

# A METHOD FOR PREDICTING THE DEFORMATION OF SWELLING CLAY SOILS AND DESIGNING SHALLOW FOUNDATIONS THAT ARE SUBJECTED TO UPLIFTING

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## **Keywords**

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## **Abstract**

*Swelling soils can be found in many parts of the world. The state of practice in this area has been changing over recent decades. The design of foundations for expansive soils is an important challenge facing engineers. The excessive damage is are, in part, due to the lack of proper design, resulting in the need for better tools for practitioners in order to assess the impact of swelling soils in typical design applications. A correct measurement of the swelling pressure is required for an accurate prediction of the heave. A theoretical model is proposed to describe the swelling potential of clay soils on the basis of their characteristics obtained from oedometer tests. This paper describes analysis of the behavior of swelling soils when moistened under buildings and structures. The methods and principles currently used for the design of structure foundations on swelling soils involve important problems due to the non-uniform deformations of these soils when subjected to structural loads. The current study was conducted to compute the uplifting of shallow foundations on swelling soils considering the water-content change as well as the contact-pressure distribution under the footing.*

## **1 INTRODUCTION**

In any geotechnical study relative to a construction project, the swelling of a soil is as important as the settlement. The dimensional variations that, result from this phenomenon, constitute a permanent challenge for design and geotechnical engineers. The durability of a structure constructed on swelling soils depends on an appropriate appreciation of the phenomenon.

The swelling of clay soils, containing smectites or illites in different quantities, is the origin of numerous problems in buildings and large structures. These disturbances are frequent in regions with a dry climate like some parts of the Caucasus parts, in Kazakhstan, Algeria, Morocco, etc.

The swelling of soils can provoke important material damages, or even partial to total rupture of the structure, when it is not considered in the design process. It is therefore important to foresee correctly the possible distortions of swelling soils, in terms of amplitude and the speed of evolution, and to analyze its influence on the serviceability and stability of the structure.

Swelling soils have been a major concern for designers over many years. Some construction procedures have been developed to limit the effects of inflation on the constructions and can be found in the classic works, of., Lancelot L et al., Mouroux et al. (in French) [19] [22],

Sorochan E.A., Mustafaev A.A. (in Russian) [27] [23], and Chen F.H. (in English) [7].

Currently, an abundance of documentation explains the mechanisms for the swelling of clays, either at the microscopic level as in test tubes tested in the laboratory or in-situ soil. Nevertheless, a survey of the behavior of the structures in contact with swelling soils constitutes a complex task and the existing methods contain some insufficiencies. Most of the research carried out, is limited to the amplitudes of the inflation of the clay soils in nature, under loads of spread foundations. Little attention has been given the propagation of the inflation phenomena in the mass of the swelling soils as a function of time.

## 2 IDENTIFICATION OF THE MATERIAL.

The studied swelling clay comes from the Urban District of Baku (Azerbaijan) where it has provoked many disturbances in the structures of a concrete channel. The tests were carried out on samples of undisturbed clay



**Figure 1.** Example of disorders in the construction of the “Samour –Apcheron” concrete channel.

soil, of the Pliocenes era, collected from the Shamour channel “Apcheron”, (Baku) in the soil mechanics laboratory of the Civil Engineering Institute of Baku (Figures 1 to 3).

### 2.1 Mineralogical analysis

A diffractometry analysis using X-rays was performed to determine the mineralogy of the studied soils. The physical and mechanical properties are given in Table 1.

A chemical analysis of the samples of inflating clays gave the following compositions:

**Table 1.** Physical and mechanical characteristics of the studied clay soils.

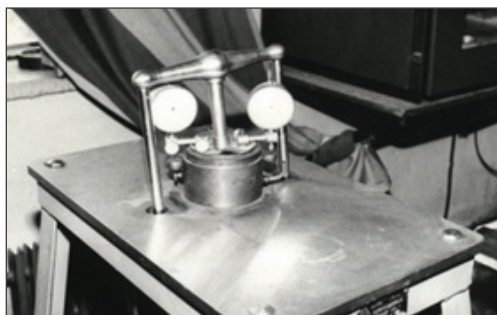
Soil characteristics	Symbols	Unit	Values
Natural water content	$W$	%	10–16
Degree of saturation	$S_r$	%	83–91
Wet Unit Weight	$\gamma_h$	kN/m <sup>3</sup>	21.6–22.5
Dry Unit Weight	$\gamma_d$	kN/m <sup>3</sup>	17.7–18.3
Specific Unit Weight	$\gamma_s$	kN/m <sup>3</sup>	27.3–27.4
Voids Ratio	$e$	%	49.6–54.2
Porosity	$n$	%	33–35
Liquid Limit	$W_L$	%	46–51
Plastic Limit	$W_P$	%	24–32
Plasticity Index	$I_P$	%	19–22
Liquidity Index	$I_L$	%	-41.0 to -74.0
Coefficient of compressibility	$a_v$	1/MPa	0.08
Modulus of distortion between 0.1– 0.2MPa	$E$	MPa	
- At natural water content			7.0–7.8
- After saturation			6.0–7.2
Cohesion	$C$	MPa	
- At natural water content			0.2–0.58
- After saturation			0.08–0.14
Internal friction angle	$\varphi$	Degree	
- At natural water content			25–31
- After saturation			17–23
Grain size distribution			
- 0.5 - 0.25 mm	%		-
- 0.25 - 0.1 mm	%		-
- 0.1 - 0.05 mm	%		18.26
- 0.05 - 0.01 mm	%		23.58
- 0.01 - 0.005 mm	%		11.79
- 0.005 - 0.001 mm	%		46.37

