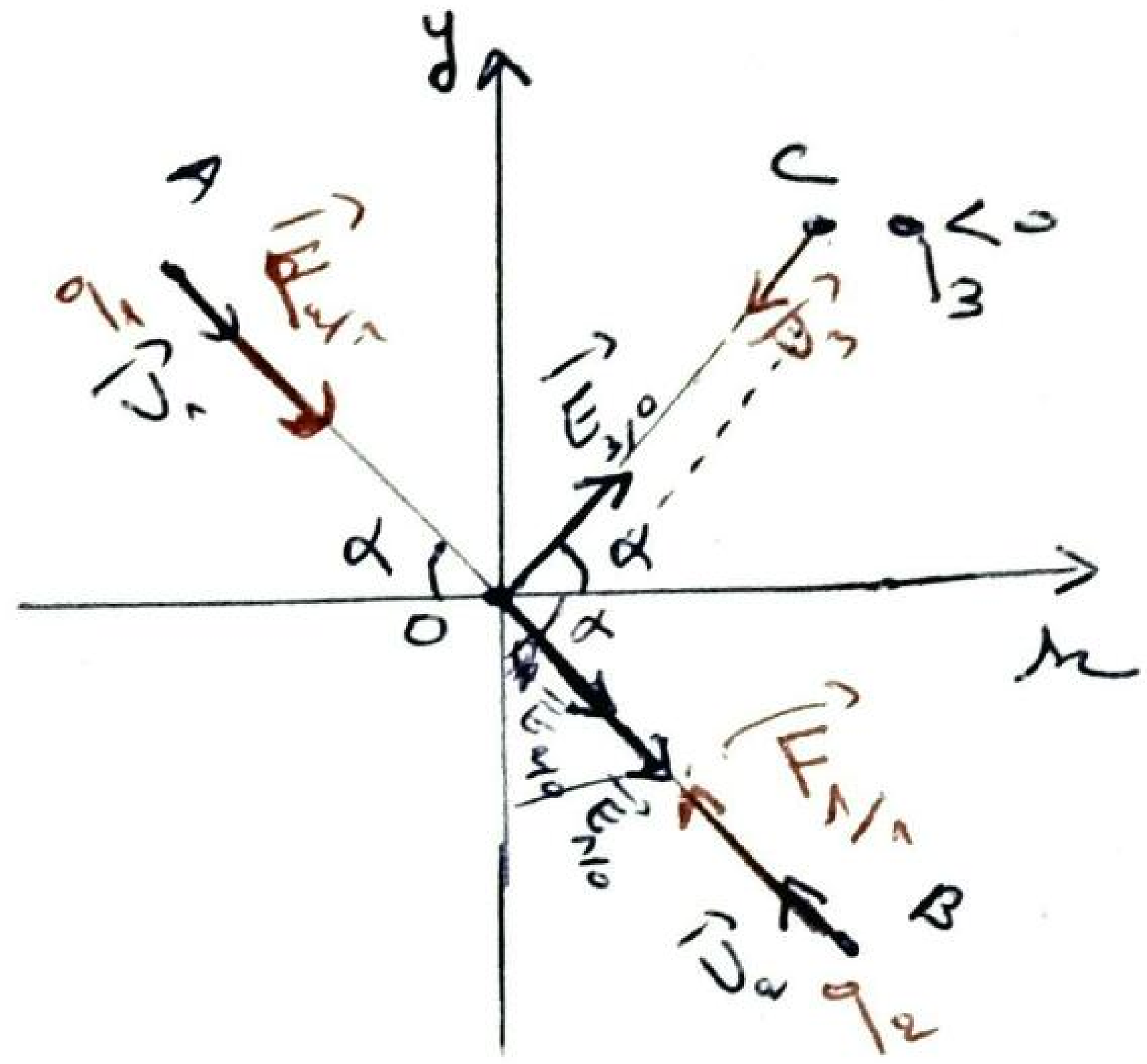
champ et potentiel électrostatique.
Distribution discrète

EXO1 $q_1 = 5\mu C$; $q_2 = -2\mu C$; $F_{1/2} = F_{2/1} = 1\,N$



1) Calculons r qui sépare ~~ces~~ deux charges :

$$F_{1/2} = k\frac{|q_1||q_2|}{r^2} \Rightarrow r^2 = k\frac{|q_1||q_2|}{F_{1/2}}$$

$$\Rightarrow r = \sqrt{\frac{k|q_1||q_2|}{F_{1/2}}} = \sqrt{\frac{9.10^9\, 5(2)10^{-6}}{1}} = 0{,}3\,M$$

$$\Rightarrow \boxed{r = 0{,}3\,m}$$

2) a) Représentation des champs électrostatique

$$\vec{E}_{3/0} = -\frac{k|q_3|}{r_3^2}\vec{U}_3 \quad \text{et} \quad \vec{E}_{1/0} = k\frac{q_1}{r_1^2}\vec{U}_1 \; ; \; \vec{E}_{2/0} = -\frac{k|q_2|}{r_2^2}\vec{U}_2$$

b) <u>Calculons $\vec{E}_{/0}$</u>

<u>1ière méthode</u>: par projection (

$\perp(ox)$: $E_{3/0}\cos\alpha + E_{1/0}\cos\alpha + E_{2/0}\cos\alpha = E_{x/0}$ avec $E_i = \frac{k|q_i|}{r_i^2}$ $i = 1,2,3$

$\perp(oy)$: $+E_{3/0}\sin\alpha - E_{1/0}\sin\alpha - E_{2/0}\sin\alpha = E_{y/0}$

et $r_1 = r_2 = r_3 \Rightarrow E_i = k\frac{|q_i|}{r^2}$

$$\Rightarrow \begin{cases} k\frac{|q_3|}{r^2}\cos\alpha + k\frac{|q_1|}{r^2}\cos\alpha + k\frac{|q_2|}{r^2}\cos\alpha = E_{x/0} = \frac{k}{r^2}\cos\alpha\left(|q_3|+|q_1|+|q_2|\right) \\ k\frac{|q_3|}{r^2}\sin\alpha - k\frac{|q_1|}{r^2}\sin\alpha - k\frac{|q_2|}{r^2}\sin\alpha = E_{y/0} \end{cases}$$

$$E_{y/0} = \frac{k}{r^2}\sin\alpha\left(|q_3| - |q_1| - |q_2|\right)$$

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$\Rightarrow$ $E_{x/o} = \frac{K}{r^2} \cos\alpha \left(|q_3| + |q_2| + |q_1|\right) = \frac{9.10^9}{\left(\frac{0,3}{2}\right)^2} \frac{\sqrt{3}}{2} (3+5+2) 10^{-6}$

