University of Batna 2

Institute of Industrial Hygiene and Safety

Module : Math 1 (L1)

Dr.Bayarassou

Chapter 2 : Sets, Relations and Functions

Introduction

This course aims to cover some basic of set theory and it's properties. Throughout this chapter, we will learn about sets, relations and functions. we will infer that sets and relations are interconnected with each other (relations define the connection between the two given sets). After that, we will delve further in relations where we define another kind that can be considered a function.

Contents

1 Sets

Definition 1.1

A set is a well-defined collection of distinct objects called elements of the set.

- Set is denoted by capital letters $A, B, C \ldots$.
- Elements of set are denoted by small letters $a, b, c \ldots$.
- In sets, order and repetition of elements don't matter.
- $\mathbf{a} \in \mathbf{A}$: a belongs to A or a is an element of A
- $\mathbf{b} \notin \mathbf{B}$: b does not belong to B or b is not an element of B

1.1 Representation of sets

There are three main ways to identify a set:

Roster Method

The set can be defined by listing all its elements, separated by commas and enclosed within braces.

Example

- $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Set Builder Method

The set can be defined by describing the elements using mathematical statements. $A = \{x \mid P(x)\}$

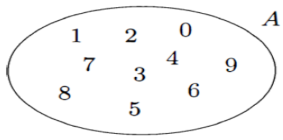
Example

- $A = \{x \in \mathbb{N} \mid 0 \leqslant x \leqslant 9\}$

Venn Diagram

A Venn Diagram is a pictorial representation of the relationships between sets.

Example



1.2 Types of sets

The sets are further categorised into different types, based on elements or types of elements. Therefore, before we state different types of sets, it is crusial to know that the number of different elements on the set is called its **cardinality** and it is denoted by $card(A)$.

Now, most types of sets are :

Empty set : A set which does not contain any element is called an empty set or void set or null set. It is denoted by $\{\}$ or $\varnothing$.

Singleton set : A set which contains a single element. For example, $A = \{4\}$

Finite sets : A set which consists of a definite number of elements. For example, $A = \{1, 2, 4, 6\}$

- A set which is not finite is called an infinite set. For example, $\mathbb{N}, \mathbb{Z}$.

Equal sets : The two sets are equal, if they have exactly the same elements. More formally, for any sets A and B , $A = B$ if and only if $\forall x,\ x \in A \Longleftrightarrow x \in B$.
Example: $\{1, 2, 4, 6\} = \{6, 2, 4, 1\}$

Equivalent sets : The two sets are equivalent, if the number of elements is the same for two different sets. For example, $A = \{1, 2, 3, 4\}$ and $B = \{9, a, 3, w\}$ are equivalent.

Subsets : The set A is a subset of B denoted by $A \subseteq B$ if and only if every element of A is olso an element of B. More formally, $A \subseteq B$ if and only if $\forall x, x \in A \implies x \in B$.

- If $A \subseteq B$, and $A \neq B$, then A is said to be a proper subset of B and it is denoted by $A \subset B$.
- $A \nsubseteq B$ is the negation of $A \subseteq B$, More formally,

$$
\begin{aligned}
A \nsubseteq B &\Longleftrightarrow \overline{A \subseteq B} \\
&\Longleftrightarrow \overline{\forall x, x \in A \implies x \in B} \\
&\Longleftrightarrow \exists x,\ x \in A \wedge x \notin B
\end{aligned}
$$

- The empty set is a subset of every set, including the empty set itself.

Examples : Let $A = \{1, 2, 4, 7\}$, $B = \{1, 2\}$ and $C = \{1, 2, 3\}$. So $B \subseteq A$ and $C \nsubseteq A$

Power Sets : If A is any set, the power set of A is the set of all subsets of A including the empty set and A itself. It is denoted by $\mathcal{P}(A)$. In other word, $\mathcal{P}(A) = \{X \mid X \subseteq A\}$

- The number of elements in the power set of A is 2^n, where n is the number of elements in set A. That is $card(\mathcal{P}(A)) = 2^n$, where $n = card(A)$
- $B \in \mathcal{P}(A)$ means $B \subseteq A$

Example : Let $A = \{1, 2\}$, then the power set of A given as: $\mathcal{P}(A) = \{\varnothing, \{1\}, \{2\}, \{1, 2\}\}$
$card(A) = 2$, and $card(\mathcal{P}(A)) = 2^{card(A)} = 2^2 = 4$.