**Table 2.** Chemical composition of the studied clays.

N° of the sample	Units	Denomination							
		Na <sup>+</sup> K	Ca <sup>++</sup>	Mg <sup>++</sup>	Cl	SO <sub>4</sub>	HCO <sub>3</sub>	CO <sub>3</sub>	PH
1	%	13.88	1.49	0.33	7.3	6.59	1.19	0.63	7.5
2	%	12.18	1.69	0.33	7.30	5.42	0.89	0.59	7.8
3	%	13.88	1.29	0.25	7.30	6.84	0.89	0.39	7.8

SiO<sub>2</sub>: 52.28 %    Al<sub>2</sub>O<sub>3</sub>: 15.27 %    Na<sub>2</sub>O: 2.73 %  
 K<sub>2</sub>O: 2.59 %    MgO: 2.45 %    CaO: 6.70 %  
 TiO<sub>2</sub>: 0.79 %    MnO<sub>2</sub>: 0.10 %    Fe<sub>2</sub>O<sub>3</sub>: 6.77 %

The chemical analysis by using the method of dosage of the elements composing the swelling clay produced the results in Table 2.

**Figure 2.** Test of inflation in a free cell oedometer (without piston).**Figure 3.** Swelling test in an oedometer under load.

### 3 EXPERIMENTAL ANALYSIS AND INTERPRETATION OF THE RESULTS

The swelling rate corresponds to the relative variation in volume (%) of a sample either unloaded or subjected to a small load (generally the weight of the piston in the oedometer) when it is put into contact with water without pressure. The pressure of swelling consists of an "osmotic" component due to the difference in

the concentration of salts of the interstitial water and a "matrix" component, governed by the initial negative interstitial pressure of the sample. In most cases "matrix" component plays a major role.

Numerous methods have been proposed in the literature to evaluate the inflation potential of a soil from measures of the plasticity parameters and grain-size distribution, for example, Delage P. et al [10], Bigot G., Zerhouni M.I., [6], Zerhouni, M.I., et al. [31], Cokca E., [8], Djedid, A., et al., [12], Cuisinier O. [9], Marcial, et al. [20], Bekkouch et al., [4] [5], Erzin Y.[14], Kariuki PC. [17], Yilmaz I. [30], Nelson, J.D. et al. [24], Tomas PJ., et al. [29]. For these authors, a very high inflation potential corresponds to a free inflation (expressed in percentage) greater than 25%, a high potential to an inflation between 5 and 20%, a medium potential to an inflation between 1.5 and 5%, and a low potential to an inflation lower than 1.5%. Johnson L.D., [16], Komine H. Ogota N [18], Miao L. et al., [21], the corresponding pressures of inflation are, respectively, superior to 300 kPa (very high potential), varying between 200 and 300 kPa (high), between 100 and 200 kPa (medium) and lower than 100 kPa (low).

Several methods exist to measure the pressure of inflation in oedometers, among which we note:

- Method of Huder and Amberg (1970).
- Method of inflation with constant volume, according to ASTM norm D 2487-06.[1].
- Method of inflation or settlement under constant load, which requires several identical samples.
- Free-swelling method followed by reloading.

The experimental studies show that the percentage of soil expansion increases proportionally with its density, its liquid limit, its clay contents, its plasticity and shrinkage indexes, as well as its pressure of pre-consolidation Rao B.H. et al., [25], Thakur VKS, et al., [28], Derriche, Z., et al., [11], Ejjaouani, H., et al. [14], and Fleurau J.M., et al., [15]. A method using the slope of the line connecting the percentage swell for various values of inundation stress was described in Nelson et al. [24]. These same studies report that the pressure of the expanding soil is

inversely proportional to its natural water content.

The analysis of the experimental results [3], allowed us to draw the curves giving the variation of the swelling potential in function of time for different values the compression stresses (Fig 4). It also allowed us to establish various other relations:

Variation of the water content after expansion as a function of the different values of the compression stresses, (Fig 5),

Variation of the expansion potential as a function of the different compression stresses, (Fig.6),

Variation of the inflation potential with the water content in oedometer tests (Fig 7).

The following section is relative to the mathematical model describing the expansion mechanism, established from the oedometric tests.

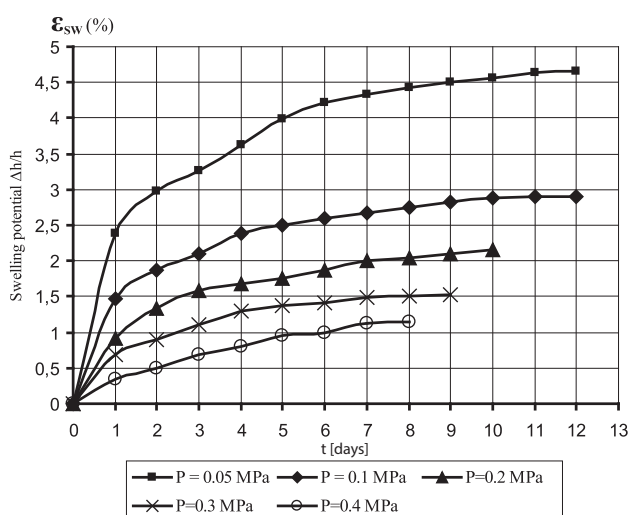


Figure 4. Variation of the swelling potential with time.