$E_{x/o} = 3464,10 \; 10^3$ V/m ou N/C

$E_{y/o} = \frac{K}{r^2} \sin\alpha \left(|q_3| - |q_2| - |q_1|\right) = \frac{9.10^9}{(0,15)^2} \frac{1}{2} (3-5-2) 10^{-6}$

$E_{y/o} = -800 \,.\, 10^3$ V/m ou N/C et $\boxed{\vec{E}_{/o} = 34,64 \; 10^5 \vec{i} - 8 \, 10^5 \vec{j}}$

$\Rightarrow E_{/o} = \sqrt{E_{x/o}^2 + E_{y/o}^2} = 35,55 \; 10^5$ V/m ou N/C $\Rightarrow \boxed{E_{/o} = 35,55 \; 10^5 \text{ N/C}}$

2ième méthode : $\vec{E}_{/o} = \vec{E}_{3/o} + \vec{E}_{1/o} + \vec{E}_{2/o}$

$\vec{E}_{3/o} = -\frac{K |q_3|}{r^2} \vec{U}_3 = -\frac{9.10^9 (3) 10^{-6}}{(0,15)^2} \vec{U}_3 = -12 \; 10^5 \vec{U}_3$

$\vec{E}_{1/o} = \frac{K |q_1|}{r^2} \vec{U}_1 = \frac{9.10^9 \; 5 \; 10^{-6}}{(0,15)^2} \vec{U}_1 = 20 \; 10^5 \vec{U}_1$

$\vec{E}_{2/o} = -\frac{K |q_2|}{r^2} \vec{U}_2 = -\frac{9.10^9 (2) 10^{-6}}{(0,15)^2} \vec{U}_2 = -8 \; 10^5 \vec{U}_2$

avec $\vec{U}_3 = \frac{\vec{r}_3}{r_3}$; $\vec{r}_3 = -r\cos 30 \, \vec{i} - r \sin 30 \, \vec{j} = -(0,15) \frac{\sqrt{3}}{2} \vec{i} - (0,15) \frac{1}{2} \vec{j}$

$\vec{r}_3 = -0,13 \, \vec{i} - 0,075 \, \vec{j}$

$\Rightarrow \vec{U}_3 = \frac{-0,13 \, \vec{i} - 0,075 \, \vec{j}}{0,15}$

$\Rightarrow \vec{U}_3 = -0,87 \, \vec{i} - 0,5 \, \vec{j}$

$\Rightarrow \vec{U}_2 = \frac{\vec{r}_2}{r_2}$; $\vec{r}_2 = -r \cos\alpha \, \vec{i} + r \sin\alpha \, \vec{j} = -(0,15) \frac{\sqrt{3}}{2} \vec{i} + (0,15) \frac{1}{2} \vec{j}$

$\Rightarrow \vec{r}_2 = -0,13 \, \vec{i} + 0,075 \, \vec{j}$ $\Rightarrow \vec{U}_2 = \frac{-0,13 \, \vec{i} + 0,075 \, \vec{j}}{(0,15)} = -0,87 \, \vec{i} + 0,5 \, \vec{j}$

$\vec{U}_1 = \frac{\vec{r}_1}{r_1}$; $\vec{r}_1 = (r \cos\alpha \, \vec{i} + r \sin\alpha \, \vec{j}) = +0,13 \, \vec{i} - 0,075 \, \vec{j}$

$\Rightarrow \vec{U}_1 = \frac{+0,13 \, \vec{i} - 0,075 \, \vec{j}}{0,15} = +0,87 \, \vec{i} - 0,5 \, \vec{j} = \vec{U}_1$

$\Rightarrow \vec{E}_{3/o} = -12.10^5 (-0,87 \, \vec{i} - 0,5 \, \vec{j})$, $\vec{E}_{2/o} = -8 \, 10^5 (-0,87 \, \vec{i} + 0,5 \, \vec{j})$;

2

$$\vec{E}_{2/0} = 20\,10^5\left(+0{,}87\,\vec{i} - 0{,}5\,\vec{j}\right)$$

$$\Rightarrow \vec{E}_{/0} = \vec{E}_{1/0} + \vec{E}_{2/0} + \vec{E}_{3/0} = 20.10^5\left(0{,}87\,\vec{i} - 0{,}5\,\vec{j}\right) - 8.10^5\left(-0{,}87\,\vec{i} + 0{,}5\,\vec{j}\right)$$

$$- 12.10^5\left(-0{,}87\,\vec{i} - 0{,}5\,\vec{j}\right)$$

$$\Rightarrow \vec{E}_{/0} = 0{,}87(10^5)\left(20 + 8 + 12\right)\vec{i} + 0{,}5\,10^5\left(-20 - 8 + 12\right)\vec{j}$$

$$= 34{,}8\,10^5\,\vec{i} - 8\,10^5\,\vec{j}$$

$$\Rightarrow E_{/0} = \sqrt{\left[(34{,}8)^2 + (-8)^2\right]10^{10}} = 35{,}7\,10^5\ N/C \text{ ou } V/m$$

3 - Calculons V_0 /

$$V_0 = V_{1/0} + V_{2/0} + V_{3/0}$$

$$V_{1/0} = K\frac{q_1}{r} = \frac{9.10^9\ 5\,10^{-6}}{0{,}15} = 300\,10^{-3} = 0{,}3$$

