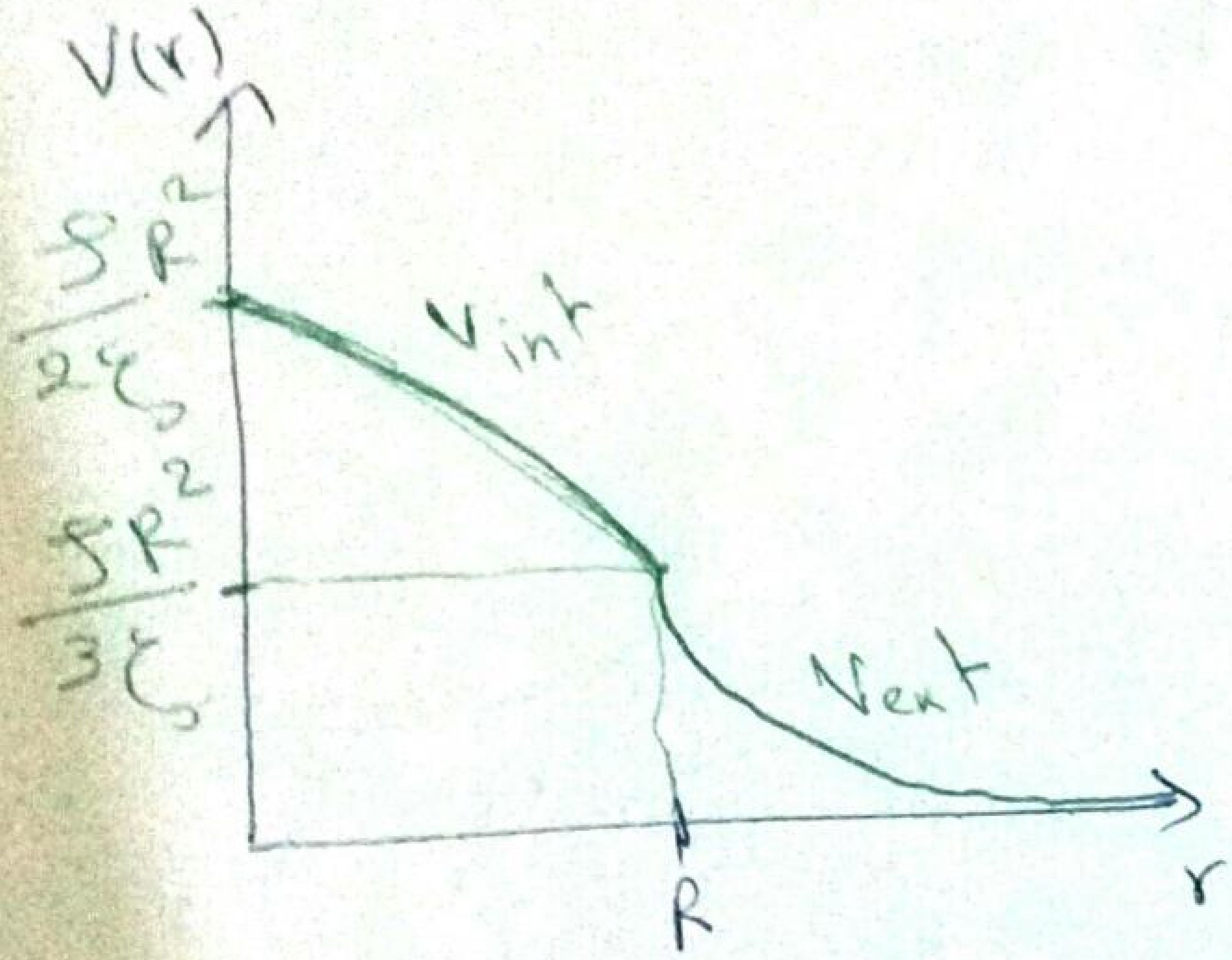
$\Rightarrow C_1 = \frac{\rho}{6\varepsilon_0} R^2 + \frac{\rho}{3\varepsilon_0} R^2 = \frac{3\rho}{6\varepsilon_0} (R^2)$; $C_1 = \frac{\rho}{2\varepsilon_0} R^2$