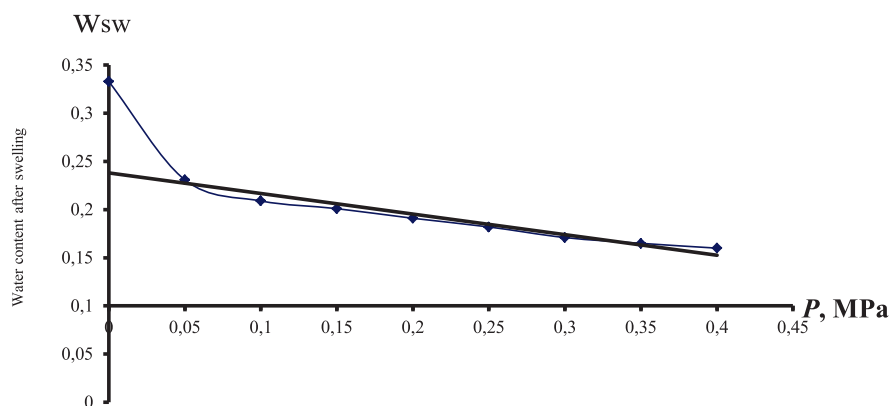


Figure 5. Variation of the water content with different load levels.

The dependence of the water content of an expansive soil, on the compression stresses (after swelling has taken place) can be expressed using an exponential function:

$$W_{sw} = W_{sw}^0 e^{(-\gamma P)} \quad (1)$$

with:  $W_{sw}^0$  : Water content without loading ( $P = 0$ )

and

$$\bar{\gamma} = \left( \frac{1}{P} \right) \ln \left( \frac{W_{sw}^0}{W_{sw}} \right) \quad (2)$$

For the interval of stresses, generally between 50 and 400 kPa in civil and industrial constructions, the dependence between the water content of the expansive soil and the compression stresses can be approximated using the relation.

$$W_{sw} = \bar{W}_{sw} - \chi P \quad (3)$$

with:

$\bar{W}_{sw}$  : Initial value of the water content given by the straight line in Figure 5.

$\chi$  : Slope of the curve in Figure 5.

The obtained values are:  $\bar{W}_{sw} = 0.24$   $\chi = 0.2 MPa^{-1}$

In this part, five models for the prediction of the clay expansion according to the geotechnical properties are used. Their formulations are given in Table 3.

Some of these models (Seed et al. and the first model of Vijayvergiya and Ghazzaly) ignore the natural water content.

These models are valid in the range of the expansion amplitude varying from the lower limit of 5 % (the point at which a soil is considered to be expansive) to the upper limit of 60% (the maximum observed expansion). This allows us to determine, from these models, minimum

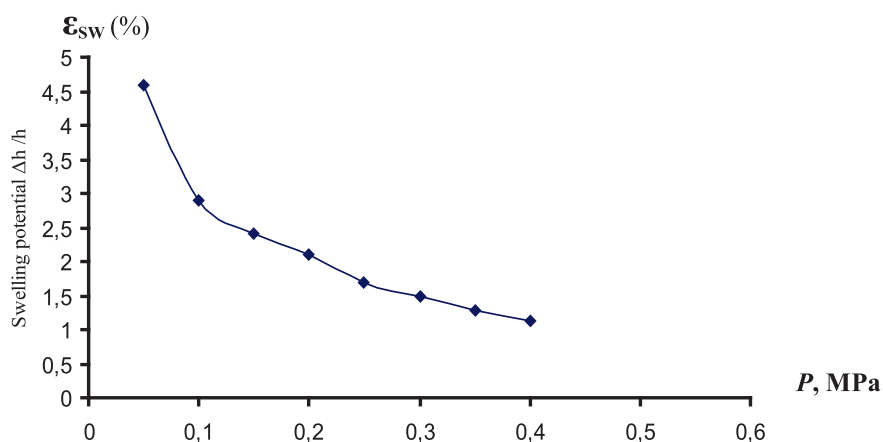


Figure 6. Variation of the inflation potential function of the different loads.

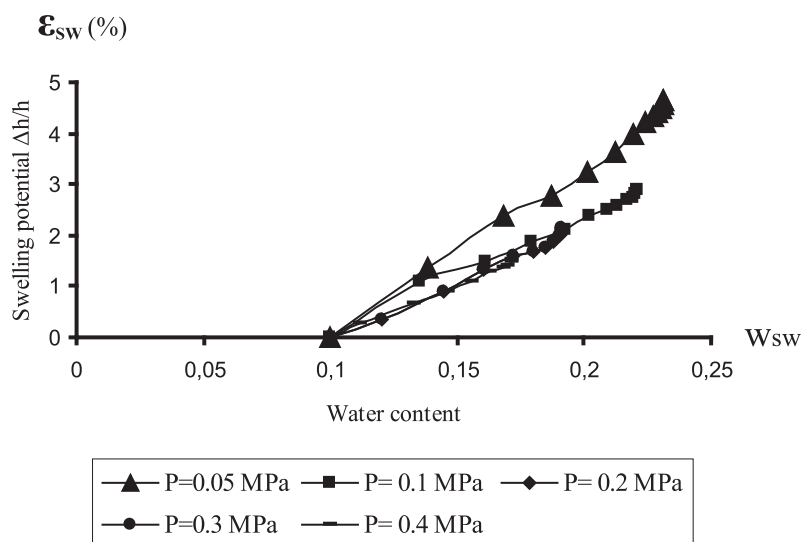


Figure 7. Variation of the swelling potential as a function of the water content.

Table 3. Tested models for predicting the inflation of the clays.

Model	Reference	Mathematical expression
Seed and al.	Seed et al. (1962)	$\varepsilon_{sw} = 2.16 \cdot 10^{-5} I_p^{2.44}$
Nayak and Christensen	Nayak and Christensen (1971)	$\varepsilon_{sw} = 2.29 \cdot 10^{-2} I_p^{1.45} \frac{C_2}{W_0} + 6.38$
Vijayvergiya and Ghazaly-1	Vijayvergiya and Ghazzaly (1973)	$\lg(\varepsilon_{sw}) = \frac{62.42\gamma_d + 0.65W_L - 130.5}{19.5}$
Vijayvergiya and Ghazaly-2	Vijayvergiya and Ghazzaly (1973)	$\lg(\varepsilon_{sw}) = \frac{0.4W_L - W_0 + 5.5}{12}$
Johnson	Johnson (1978)	$\varepsilon_{sw} = -9.18 + 1.5546I_p + 0.08424Z +$ $+ 0.1W_0 - 0.0432W_0 I_p - 0.01215Z I_p$

water content comparable to the shrinkage limit, and maximum water content comparable to the liquid limit.