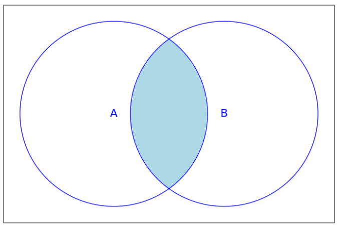
1.3 Set Operations

Let E be a set, and $A, B \in \mathcal{P}(E)$:

Intersection: The intersection of A and B denoted by $A \cap B$ is the set of all elements that are in both A and B. That is, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

- $A \cap B \subseteq A$
- $A \cap B \subseteq B$

Example : $\{1,2,3,4\} \cap \{3,4,5,6\} = \{3,4\}$

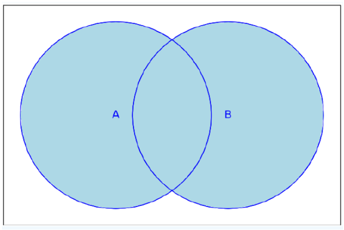


Venn Diagram for $A \cap B$

Union: The union of A and B denoted by $A \cup B$ is the set of all elements that are in A or in B or in both A and B. That is, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

- $A \subseteq A \cup B$
- $B \subseteq A \cup B$

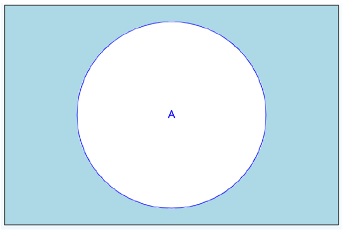
Example : $\{1,2,3,4\} \cup \{3,4,5,6\} = \{1,2,3,4,5,6\}$



Venn Diagram for $A \cup B$

Complement: The complement of A relative to E denoted by $\complement_E(A)$ is the set of elements that are in E and not in A. That is, $\complement_E(A) = \{x \in E \text{ , } x \notin A\}$.

- $x \in \complement_E(A) \iff x \notin A$

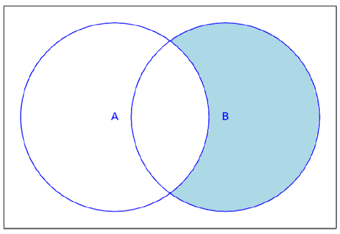


Venn Diagram for $\complement_E(A)$

Difference: The difference of A and B is the set of elements that are in A but not in B.
That is, $A \backslash B = \{x \mid x \in A \wedge x \notin B\}$.

- $A \subseteq B \iff A \backslash B = \varnothing$

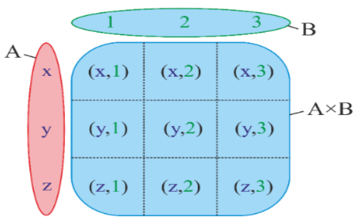
Example : $\{1,2,3,4\} \backslash \{3,4,5,6\} = \{1,2\}$.



Venn Diagram for $A \backslash B$

Cartesian product: The Cartesian product of A and B denoted by $A \times B$ is the set of all ordered pairs.
That is $A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$.
Exapmle : Let $A = \{1,2,3\}$ and $B = \{4,5\}$.
Then $A \times B = \{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$.

- $card(A \times B) = card(A) \times card(B)$



Representation of $A \times B$

Properties of sets

Let E be a set, and $A, B \in \mathcal{P}(E)$:

1. **C**ommutative property
   (a) $A \cup B = B \cup A$
   (b) $A \cap B = B \cap A$
2. **A**ssociative property
   (a) $A \cap (B \cap C) = (A \cap B) \cap C$
   (b) $A \cup (B \cup C) = (A \cup B) \cup C$
3. **D**istributive property
   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
4. **D**eMorgan's property
   (a) $\complement_E(A \cup B) = \complement_E(A) \cap \complement_E(B)$
   (b) $\complement_E(A \cap B) = \complement_E(A) \cup \complement_E(B)$

1.4 Proofs on sets

Set theory is a branch of mathematical logic. Therefore, we will focus on proof methods to show set relationships (students may need to refer back to Chapter 1 to refresh their memory) . We will learn how to prove one set is a subset of another and how to prove two sets are equal:

1. To prove that $A \subseteq B$, we must show that if $x \in A$, then $x \in B$.
2. To prove that $A = B$, we must show that $A \subseteq B$ and $B \subseteq A$.