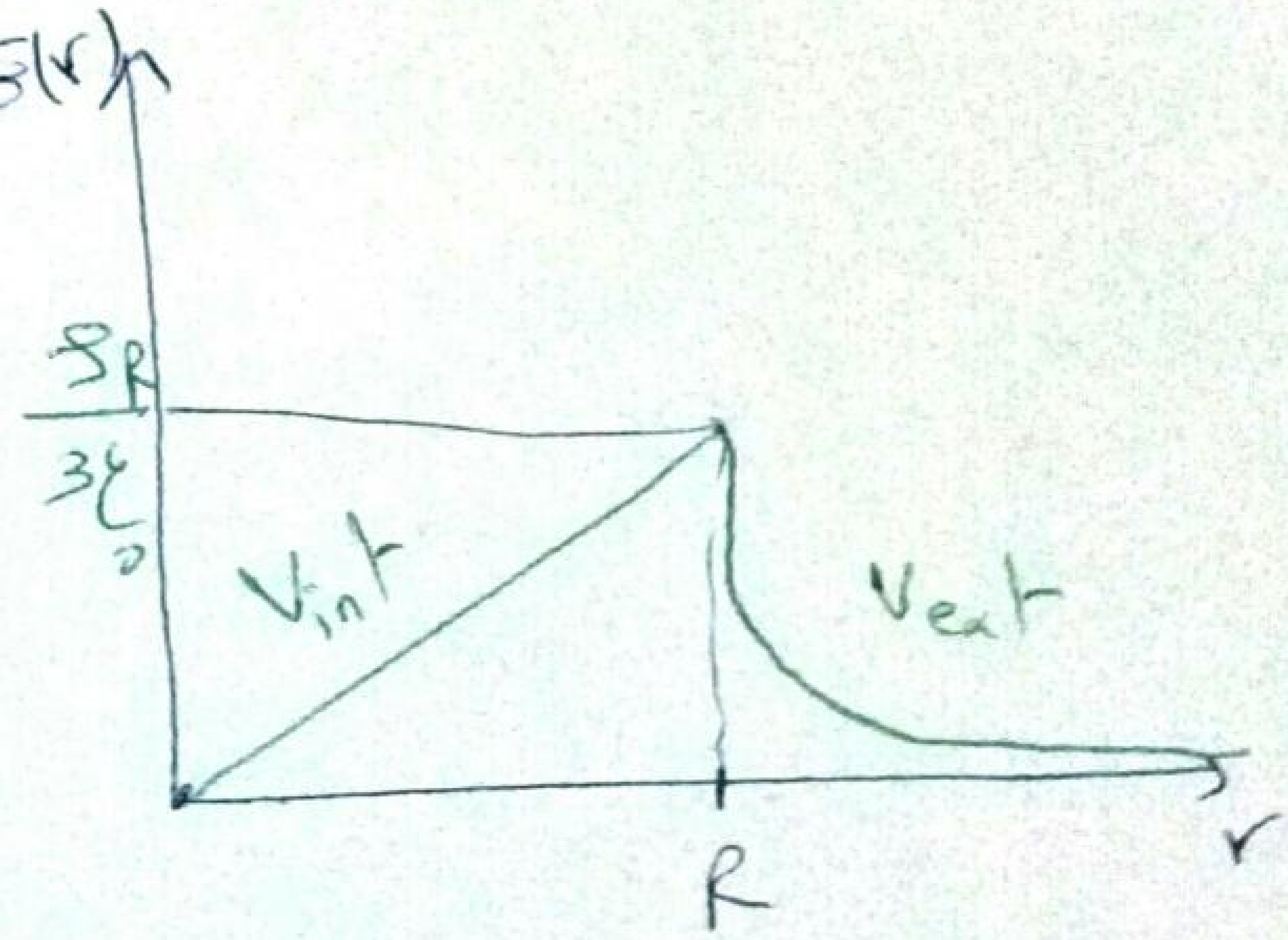
Suite d'Exo 4

3) Traçons $V(r)$ et $E(r)$

Pour $r<R$: $E_{int}(r) = \frac{\rho}{3\varepsilon_0} r$; $V_{int}(r) = -\frac{\rho}{6\varepsilon_0} r^2 + \frac{\rho R^2}{2\varepsilon_0}$

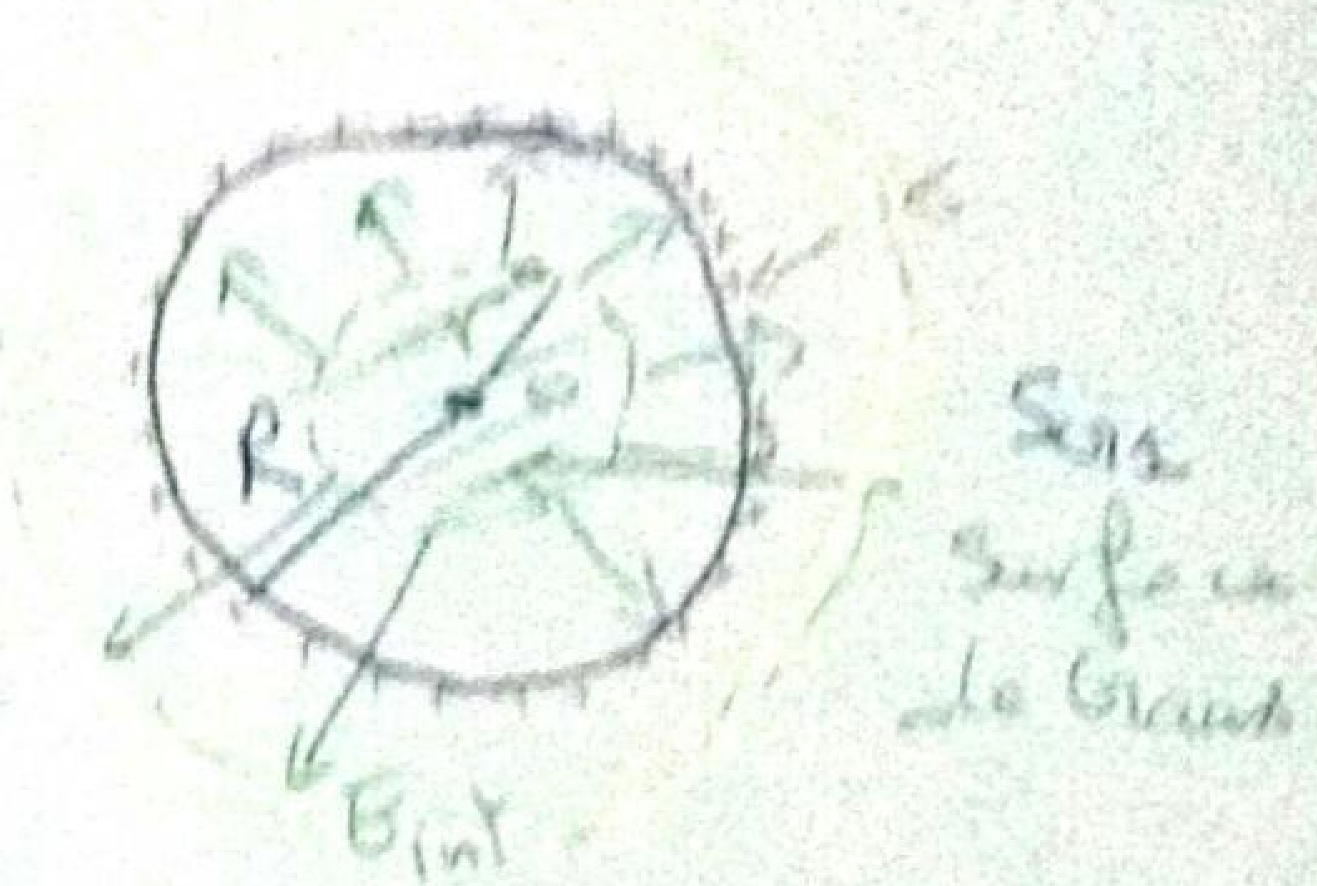
pour $r>R$: $E_{ext} = \frac{\rho}{3\varepsilon_0}\frac{R^3}{r^2}$; $V_{ext} = \frac{\rho}{3\varepsilon_0} R^3 \frac{1}{r}$