Mathematically, the prediction models should constitute functional relations between a dependent variable, the percentage of the expansion, and some independent explicative variables.

#### 4 PROPOSITION OF A MODEL FOR THE EXPANSION POTENTIAL OF THE STUDIED CLAYS

The analytical results obtained show that, the variation of the expansion potential is dependent on the value of the compression stresses and the variations of the water content during the swelling process, and is governed by:

$$\varepsilon_{sw} = \varepsilon_{sw}^0 \left( 1 - \frac{P}{P_{sw}} \right) \left[ (\bar{W}_{sw} - W_0) - \chi P \right] \quad (4)$$

This analytical expression (4) gives, with good precision, the value of the swelling potential of clay soils on the basis of their properties obtained from oedometer tests. This formula contains the specific features of the expansive soils:

- $\varepsilon_{sw}$  : Expansion potential.
- $\varepsilon_{sw}^0$  : Expansion potential without loading.
- $P_{sw}$  : Expansion pressure.
- $W_0$  : Initial water content.
- $\bar{W}_{sw}$  : Initial value of water content given by the graph  $W_{sw}=f(P)$  (Fig 5).
- $\chi$  : Slope of the curve in Figure 5.

This expression for the expansion potential is different from that found in the literature, by including a good approximation of its nonlinear relation with loading. The power of the load  $P$  in equation (4) does not exceed 2.

#### 5 DESIGN SHALLOW FOUNDATIONS ON EXPANSIVE SOILS

The stress state of the soil foundation is determined by computations calculations based on rules that consider the effect of the pressure of the weight of soil  $\sigma_{z,g}$ , the weight of the structure  $\sigma_{z,p}$  and the supplementary vertical pressure due to the influence of the weight of the non-wetted soil at the boundaries of the wet surfaces.

In determining the value of the uplift of the foundations after swelling the soil depending on the swelling pressure and the stress state of the sub grade, it is necessary to

find the position of the boundaries of the swelling zones (II) (Fig. 6).

The main method for determining the boundaries of the areas of the swelling soil is as follows: In general the depth is determined, as the sum of the stresses due to the action of the weight of the soil, the live load and the additional vertical stress due to the non-wetted portion of the soil mass balancing the swelling pressure.

The probable swelling zone of the soil can be between the obtained depths.

Taking this into account, the boundaries of the areas of the swelling soil are determined from the following equations:

$$\sigma_{z,tot} = \sigma_{z,p} + \sigma_{z,g} + \sigma_{z,ad} \approx P_{sw} \quad (5)$$

Under the conditions of the in-situ pressure, the deformation due to swelling is taken through dimensional compression conditions within the swelling soil layer, where the condition  $\sigma_{z,g} \leq P$  is satisfied.

From the top of the soil layer, we divide the wet soil layer of thickness  $d_z$  into elementary slices.

The relative deformation of the swelling soils takes the following form:

$$\varepsilon_{sw} = \frac{\Delta d_z}{d_z} = \varepsilon_{sw}^0 \left( 1 - \frac{p}{P_{sw}} \right) \left[ (\bar{w}_{sw} - w_n) - \chi p \right] \quad (6)$$

Based on the results of experimental research [3], the relation (6) can be simplified as follows:

$$\varepsilon_{sw} = b_0 - b_1 p + b_2 p^2 \quad (7)$$

where:

$$b_0 = \varepsilon_{sw}^0 \left( \bar{w}_{sw} - w_n \right)$$

$$b_1 = \left( \frac{\varepsilon_{sw}^0}{P_{sw}} \right) \left[ (\bar{w}_{sw} - w_n) + \chi P_{sw} \right]$$

$$b_2 = \frac{\varepsilon_{sw}^0}{P_{sw}} \chi$$

Note that the relation (7), which incorporates the coefficients  $b_0$ ,  $b_1$  and  $b_2$ , includes important properties of the swelling soils.

Under the conditions of natural tension, changes to the compressive stresses can be determined using the classic formulas of soil mechanics  $\sigma_{z,g} = \gamma_{sw} \times z$ . Then, from (4) we will have:

$$\varepsilon_{sw} = b_0 - b_1 \gamma_{sw} z + b_2 \gamma_{sw}^2 z^2 \quad (8)$$

where:  $\gamma_{sw}$  : is the: unit weight of the swelling soils.

The final value of the absolute swelling strain of the soil under conditions of natural tension is determined using the following expression:

$$S_{sw,g} = \int_{Z_B}^{Z_H} \varepsilon_{sw}(p) dz \quad (9)$$

where:  $Z_B$  et  $Z_H$ : Represent the lower and upper limits of the swelling soil, respectively.

Substituting (8) into (9) and integrating, we obtain:

$$S_{sw,g} = b_o (Z_H - Z_B) - \frac{1}{2} b_1 \gamma_{sw} (Z_H^2 - Z_B^2) + \frac{1}{3} b_2 \gamma_{sw}^2 (Z_H^3 - Z_B^3) \quad (10)$$

In the particular case, when the stress due to the self weight of the soil, does not exceed the value of the swelling pressure across the entire thickness of the swelling soil mass and with, ( $\sigma_{z,g} < p_{sw}$ ) along with ( $0 < Z \leq H_{sw}$ ):

For:  $Z_B = 0$ ;  $Z_H = H_{sw}$ , expression (10) determines the deformation due to swelling in conditions of natural tension becomes:

$$S_{sw,g} = b_o H_{sw} - \frac{1}{2} b_1 \gamma_{sw} H_{sw}^2 + \frac{1}{3} b_2 \gamma_{sw}^2 H_{sw}^3 \quad (11)$$

where:  $H_{sw}$ : Thickness of the layer of swelling soil.