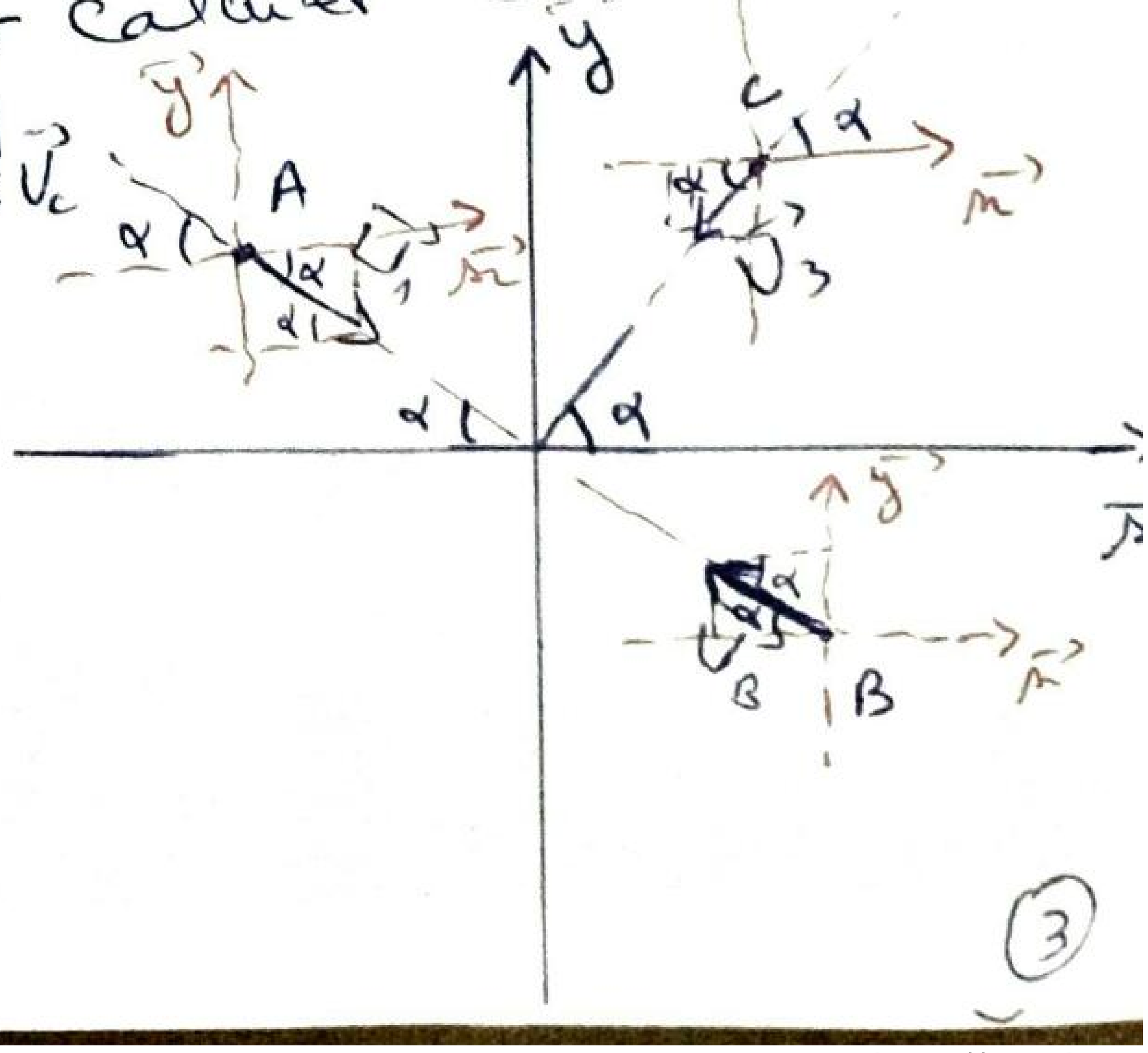
$$V_{2/0} = \frac{K\ q_2}{r} = \frac{9.10^9(-2)10^{-6}}{0{,}15} = -120\,10^{-3} = 0{,}12$$

$$V_{3/0} = \frac{K\ q_3}{r} = \frac{9.10^9(-3)10^{-6}}{0{,}15} = -0{,}18$$

$$\Rightarrow V_0 = 0{,}3 - 0{,}12 - 0{,}18 = 0{,}0 \Rightarrow \boxed{V_0 = 0}$$

~~2ème~~ 3ième méthode : Pour calculer $\vec{E}$

$$\vec{E}_A = \frac{K|q_A|}{r_A^2}\vec{U}_A \ ;\ \vec{E}_B = -\frac{K|q_B|}{r_B^2}\vec{U}_B \ ;\ \vec{E}_C = \frac{-K|q_C|}{r_C^2}\vec{U}_C$$