Exo 3

1- détermination de $\vec{E}_n(r)$ en utilisant du théorème de Gauss



$$\Phi = \oiint E \, ds_n = \frac{Q_{int}}{\varepsilon_0}$$

• 1ier cas : $r < R$

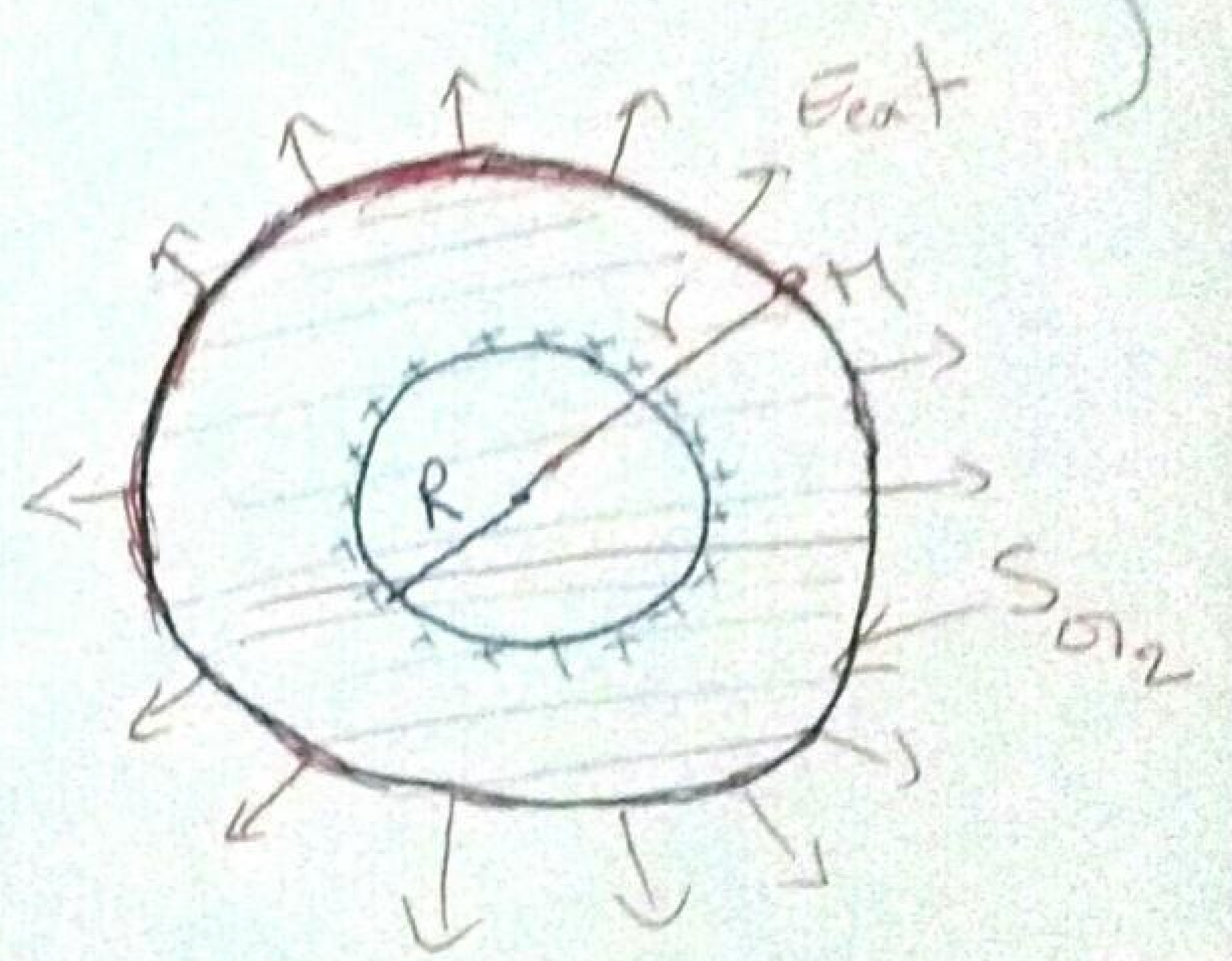
$$\oiint E_{int} \, ds_n = E_{int} \, S_{\sigma_1} = E_{int} \, 4\pi r^2$$