### 5.1 Computations of the uplift of the foundation of a building or structure, founded on swelling clay soils.

The general case must take into account the action of the three normal stress components. But to simplify the problem, as supported by the standards of literature, swelling deformation is determined in relation to the conditions of uniform compression.

As noted is above, the deformation due to swelling in the foundations of buildings and structures occurs in the area of swelling, where it meets the following condition:

$$\sigma_{z,tot} \leq p_{sw}$$

Hence, the equations for determining the formulas, for the uplift of the soil mass, are the transformation of expression (4) into a polynomial.

The compressive stress to the depth of the foundation is taken by Sadjin V.S. [26] as.

$$\sigma_{z,p} = \sigma_o e^{-\frac{1}{2}Z \left( \frac{1}{b} + \frac{1}{a} \right)}$$

where  $\sigma_o = \sigma_{moy} - \gamma_{sw} D_f$  (12)

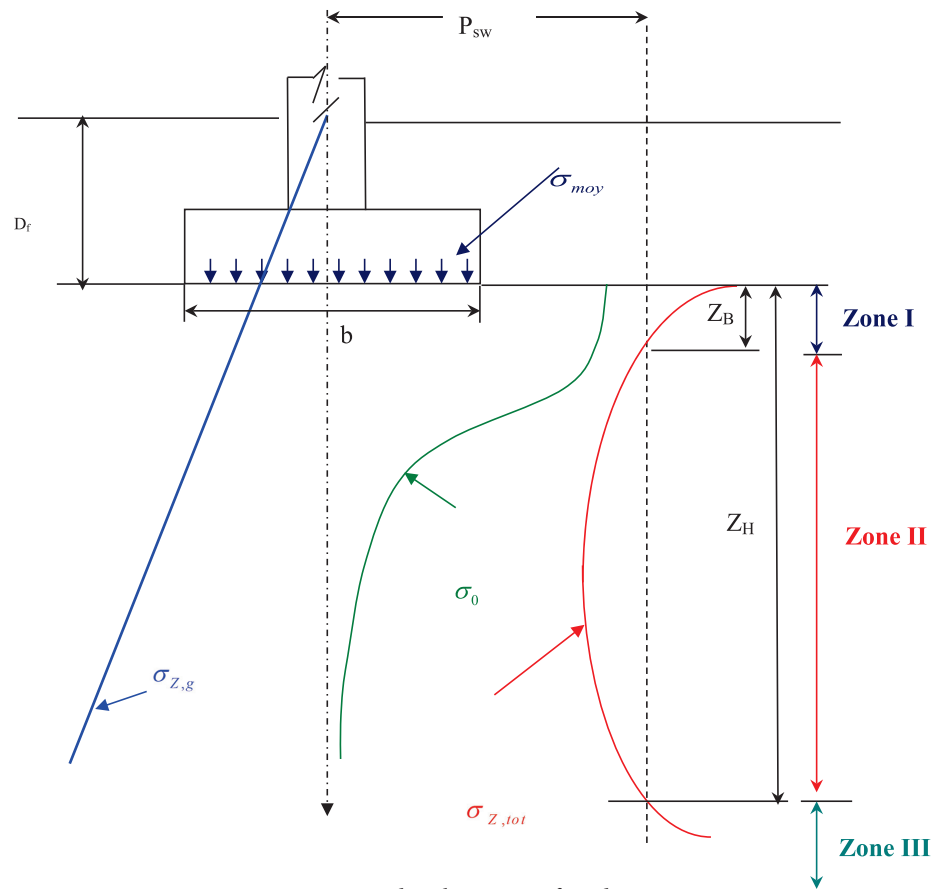


Figure 8. Stress distribution in a foundation.

$\sigma_{moy}$  : Mean stress at the bottom of the foundation.  
 $D_f$  : Depth at the bottom of the foundation.  
 $b, a$  : Dimensions of the bottom of the foundation.

Thus the sum of the pressure within the studied depth of the foundation takes the following form:

$$\sigma_{z,tot} = \rho = \gamma_{sw} (Z + D_f) + \sigma_o e^{-\frac{1}{2}Z(\frac{1}{b} + \frac{1}{a})} \quad (13)$$

Taking into account (13), the expression (4) takes the following form:

$$\varepsilon_{sw} = b_o - b_1(Z + D_f) - b_1\sigma_o e^{-\frac{1}{2}Z(\frac{1}{b} + \frac{1}{a})} + b_2 \left[ \gamma_{sw} (Z + D_f) + \sigma_o e^{-\frac{1}{2}Z(\frac{1}{b} + \frac{1}{a})} \right]^2 \quad (14)$$

The final value of the deformation due to the uplift of the foundations of buildings and structures is determined using formula (9).