$$\vec{U}_A = \cos\alpha\,\vec{i} - \sin\alpha\,\vec{j}$$

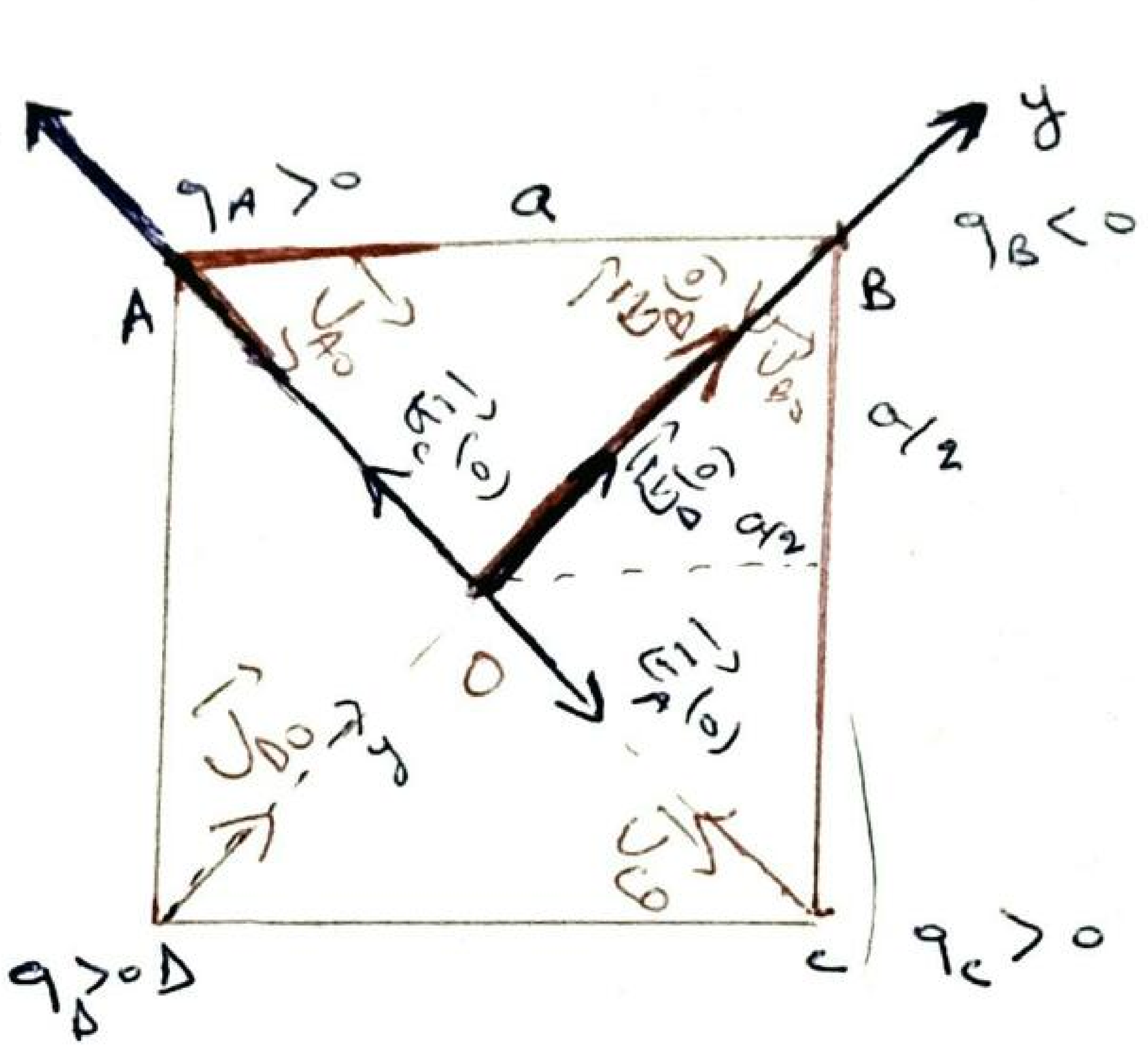
$$\vec{U}_B = -\cos\alpha\,\vec{i} + \sin\alpha\,\vec{j}$$

$$\vec{U}_C = -\cos\alpha\,\vec{i} - \sin\alpha\,\vec{j}$$

Exo 2 $a = 2m$; $q_A = +2.10^{-8}C$; $q_B = -8\,10^{-8}C$; $q_C = +2.10^{-8}C$; $q_D = 4.10^{-8}C$

1) Donnons l'expression de $\vec{E}(O)$

$$\vec{E}(O) = \vec{E}_A(O) + \vec{E}_B(O) + \vec{E}_C(O) + \vec{E}_D(O)$$



1ière méthode : de projection

$\perp(Ox)$: $E_C(O) - E_A(O) = E_x$

$\perp(Oy)$: $E_D(O) + E_B(O) = E_y$

$$\Rightarrow E_C(O) - E_A(O) = \frac{K|q_C|}{r_C^2} - \frac{K|q_A|}{r_A^2} = E_x$$

$$r_{AO} = r_{CO} = r_{BO} = r_{DO} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{2}}$$

avec $q_C = q_A \Rightarrow \boxed{E_C(O) - E_A(O) = E_x = 0}$

$$\perp(Oy) : E_y = E_D(O) + E_B(O) = \frac{K|q_D|}{\left(\frac{a}{\sqrt{2}}\right)^2} + \frac{K|q_B|}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{2K}{a^2}\left(|q_D| + |q_B|\right)$$

$$\Rightarrow E_y = \frac{2K}{a^2}(8+4)10^{-8} = \frac{2.9.10^{9}(12)10^{-8}}{(2)^2} = 540\ N/C$$

$$\Rightarrow \boxed{E(O) = E_y = 540\ N/C}$$

$$\boxed{\vec{E}(O) = E_y\,\vec{j} = 540\,\vec{j}}$$

2ième méthode : $\vec{E}_A(O) = \frac{K|q_A|}{r_{AO}^2}\vec{U}_{AO}$; $\vec{E}_B = -\frac{K|q_B|}{r_{BO}^2}\vec{U}_{BO}$; $\vec{E}_C = \frac{K|q_C|}{r_{CO}^2}\vec{U}_{CO}$