$$Q_{int} = 0 \quad \Rightarrow \quad E_{int} = 0$$

• 2ième cas $r > R$

$$\oiint E_{ext} \, ds = \frac{Q_{int}}{\varepsilon_0}$$

$$\oiint E_{ext} \, ds = E_{ext} \, S_{\sigma_2} = E_{ext} \, 4\pi r^2$$



$$Q_{int} = \sigma \oiint ds = \sigma S = \sigma \, 4\pi R^2$$

$$\Rightarrow E_{ext} \, 4\pi r^2 = \frac{\sigma \, 4\pi R^2}{\varepsilon_0}$$

$$\Rightarrow E_{ext} = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2} \quad \text{et} \quad \vec{E}_{ext} = \frac{\sigma}{\varepsilon_0} \frac{R^2}{r^2} \vec{e_r}$$

2- Déduction de $V(r)$; $\vec{E} = -\,\text{grad}\,V = -\frac{dV}{d\vec{l}}$

• 1ier cas $r < R$

$$\vec{E}_{int} = -\frac{dV_{int}}{d\vec{l}} \Rightarrow dV_{int} = -\vec{E}_{int} \, d\vec{l} \Rightarrow V_{int} = \int \vec{E}_{int} \cdot d\vec{l} = \int E_{int} \, dr$$

$$V_{int} = \int 0 \, dr = cte$$

$$\boxed{V_{int} = C_1 = cte} \quad \text{pour } r < R$$

2ième cas $r > R$

$$V_{ext} = -\int E_{ext}\, dr = -\int \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}\, dr = \frac{-\sigma R^2}{\varepsilon_0}\int r^{-2}\, dr$$

$$= -\frac{\sigma R^2}{\varepsilon_0}\frac{r^{-1}}{(-1)} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r}$$

$$\Rightarrow \boxed{V_{ext} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r} + C_2}$$

détermination de C_1 et C_2

pour $r \longrightarrow \infty$ on a $V_{ext} = 0 \Rightarrow C_2 = 0$

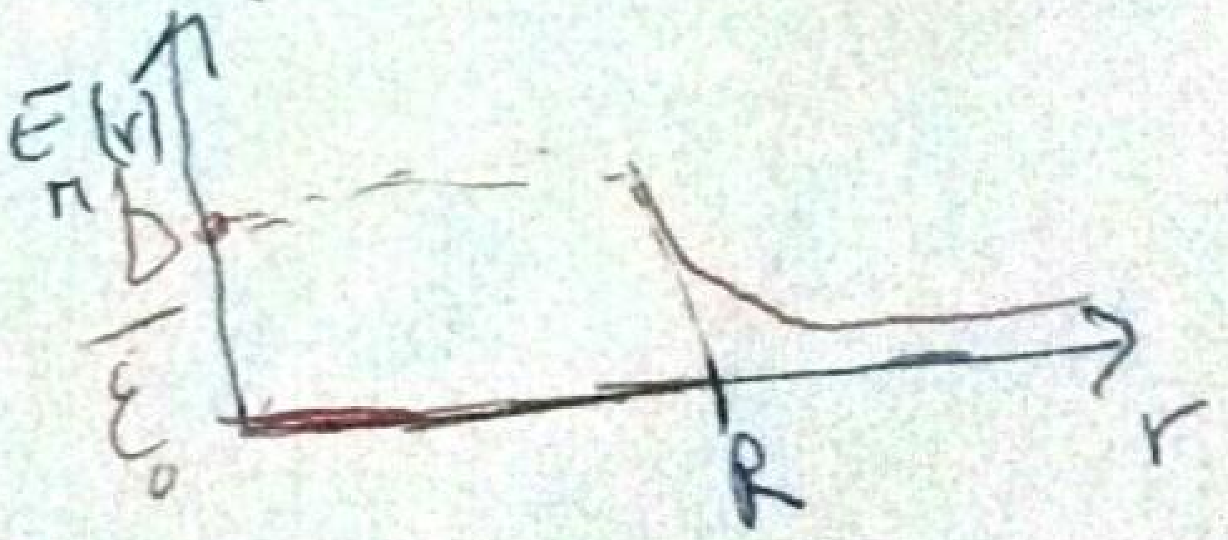
alors $\boxed{V_{ext} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r}}$

pour $r = R$

$$\lim_{r \to R} V_{ext} = V_{int} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{R} = \frac{\sigma R}{\varepsilon_0} = C_1$$