Substituting (14) into (9), we obtain:

$$S_{sw,p} = \int_{Z_B}^{Z_H} \left\{ b_o - b_1\gamma_{sw} (Z + D_f) - b_1\sigma_o e^{-\frac{1}{2}Z(\frac{1}{b} + \frac{1}{a})} + b_2 \left[ \gamma_{sw} (Z + D_f) + \sigma_o e^{-\frac{1}{2}Z(\frac{1}{b} + \frac{1}{a})} \right]^2 \right\} d_z$$

After necessary transformations, we obtain the relation for the uplift of rectangular foundation bearing on the swelling soil:

$$\begin{aligned} S_{sw,p} &= (b_o - b_1\gamma_{sw} D_f + b_2\gamma_{sw}^2 D_f^2)(Z_H - Z_B) \\ &- \frac{1}{2}\gamma_{sw} (b_1 - 2b_2\gamma_{sw} D_f)(Z_H^2 - Z_B^2) + \frac{1}{3}\gamma_{sw}^2 b_2 (Z_H^3 - Z_B^3) \\ &+ \frac{2\sigma_o (b_1 - 2b_2\gamma_{sw} D_f)}{\frac{1}{b} + \frac{1}{a}} \left[ e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})Z_H} - e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})Z_B} \right] \\ &+ \frac{b_2\sigma_o^2}{\frac{1}{b} + \frac{1}{a}} \left[ e^{-(\frac{1}{b} + \frac{1}{a})Z_B} - e^{-(\frac{1}{b} + \frac{1}{a})Z_H} \right] \\ &+ 4b_2\gamma_{sw}\sigma_o \left\{ \left[ \frac{Z_B}{\frac{1}{b} + \frac{1}{a}} + \frac{2}{\left(\frac{1}{b} + \frac{1}{a}\right)^2} \right] e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})Z_B} \right. \\ &\left. - \left[ \frac{Z_H}{\frac{1}{b} + \frac{1}{a}} + \frac{2}{\left(\frac{1}{b} + \frac{1}{a}\right)^2} \right] e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})Z_H} \right\} \quad (15) \end{aligned}$$

In the particular case, when all the soil layer is a swelling clay,  $\sigma_{z,tot} \leq p_{sw}$  and in accordance with  $Z_B=0$  and  $Z_H=H_{sw}$ , formula (15) takes the form:

$$\begin{aligned} S_{sw,p} &= (b_o - b_1\gamma_{sw} D_f + b_2\gamma_{sw}^2 D_f^2)H_{sw} \\ &- \frac{1}{2}\gamma_{sw} (b_1 - 2b_2\gamma_{sw} D_f)H_{sw}^2 + \frac{1}{3}b_2\gamma_{sw}^2 H_{sw}^3 \\ &+ \frac{2\sigma_o (b_1 - 2b_2\gamma_{sw} D_f)}{\frac{1}{b} + \frac{1}{a}} \left[ e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})H_{sw}} - 1 \right] \\ &+ \frac{b_2\sigma_o^2}{\frac{1}{b} + \frac{1}{a}} \left[ 1 - e^{-(\frac{1}{b} + \frac{1}{a})H_{sw}} \right] \\ &+ 4b_2\gamma_{sw}\sigma_o \left\{ \left[ \frac{2}{\left(\frac{1}{b} + \frac{1}{a}\right)^2} - \frac{H_{sw}}{\frac{1}{b} + \frac{1}{a}} + \frac{2}{\left(\frac{1}{b} + \frac{1}{a}\right)^2} \right] e^{-\frac{1}{2}(\frac{1}{b} + \frac{1}{a})H_{sw}} \right\} \quad (16) \end{aligned}$$

### Case of a square foundation

For a square foundation  $a = b$ , and in accordance with the formula (15) we have:

$$\begin{aligned} S_{sw,p} &= (b_o - b_1\gamma_{sw} D_f + b_2\gamma_{sw}^2 D_f^2)(Z_H - Z_B) \\ &- \frac{1}{2}\gamma_{sw} (b_1 - 2b_2\gamma_{sw} D_f)(Z_H^2 - Z_B^2) \\ &+ \frac{1}{3}b_2\gamma_{sw}^2 (Z_H^3 - Z_B^3) + b\sigma_o (b_1 - 2b_2\gamma_{sw} D_f) \left( e^{-\frac{Z_H}{b}} - e^{-\frac{Z_B}{b}} \right) \\ &+ \frac{1}{2}bb_2\sigma_o^2 \left( e^{-\frac{2Z_B}{b}} - e^{-\frac{2Z_H}{b}} \right) \\ &+ 2bb_2\gamma_{sw}\sigma_o \left[ (Z_B + b) e^{-\frac{Z_B}{b}} - (Z_H + b) e^{-\frac{Z_H}{b}} \right] \quad (17) \end{aligned}$$

In the particular case, when the entire thickness of the soil layer is a swelling clay,  $\sigma_{z,tot} \leq p_{sw}$ ; and  $Z_B=0$ ,  $Z_H=H_{sw}$ ; and formula (17) takes the form:

$$\begin{aligned} S_{sw,p} &= (b_o - b_1\gamma_{sw} D_f + b_2\gamma_{sw}^2 D_f^2)H_{sw} \\ &- \frac{1}{2}\gamma_{sw} (b_1 - 2b_2\gamma_{sw} D_f)H_{sw}^2 + \frac{1}{3}b_2\gamma_{sw}^2 H_{sw}^3 \\ &+ b\sigma_o (b_1 - 2b_2\gamma_{sw} D_f) \left( e^{-\frac{H_{sw}}{b}} - 1 \right) + \frac{1}{2}bb_2\sigma_o^2 \left( 1 - e^{-\frac{2H_{sw}}{b}} \right) \\ &+ 2bb_2\gamma_{sw}\sigma_o \left[ b - (H_{sw} + b) e^{-\frac{H_{sw}}{b}} \right] \quad (18) \end{aligned}$$



### Case of continuous footing

For a continuous footing ( $a \rightarrow \infty$ ) formula (15) takes the form:

$$\begin{aligned}
 S_{sw,p} = & (b_o - b_1 \gamma_{sw} D_f + b_2 \gamma_{sw}^2 D_f^2) (Z_H - Z_B) \\
 & - \frac{1}{2} \gamma_{sw} (b_1 - 2 b_2 \gamma_{sw} D_f) (Z_H^2 - Z_B^2) \\
 & + \frac{1}{3} b_2 \gamma_{sw}^2 (Z_H^3 - Z_B^3) \\
 & + 2 b \sigma_o (b_1 - 2 b_2 \gamma_{sw} D_f) \left( e^{-\frac{Z_H}{2b}} - e^{-\frac{Z_B}{2b}} \right) \\
 & + b b_2 \sigma_o^2 \left( e^{-\frac{Z_B}{b}} - e^{-\frac{Z_H}{b}} \right) \\
 & + 4 b b_2 \gamma_{sw} \sigma_o \left[ (Z_B + 2b) e^{-\frac{Z_B}{2b}} - (Z_H + 2b) e^{-\frac{Z_H}{2b}} \right]
 \end{aligned} \tag{19}$$