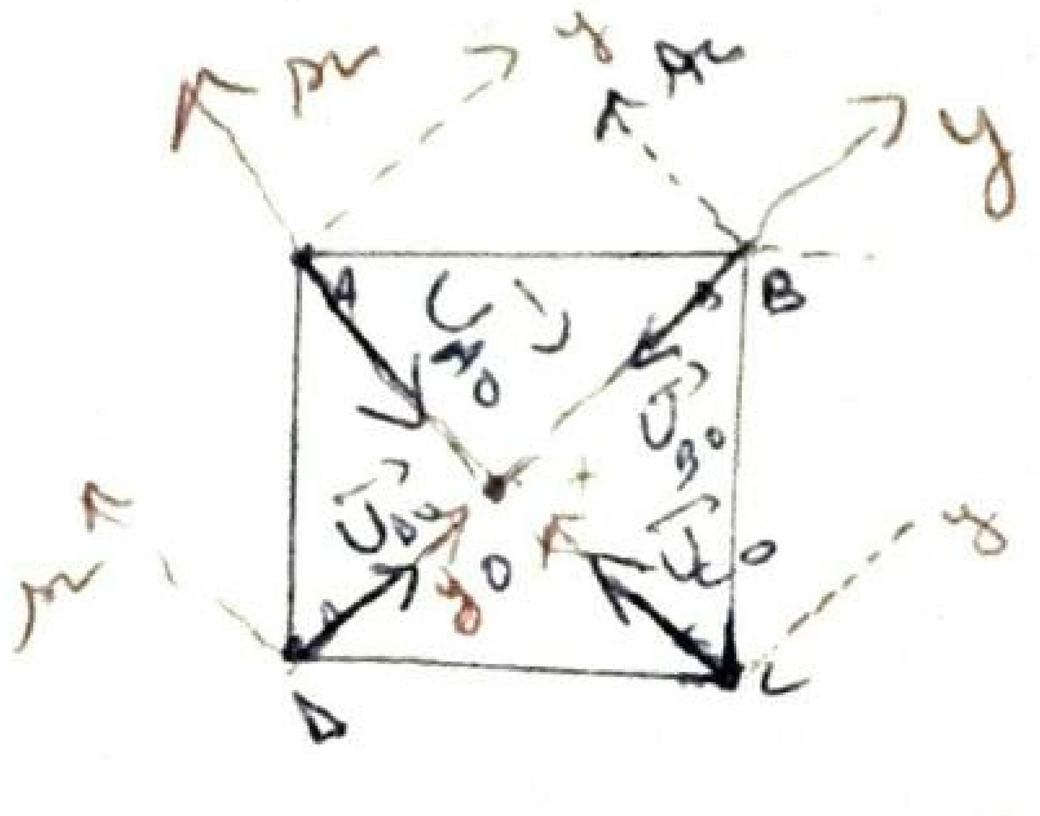
$$\vec{E}_D(O) = \frac{K|q_D|}{r_{DO}^2}\vec{U}_{DO}$$

$\vec{U}_{AO} = -\vec{i}$

ou bien $\vec{U}_A = \frac{\vec{r}_{AO}}{r_{AO}}$ avec $\vec{r}_{AO} = -\vec{r}_{OA} = -\frac{a}{\sqrt{2}}\vec{i}$

$\vec{U}_{BO} = -\vec{j}$

ou $\vec{U}_{BO} = \frac{\vec{r}_{BO}}{r_{BO}}$ avec $\vec{r}_{BO} = -\vec{r}_{OB} = -\frac{a}{\sqrt{2}}\vec{j}$



$$\begin{cases} \vec{U}_{CO} = +\vec{i} \\ \text{ou bien} \\ \vec{U}_{CO} = \dfrac{\vec{r}_{CO}}{r_{CO}} \text{ avec } \vec{r}_{CO} = -\vec{r}_{OC} = -\begin{pmatrix} -\frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} = \dfrac{a}{\sqrt{2}}\vec{i} \\ \Rightarrow \vec{U}_{CO} = \dfrac{a}{\sqrt{2}}\vec{i} / a/\sqrt{2} = \vec{i} \end{cases}$$

$$\begin{cases} \vec{U}_{DO} = +\vec{j} \text{ ou bien } \vec{U}_{DO} = \dfrac{\vec{r}_{DO}}{r_{DO}} \text{ avec } \vec{r}_{DO} = -\vec{r}_{OD} = -\begin{pmatrix} +0 \\ -\frac{a}{\sqrt{2}} \end{pmatrix} = \frac{a}{\sqrt{2}}\vec{j} \\ \Rightarrow \vec{U}_{DO} = \dfrac{\frac{a}{\sqrt{2}}\vec{j}}{\frac{a}{\sqrt{2}}} = \vec{j} \end{cases}$$

$$\Rightarrow \vec{E}_A(0) = K\frac{|q_A|}{r_A^2}\vec{U}_A = K\frac{|q_A|}{\left(\frac{a}{\sqrt{2}}\right)^2}(-\vec{i}) \quad ; \quad \vec{E}_B(0) = -K\frac{|q_B|}{r_B^2}\vec{U}_B = +K\frac{|q_B|}{\left(\frac{a}{\sqrt{2}}\right)^2}\vec{j}$$

$$\vec{E}_C(0) = \frac{K|q_C|}{r_C^2}\vec{U}_C = \frac{K|q_C|}{\left(\frac{a}{\sqrt{2}}\right)^2}(\vec{i}) \quad ; \quad \vec{E}_D(0) = K\frac{|q_D|}{r_C^2}\vec{U}_D = K\frac{|q_D|}{\left(\frac{a}{\sqrt{2}}\right)^2}\vec{j}$$

$$\Rightarrow \vec{E}(0) = \frac{2K}{a^2}\left(\cancel{-|q_A|\vec{i}} + |q_B|\vec{j} + \cancel{|q_C|\vec{i}} + |q_D|\vec{j}\right) \text{ avec } q_C = q_A$$

$$\Rightarrow \vec{E}(0) = \frac{2K}{a^2}\left(|q_B| + |q_D|\right) = \frac{2.9.10^{9}}{(2)^2}(8+4)10^{-8} = 540\ V/m$$

2) la valeur de V(0) crée par ces quatres charges au (0)

$$V(0) = V_A(0) + V_B(0) + V_D(0) + V_C(0) = \sum_{i=1}^{4} V_i(0)$$

$$V_A(0) = \frac{K q_A}{r_{AO}} = 9.10^{+9}\frac{.2.10^{-8}}{a/\sqrt{2}} = \frac{180}{2/\sqrt{2}}\ (V)$$

$$V_B(0) = K\frac{q_B}{r_{BO}} = \frac{9.10^{+9}.(-8.10^{-8})}{2/\sqrt{2}} = \frac{-720}{2/\sqrt{2}}$$