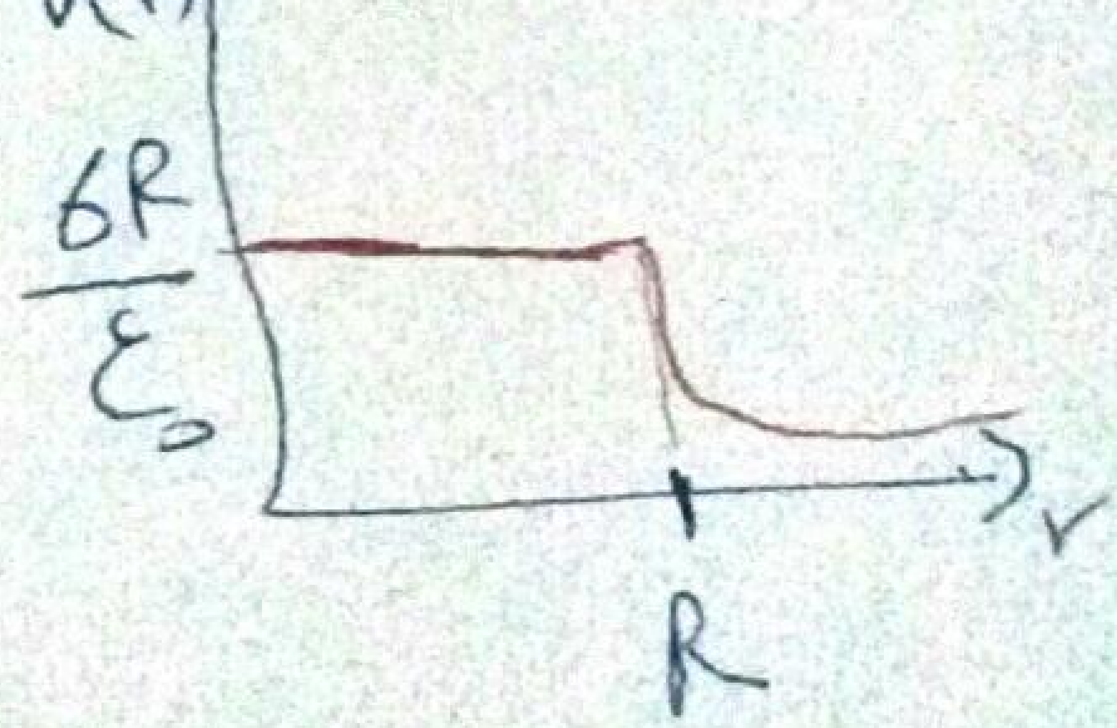
$$\Rightarrow \boxed{V_{int} = C_1 = \frac{\sigma R}{\varepsilon_0}}$$