In the particular case, when the entire thickness of the soil layer is a swelling clay.  $\sigma_{z,tot} \leq p_{sw}$ ; Formula (19) takes the form:

$$\begin{aligned}
 S_{sw,p} = & (b_o - b_1 \gamma_{sw} D_f + b_2 \gamma_{sw}^2 D_f^2) H_{sw} \\
 & - \frac{1}{2} \gamma_{sw} (b_1 - 2 b_2 \gamma_{sw} D_f) H_{sw}^2 + \frac{1}{3} b_2 \gamma_{sw}^2 H_{sw}^3 \\
 & + 2 b \sigma_o (b_1 - 2 b_2 \gamma_{sw} D_f) \left( e^{-\frac{H_{sw}}{2b}} - 1 \right) + b b_2 \sigma_o^2 \left( 1 - e^{-\frac{H_{sw}}{b}} \right) \\
 & + 4 b b_2 \gamma_{sw} \sigma_o \left[ 2b - (H_{sw} + 2b) e^{-\frac{H_{sw}}{2b}} \right]
 \end{aligned} \tag{20}$$

### Case of a circular foundation

For circular foundations with diameter  $d$ , ( $b = a = 0.886 d$ ), formula (15) takes the form:

$$\begin{aligned}
 S_{sw,p} = & (b_o - b_1 \gamma_{sw} D_f + b_2 \gamma_{sw}^2 D_f^2) (Z_H - Z_B) \\
 & - \frac{1}{2} \gamma_{sw} (b_1 - 2 b_2 \gamma_{sw} D_f) (Z_H^2 - Z_B^2) + \frac{1}{3} b_2 \gamma_{sw}^2 (Z_H^3 - Z_B^3) \\
 & + 0.886 d \sigma_o (b_1 - 2 b_2 \gamma_{sw} D_f) \left( e^{-\frac{1.129 Z_H}{d}} - e^{-\frac{1.129 Z_B}{d}} \right) \\
 & + 0.443 d \sigma_o^2 \left( e^{-\frac{2.257 Z_B}{d}} - e^{-\frac{2.257 Z_H}{d}} \right) \\
 & + 1.772 b_2 \gamma_{sw} \sigma_o \left[ (Z_B + 0.886 d) e^{-\frac{1.129 Z_B}{d}} \right. \\
 & \left. - (Z_H + 0.886 d) e^{-\frac{1.129 Z_H}{d}} \right]
 \end{aligned} \tag{21}$$

In the particular case, when the entire thickness of the soil layer is a swelling clay.  $\sigma_{z,tot} \leq p_{sw}$ ; formula (21) takes the form:

$$\begin{aligned}
 S_{sw,p} = & (b_o - b_1 \gamma_{sw} D_f + b_2 \gamma_{sw}^2 D_f^2) H_{sw} \\
 & - \frac{1}{2} \gamma_{sw} (b_1 - 2 b_2 \gamma_{sw} D_f) H_{sw}^2 + \frac{1}{3} b_2 \gamma_{sw}^2 H_{sw}^3 \\
 & + 0.886 d \sigma_o (b_1 - 2 b_2 \gamma_{sw} D_f) \left( e^{-\frac{1.129 H_{sw}}{d}} - 1 \right) \\
 & + 0.443 d b_2 \sigma_o^2 \left( 1 - e^{-\frac{2.257 H_{sw}}{d}} \right) \\
 & + 1.772 d b_2 \gamma_{sw} \sigma_o \left[ 0.886 d - (H_{sw} + 0.886 d) e^{-\frac{1.129 H_{sw}}{d}} \right]
 \end{aligned} \tag{22}$$

## 6 APPLICATION

### For a continuous footing

Data:

$$\begin{aligned}
 \gamma_{sw} = & 2.125 \text{ t/m}^3 ; D_f = 1 \text{ m} ; b = 1 \text{ m} ; H_{sw} = 12 \text{ m} ; \\
 \sigma_o = & 10 \text{ t/m}^2 ; w_n = 14.1\% ; \varepsilon_{sw}^o = 22\%
 \end{aligned}$$

Computations of the coefficients:  $b_0$ ,  $b_1$  and  $b_2$ :

$$b_o = \varepsilon_{sw}^o \left( \bar{w}_{sw} - w_n \right) = 0.22 (0.212 - 0.141) = 0.01562$$

$$\begin{aligned}
 b_1 = & \left( \frac{\varepsilon_{sw}^o}{p_{sw}} \right) \left[ (\bar{w}_{sw} - w_n) + \chi p_{sw} \right] \\
 = & \left( \frac{0.22}{2.18} \right) \left[ (0.212 - 0.141) + 0.007 \times 2.18 \right] \\
 = & 8.7 \cdot 10^{-3} \text{ cm}^2 / \text{daN} = 8.7 \cdot 10^{-4} \text{ m}^2 / \text{t}
 \end{aligned}$$

$$\text{With: } \chi = \frac{0.212 - 0.198}{2 - 0} = 0.007 \text{ cm}^2 / \text{daN}$$

$$\begin{aligned}
 b_2 = & \frac{\varepsilon_{sw}^o}{p_{sw}} \chi = \frac{0.22}{2.18} \times 0.007 = 7.06 \cdot 10^{-4} \text{ cm}^4 / \text{daN}^2 \\
 = & 7.06 \cdot 10^{-6} \text{ m}^4 / \text{t}^2
 \end{aligned}$$