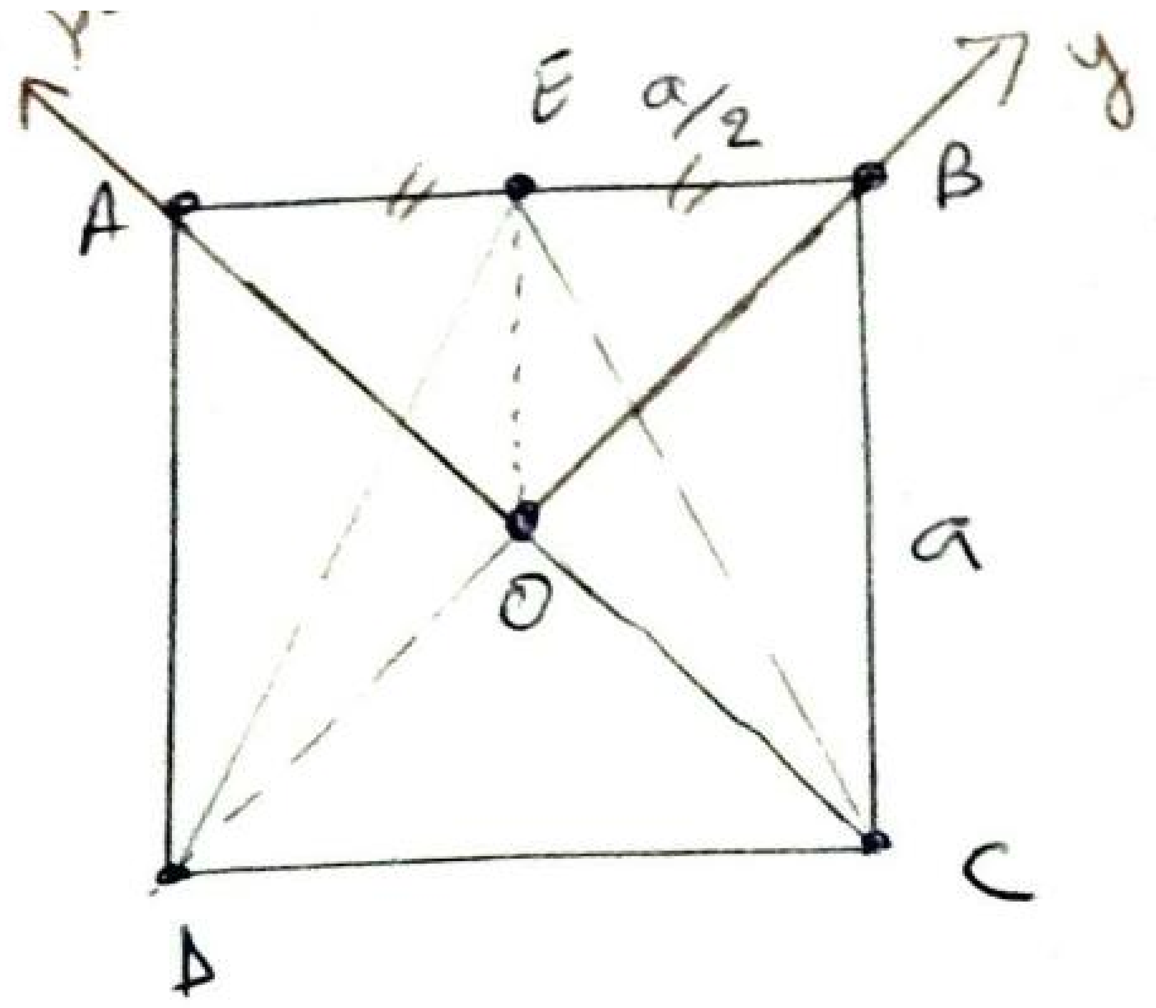
$$V_C(0) = K\frac{q_C}{r_{CO}} = \frac{9.10^{+9}.(2)10^{-8}}{2/\sqrt{2}} = \frac{180}{2/\sqrt{2}}$$

$$V_D(0) = \frac{K q_D}{r_{DO}} = \frac{9.10^{9}.(4).10^{-8}}{2/\sqrt{2}} = \frac{360}{2/\sqrt{2}}$$

$$\Rightarrow \sum_{i=1}^{0} V_i = \boxed{0 = V(0)}$$

suite d'Exo 2

3) Calculons $V(E)$ au point E



$V(E) = V_A(E) + V_B(E) + V_C(E) + V_D(E)$

$V_A(E) = K\frac{q_A}{r_{AE}}$ et $V_B(E) = K\frac{q_B}{r_{BE}}$

avec $r_{AE} = r_{BE} = a/2 = \frac{2}{2} = 1\,m$

$V_C(E) = \frac{K q_C}{r_{CE}}$ et $V_D = K\frac{q_D}{r_{DE}}$

$\Rightarrow r_{CE} = r_{DE} = \sqrt{(a/2)^2 + a^2} = \sqrt{\frac{5a^2}{4}} = \frac{a}{2}\sqrt{5}$

$\Rightarrow V(E) = K\frac{q_A}{r_{AE}} + K\frac{q_B}{r_{BE}} + K\frac{q_C}{r_{CE}} + K\frac{q_D}{r_{DE}}$

$= \frac{K}{a/2}\left(q_A + q_B + \frac{q_C}{\sqrt5} + \frac{q_D}{\sqrt5}\right) = \frac{9.10^9}{1}\left(2\,10^{-8} - 8\,10^{-8} + \frac{2.10^{-8}}{\sqrt5} + \frac{4\,10^{-8}}{\sqrt5}\right]$