3) Traçons les courbes $V(r)$ et $E(r)$



- Pour $r < R$

$V_{int} = \frac{\sigma R}{\varepsilon_0}$ et $E_{int} = 0$



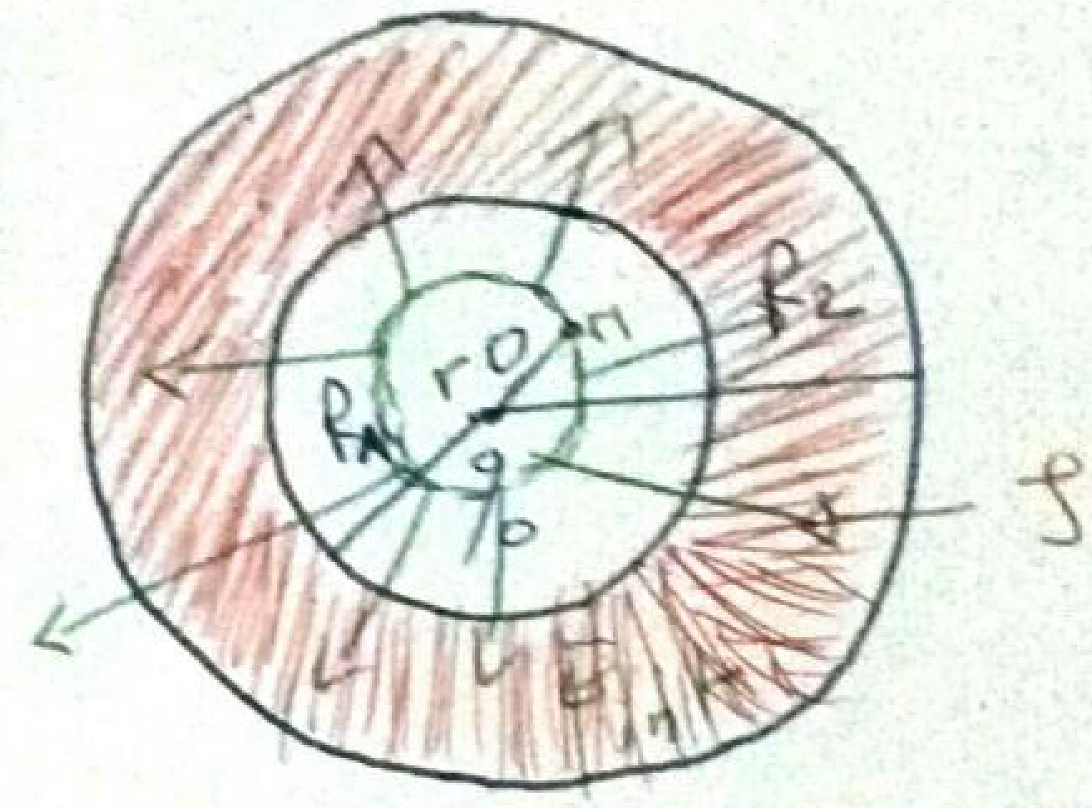
- Pour $r > R$

$V_{ext} = \frac{\sigma R^2}{\varepsilon_0}\frac{1}{r}$ et $E_{ext} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r^2}$

$V_{ext}(r=R) = \frac{\sigma R}{\varepsilon_0}$ et $E_{ext}(r=R) = \frac{\sigma}{\varepsilon_0}$

Exo 5

1) Calculons $\vec{E}_M(r)$



1ier cas $r < R_1$

$\phi = \oiint \vec{E}_{int} \cdot \vec{dS}_{\sigma_1} = \frac{Q_{int}}{\varepsilon_0}$

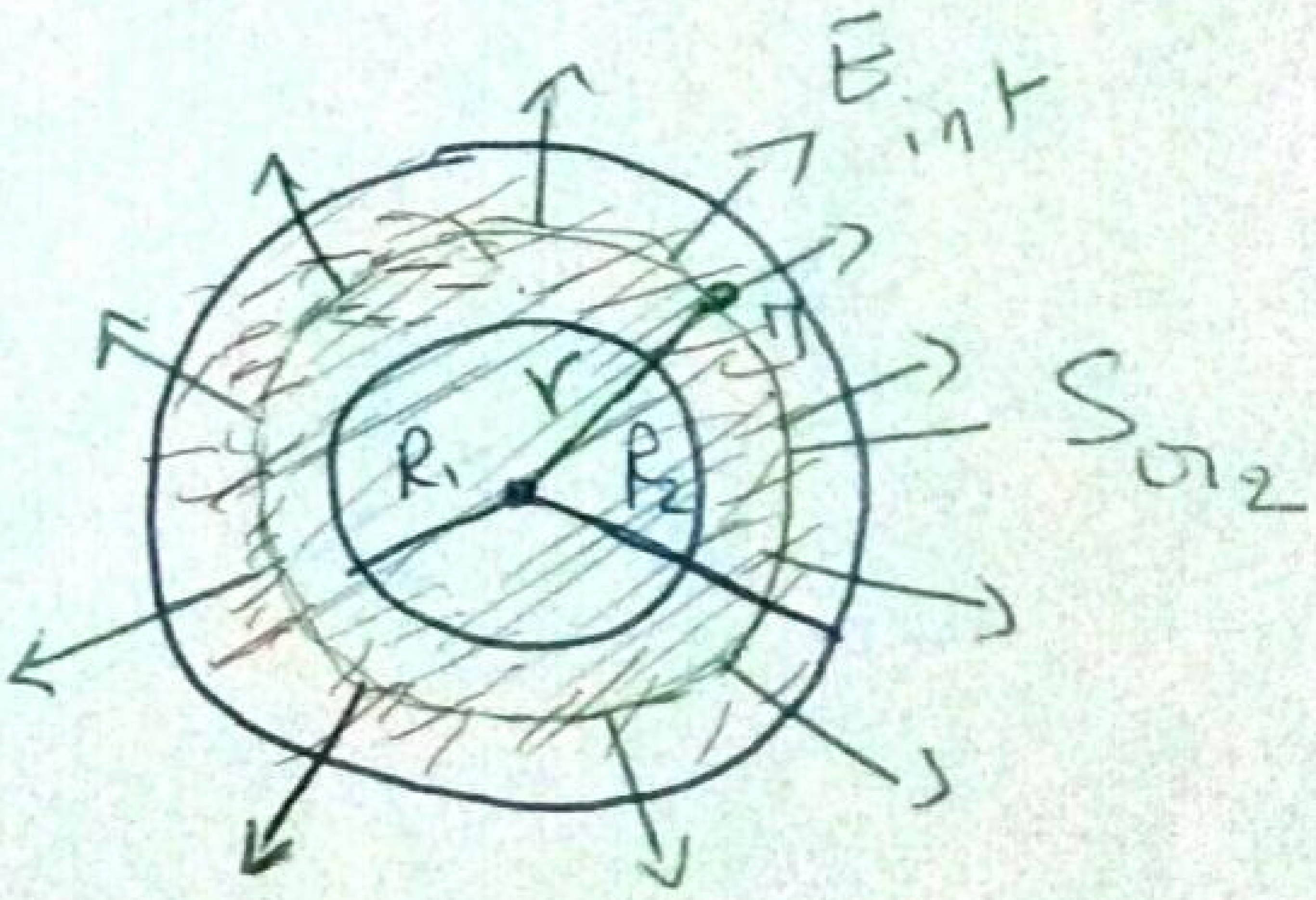
$\oiint \vec{E}_{int} \, \vec{dS}_{\sigma_1} = \oiint E_{int} \, dS_{\sigma_1}$