By substituting the values of the coefficients  $b_0$ ,  $b_1$  and  $b_2$  into formula (17), we will obtain the uplift deformation of the foundation:

$$S_{sw,p} = (0.01562 - 8.7 \cdot 10^{-4} \times 2.125 \times 1 + 7.06 \cdot 10^{-6} \times 2.125^2 \times 1^2)12 - \frac{1}{2} \times 2.125 \times (8.7 \cdot 10^{-4} - 2 \times 7.06 \cdot 10^{-6} \times 2.125 \times 1)12^2 + \frac{1}{3} \times 7.06 \cdot 10^{-6} \times 2.125^2 \times 12^3 + 2 \times 1 \times 10(8.7 \cdot 10^{-4} - 2 \times 7.06 \cdot 10^{-6} \times 2.125 \times 1) \left( e^{\frac{12}{2 \times 1}} - 1 \right) + 1 \times 7.06 \cdot 10^{-6} \times 10^2 \left( 1 - e^{\frac{12}{2 \times 1}} \right) + 4 \times 1 \times 7.06 \cdot 10^{-6} \times 2.125 \times 10 \left[ 2 \times 1 - (12 + 2 \times 1) e^{\frac{12}{2 \times 1}} \right] = 0.0404 m$$

$$\Rightarrow S_{sw,p} = 0.0404 \text{ m} = 4.04 \text{ cm}$$

If we take  $b = 2 \text{ m}$ , the value of the uplifting becomes:

$$\Rightarrow S_{sw,p} = 0.028 \text{ m} = 2.80 \text{ cm}$$

### Case of a square foundation

Data:

$$\gamma_{sw} = 2.125 \text{ t/m}^3 ; D_f = 1 \text{ m} ; b = 1 \text{ m} ; H_{sw} = 12 \text{ m} ; \sigma_o = 10 \text{ t/m}^2 ; w_n = 14.1\% ; \varepsilon_{sw}^o = 22\%$$

Computing the coefficients:  $b_0$ ,  $b_1$  and  $b_2$ :

$$b_0 = \varepsilon_{sw}^o \left( \bar{w}_{sw} - w_n \right) = 0.22 (0.212 - 0.141) = 0.01562$$

$$b_1 = \left( \frac{\varepsilon_{sw}^o}{p_{sw}} \right) \left[ \left( \bar{w}_{sw} - w_n \right) + \chi p_{sw} \right]$$

$$= \left( \frac{0.22}{2.18} \right) \left[ (0.212 - 0.141) + 0.007 \times 2.18 \right]$$

$$= 8.7 \cdot 10^{-3} \text{ cm}^2 / \text{daN} = 8.7 \cdot 10^{-4} \text{ m}^2 / \text{t}$$

$$\text{With: } \chi = \frac{0.212 - 0.198}{2 - 0} = 0.007 \text{ cm}^2 / \text{daN}$$

$$b_2 = \frac{\varepsilon_{sw}^o}{p_{sw}} \chi = \frac{0.22}{2.18} \times 0.007 = 7.06 \cdot 10^{-4} \text{ cm}^4 / \text{daN}^2$$

$$= 7.06 \cdot 10^{-6} \text{ m}^4 / \text{t}^2$$

By substituting the values of the coefficients  $b_0$ ,  $b_1$  and  $b_2$  into formula (5) we will evaluate the uplift deformation of the foundation.

$$S_{sw,p} = (0.01562 - 8.7 \cdot 10^{-4} \times 2.125 \times 1 + 7.06 \cdot 10^{-6} \times 2.125^2 \times 1^2)12 - \frac{1}{2} \times 2.125 \times (8.7 \cdot 10^{-4} - 2 \times 7.06 \cdot 10^{-6} \times 2.125 \times 1)12^2 + \frac{1}{3} \times 7.06 \cdot 10^{-6} \times 2.125^2 \times 12^3 + 1 \times 10(8.7 \cdot 10^{-4} - 2 \times 7.06 \cdot 10^{-6} \times 2.125 \times 1) \left( e^{\frac{12}{1}} - 1 \right) + \frac{1}{2} \times 1 \times 7.06 \cdot 10^{-6} \times 10^2 \left( 1 - e^{\frac{2 \times 12}{1}} \right) + 2 \times 1 \times 7.06 \cdot 10^{-6} \times 2.125 \times 10 \left[ 1 - (12 + 1) e^{\frac{12}{1}} \right] = 0.0477 m$$

$$\Rightarrow S_{sw,p} = 0.0477 \text{ m} = 4.77 \text{ cm}$$

## 7 CONCLUSIONS

Based on an analysis of the experimental results presented here, the following conclusions can be drawn. A theoretical model was formulated to describe the swelling potential of clay soils on the basis of their features obtained from oedometer tests. For expansive soils, designers are only interested in the measurable quantities which are generally the pressure and the amplitude of the expansion. A model has been proposed for the expansion potential of the studied clays. On the basis of the non linearity of the physical mechanism of distortion, the method developed in the present work, relates the phenomenon of expansion with the distribution of the contact pressure, the soil properties, as well as the type and the value of the external loading and allows the design of lifting shallow foundations on expansive soils,

On the basis of the nonlinearity of the deformation mechanism of swelling clays, the method developed in the present work permits a calculation of the lifting surface of different types of foundations on expansive soils.

The rise of the foundations, based on expansive soils, depends not only on the expansiveness of the clay soils but also on the magnitude of the external loads transmitted to the ground by the buildings.

The obtained results open up new perspectives for methods to design foundations on expansive clay soils.

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