$= 9.10^9\left(2 - 8 + 0{,}8944 + 1{,}7888\right)10^{-8} = -298{,}512\,(V)$

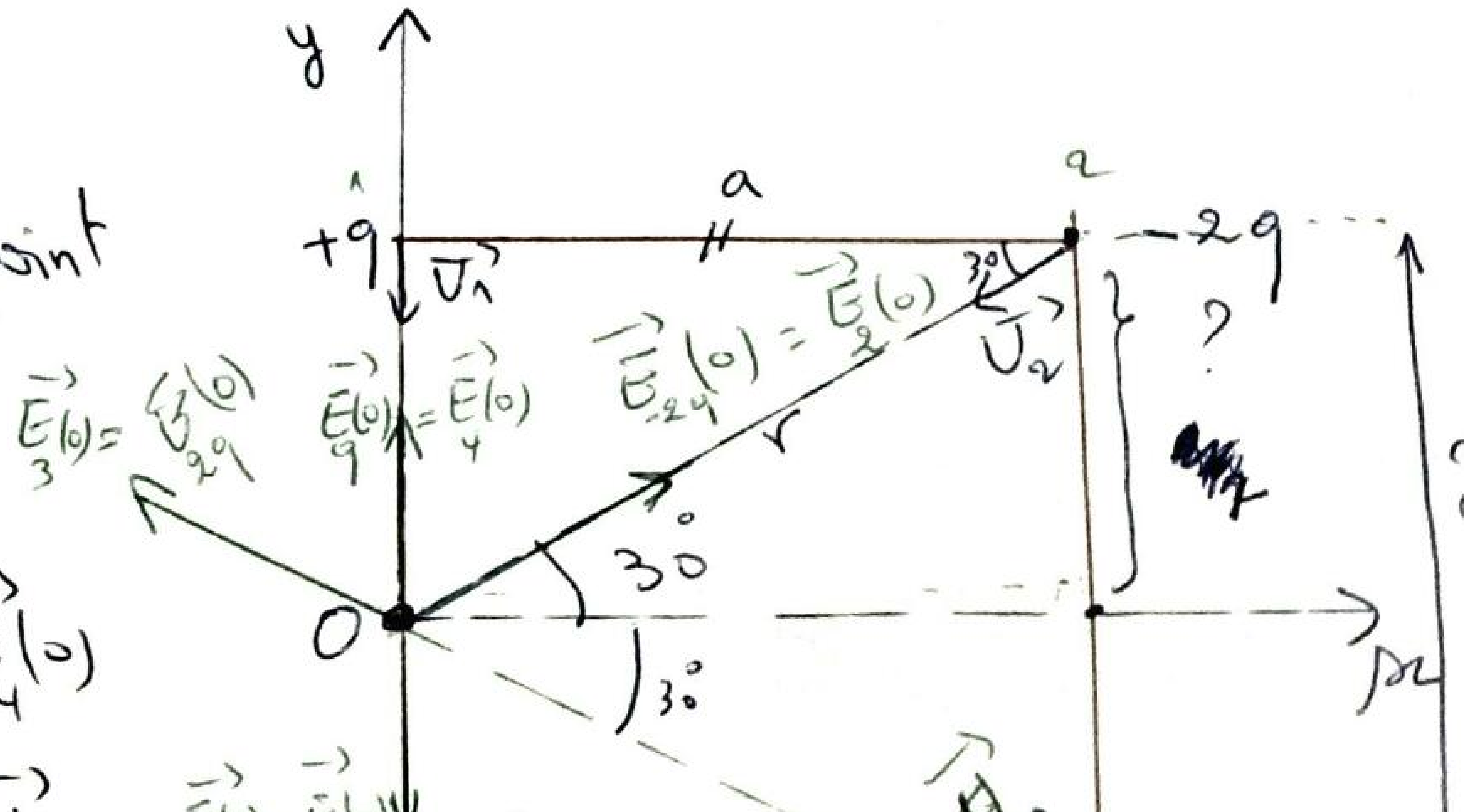
4) Déduction de $\vec{F}(0)$

$\vec{F}(0) = q(0)\,\vec{E}(0) = 2.10^{-8}(540)\vec{j} = 0{,}1080\,10^{-4}\vec{j}$

EXO 3

Calculons le champ et le potentiel électrique au point O avec $q = 10^{-9}\,C$ et

$a = 5cm = 5.10^{-2}\,m$

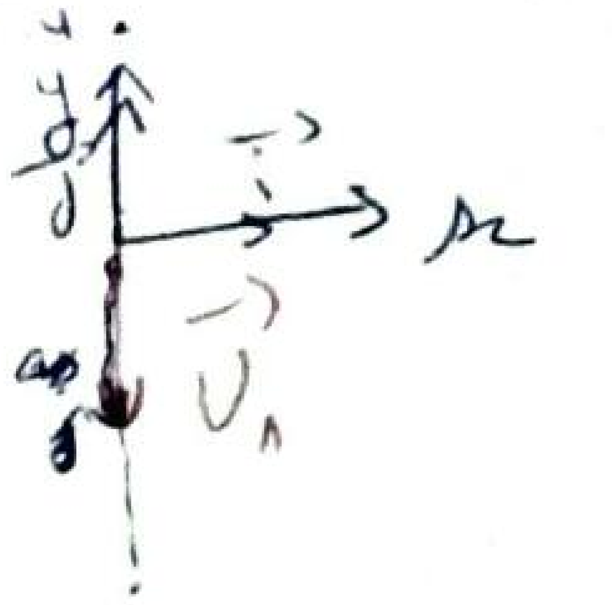


$\vec{E}(0) = \vec{E}_1(0) + \vec{E}_2(0) + \vec{E}_3(0) + \vec{E}_4(0)$

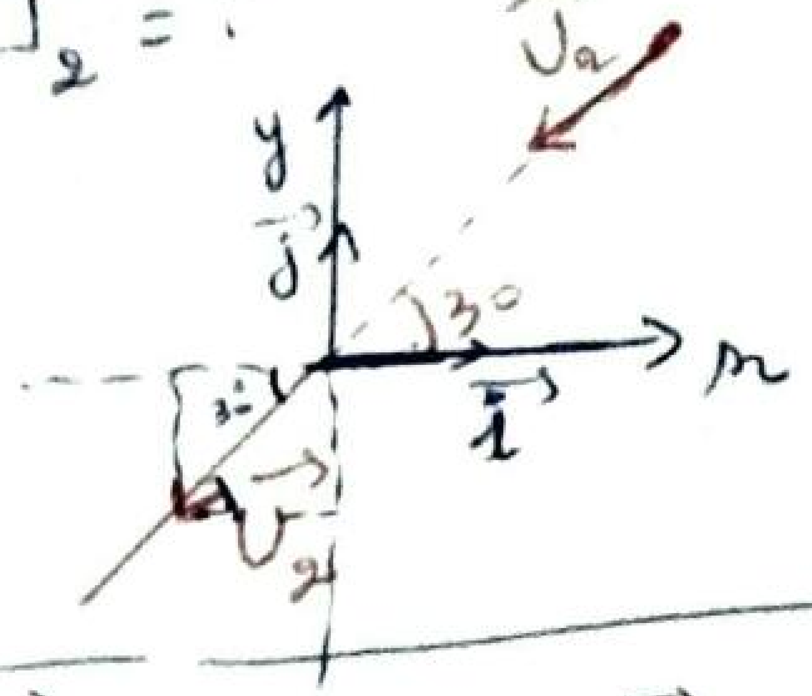
$\vec{E}_1(0) = \frac{kq_1}{r_{10}^2}\vec{U}_1$; $\vec{E}_{20}(0) = \frac{Kq_2}{r_{20}^2}\vec{U}_2$

$\vec{E}_{30} = \frac{Kq_3}{r_{30}^2}\vec{U}_3$; $\vec{E}_{40}(0) = \frac{Kq_4}{r_{40}^2}\vec{U}_4$.