$= E_{int} S_{\sigma_1} = E_{int} (4\pi r^2)$ surface de gauss

$Q_{int} = 0 \Rightarrow E_{int} 4\pi r^2 = \frac{0}{\varepsilon_0} = 0 \Rightarrow \boxed{E_{int} = 0}$

2ième cas $R_1 < r < R_2$

$\phi = \oiint E_{int} dS_{\sigma_2} = Q_{int}/\varepsilon_0$



$\oiint E_{int} dS_{\sigma_2} = E_{int} S_{\sigma_2}$

$= E_{int} 4\pi r^2$

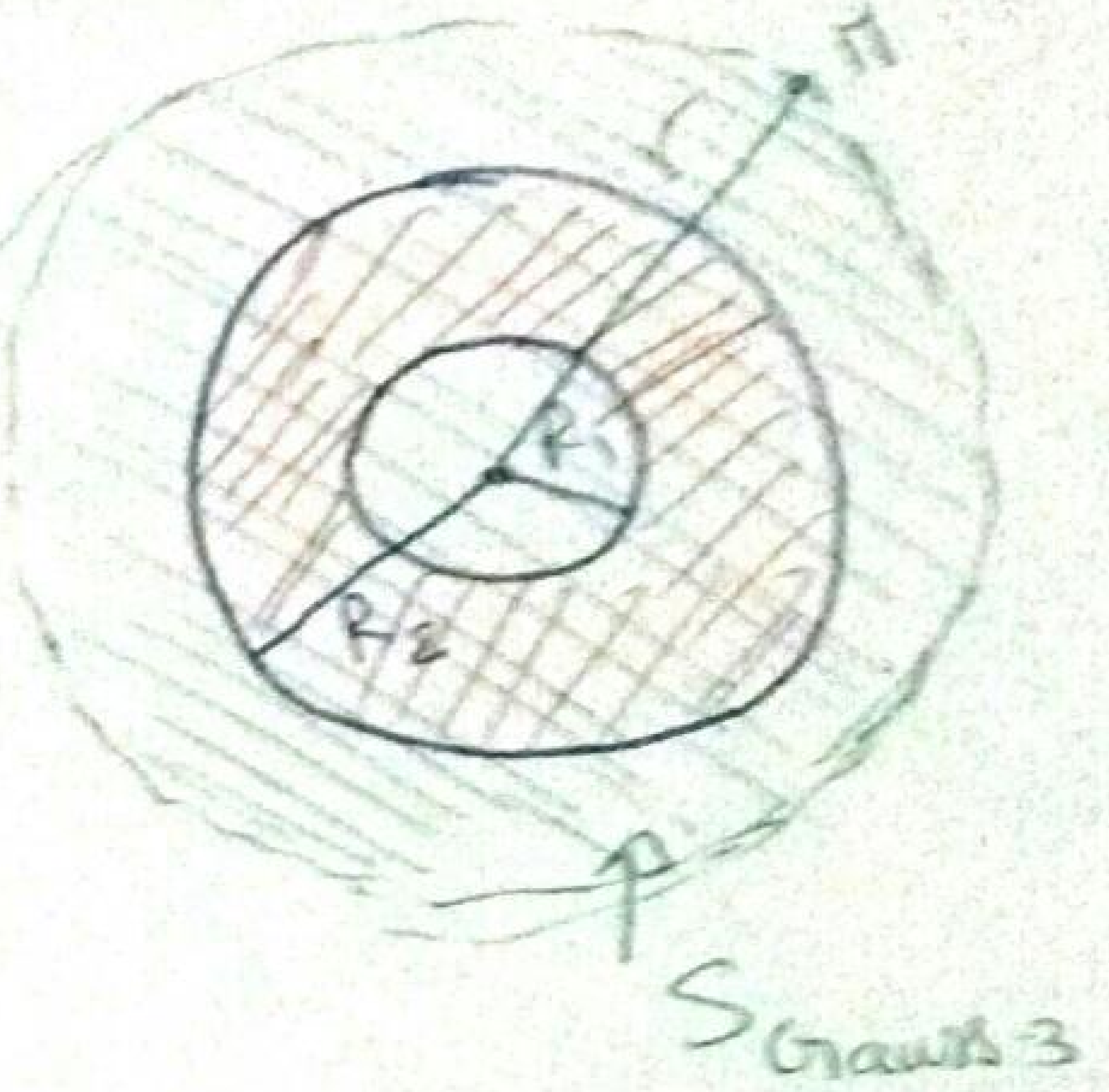
$Q_{int} = \rho \iiint dV = \rho V$

$= \rho \frac{4}{3}\pi (r^3 - R_1^3)$

$\Rightarrow E_{int} \cancel{4\pi} r^2 = \frac{\rho}{\varepsilon_0} \frac{\cancel{4}}{3} \cancel{\pi} (r^3 - R_1^3)$

$\Rightarrow E_{int} = \frac{\rho}{3\varepsilon_0} \frac{(r^3 - R_1^3)}{r^2}$ et $\vec{E}_{int} = \frac{\rho}{3\varepsilon_0}\left(\frac{r^3 - R_1^3}{r^2}\right)\vec{e_r}$

$3^{ième}$ cas $r > R_2$



$$\oiint E_{ext}\, dS_{\sigma_3} = \frac{Q_{int}}{\varepsilon_0}$$

$$\oiint E_{ext}\, dS_{\sigma_3} =$$

$$E_{ext}\, S_{\sigma_3} = E_{ext}\, 4\pi r^2$$

$$Q_{int} = \rho \iiint V = \rho V$$

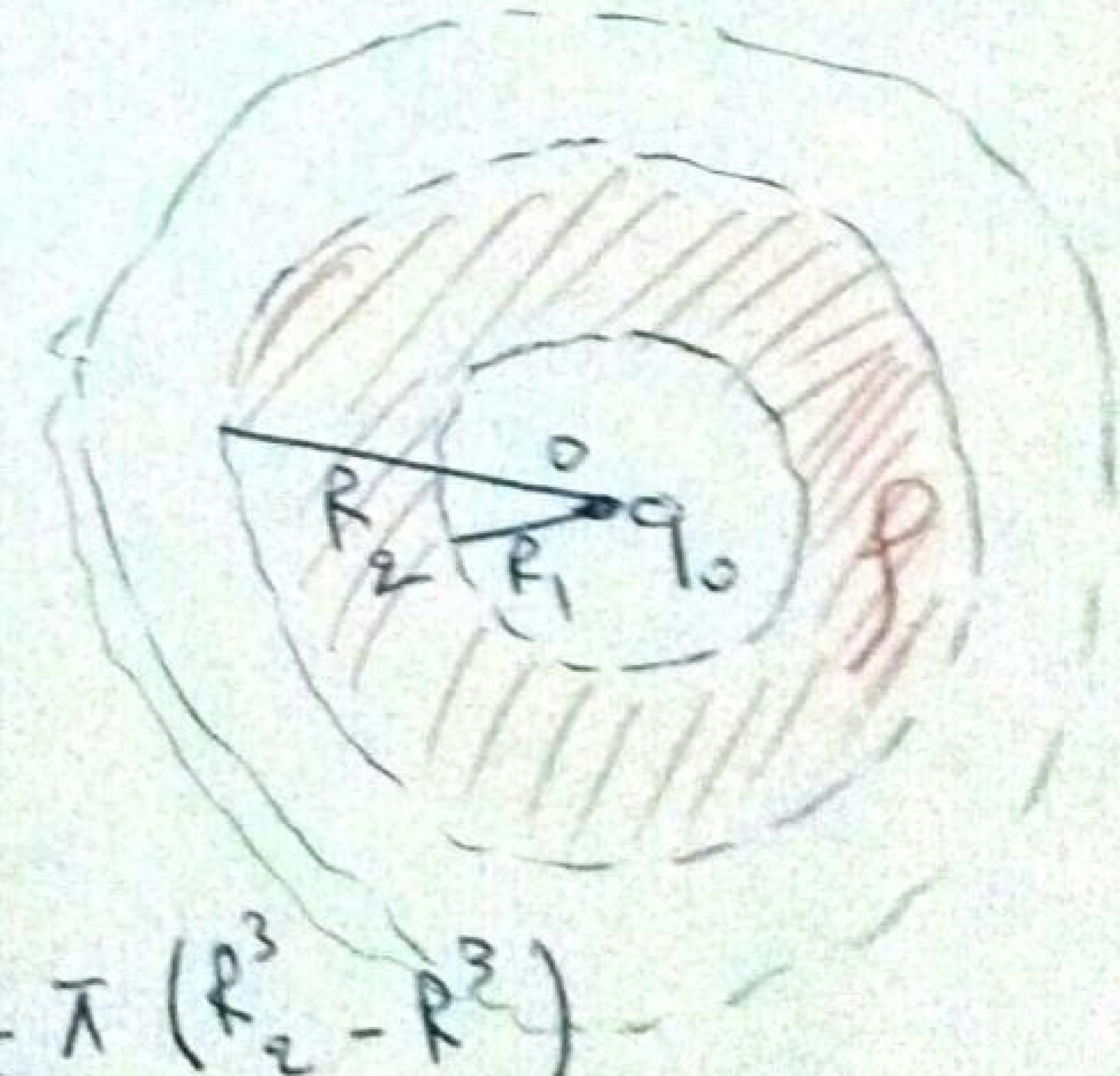
$$= \rho\, \frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$

$$E_{ext}\, S_{\sigma_3} = \frac{Q_{int}}{\varepsilon_0} \iff E_{ext}\, \cancel{4\pi} r^2 = \frac{\rho}{\varepsilon_0}\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$

$$E_{ext} = \frac{\rho}{3\varepsilon_0}\frac{\left(R_2^3 - R_1^3\right)}{r^2} \quad \text{et} \quad \vec{E}_{ext} = \frac{\rho}{3\varepsilon_0}\frac{\left(R_2^3 - R_1^3\right)}{r^2}\vec{e_r}$$

2) On ajoute au centre O une charge ponctuelle

$$q_0 = -\rho\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$



$$Q_{int} = Q_{int\,sphère} + q_0$$

$$= \rho\frac{4}{3}\pi\left(R_2^3 - R_1^3\right) - \rho\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)$$

$$= 0$$

$$\Rightarrow \boxed{\vec{E}_M(r) = \vec{0}}$$