$\boxed{\vec{U}_1 = -\vec{j}}$

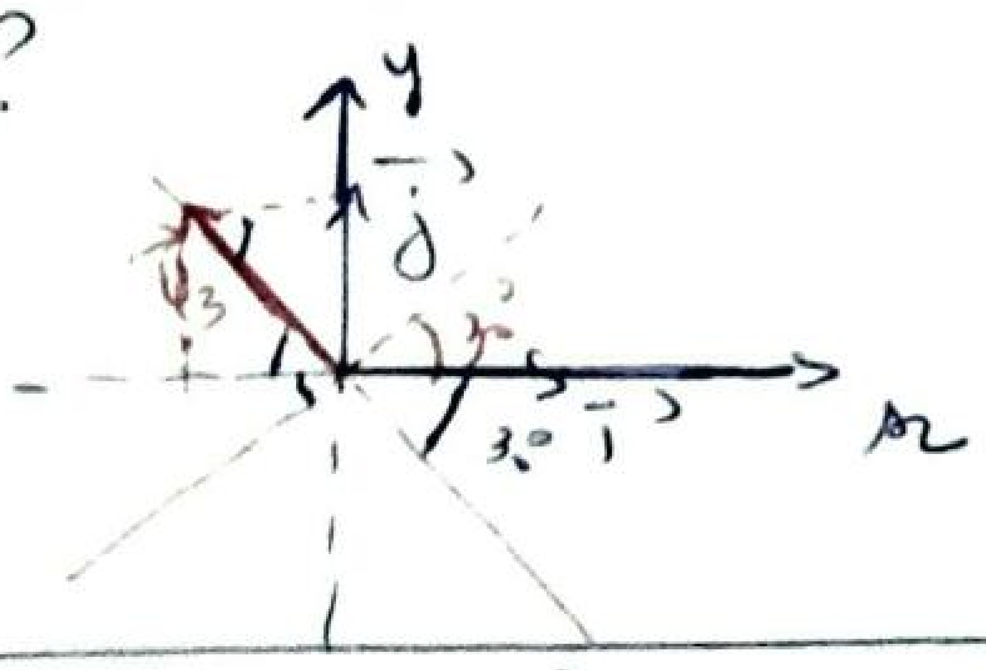


$\vec{U}_2 = ?$



$$\boxed{\begin{aligned}\vec{U}_2 &= -\cos 30°\,\vec{i} - \sin 30°\,\vec{j}\\ &= -\frac{\sqrt{3}}{2}\vec{i} - \frac{1}{2}\vec{j}\end{aligned}}$$

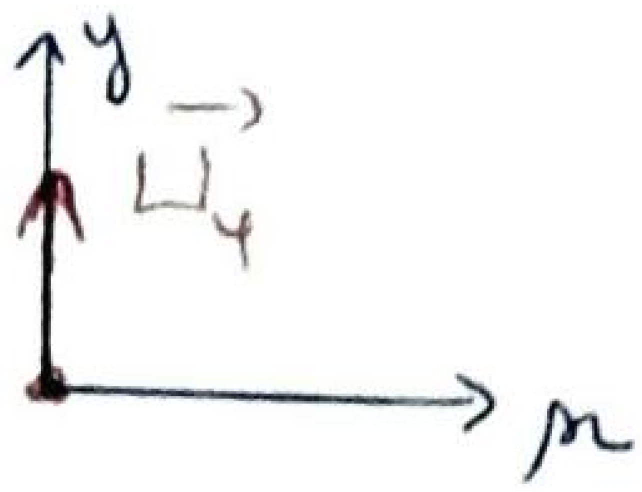
$\vec{U}_3 = ?$



$$\boxed{\begin{aligned}\vec{U}_3 &= -\cos 30°\,\vec{i} + \sin 30°\,\vec{j}\\ &= -\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\end{aligned}}$$

$\vec{U}_4 = ?$

$\vec{U}_4 = +\vec{j}$



d'autre part :

$r_{10} = r_{40} = r \sin 30°$ avec $r = \dfrac{a}{\cos 30°} = \dfrac{a}{\sqrt{3}/2} = \dfrac{2a}{\sqrt{3}} \Rightarrow r_{10} = r_{40} = \dfrac{2a}{\sqrt{3}}\,\dfrac{1}{2} = \dfrac{a}{\sqrt{3}}$

$r_{20} = r_{30} = r = \dfrac{a}{\cos 30°} = \dfrac{2a}{\sqrt{3}}$

$$\Rightarrow \vec{E}(0) = \cancel{\frac{kq}{\left(\frac{a}{\sqrt{3}}\right)^2}(-\vec{j})} + \frac{k(-2q)}{\left(\frac{2a}{\sqrt{3}}\right)^2}\left(\cancel{-\frac{\sqrt{3}}{2}\vec{i}} - \frac{1}{2}\vec{j}\right) + \frac{k(2q)}{\left(\frac{2a}{\sqrt{3}}\right)^2}\left(\cancel{-\frac{\sqrt{3}}{2}\vec{i}} + \frac{1}{2}\vec{j}\right)$$

$$+ \cancel{\frac{kq}{\left(\frac{a}{\sqrt{3}}\right)^2}(+\vec{j})}$$

$$\vec{E}(0) = \frac{kq3}{4a^2}\vec{j} + \frac{k3q}{4a^2}\vec{j} = \frac{kq3}{2a^2}\vec{j} = \frac{9.10^9(10^{-9})3}{2(5.10^{-2})^2} = 5400\ N/C \text{ ou } V/m$$