Therefore, we just need to prove the implication statement using direct proof, proof by contrapositive and even then proof by contradiction.

Example of Proof

- Prove that if A, B and C are sets, then $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  Proof : First, we prove $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.
  We use direct proof. Assume $x \in A \cap (B \cup C)$. Based on this assumption, we must now show that $x \in (A \cap B) \cup (A \cap C)$
  Thus :
  $$\begin{aligned} x \in A \cap (B \cup C) &\implies x \in A \wedge x \in (B \cup C) \\ &\implies x \in A \wedge (x \in B \vee x \in C) \\ &\implies (x \in A \wedge x \in B) \vee (x \in A \wedge x \in C) \\ &\implies (x \in A \cap B) \vee (x \in A \cap C) \\ &\implies x \in (A \cap B) \cup (A \cap C) \end{aligned}$$

  We've shown that $x \in A \cap (B \cup C)$ implies $x \in (A \cap B) \cup (A \cap C)$, so therefore $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.
  Next, to show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$, we use the same argument as above. ■

Exercise : Let E be a set, and $A, B \in \mathcal{P}(E)$. Show that,

1. $A \subseteq B \implies \mathcal{C}_E(B) \subseteq \mathcal{C}_E(A)$

2 Relations

Definition 2.1

Let A and B are two non empty sets. A relation $\mathcal{R}$ from A to B is a subset of $A \times B$.

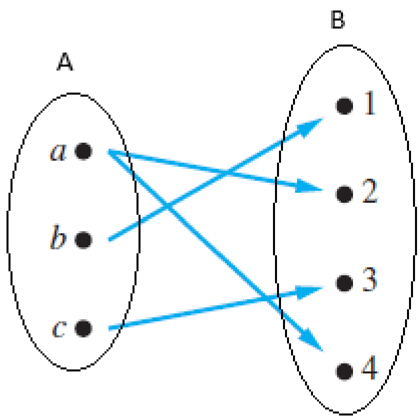
- In other words, a relation from A to B is a set R of ordered pairs where the first element of each ordered pair comes from A and the second element comes from B.
- If $(a, b) \in \mathcal{R}$, then we say that a is related to b by $\mathcal{R}$.
- $a\mathcal{R}b$ write to express that $(a, b) \in \mathcal{R}$ and $a \not{\mathcal{R}} b$ to express that $(a, b) \notin \mathcal{R}$.
- A relation on a set A is a subset of $A \times A$.

Example :
Let A be the set $\{1, 2, 3, 4\}$ and the relation $\mathcal{R}$ defined on a set A by $\mathcal{R} = \{(a, b) \mid a \text{ devides } b\}$. Thus,

$$\mathcal{R} = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

Representing relations : Relations can be represented graphically, using arrows to represent ordered pairs. In other words, to connecte the domain and codomain elements. A relation can relate an element with more than one value.
Let $E = \{a, b, c, \}$, $F = \{1, 2, 3, 4\}$ and the relation defined as

$$\mathcal{R} = \{(a, 2), (a, 4), (b, 1), (c, 3)\}$$



2.1 Properties of Relations

There are several properties that are used to classify relations on a set. We will introduce the most important of these here.

1. **Reflexive :** A relation $\mathcal{R}$ on a set A is called *reflexive* if $(a,a) \in \mathcal{R}$ for every element $a \in A$.
   In other words, $\forall a \in A,\ (a,a) \in \mathcal{R}$
   Examples : Consider the following relations on $\{1,2,3,4\}$
   $\mathcal{R}_1 = \{(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)\}$
   $\mathcal{R}_2 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$
   The relation $\mathcal{R}_1$ is reflexive because it both contain all pairs of the form (a,a), while $\mathcal{R}_2$ is not reflexive.

2. **Symmetric :** A relation $\mathcal{R}$ on a set A is called *symmetric* if $(b,a) \in \mathcal{R}$ whenever $(a,b) \in \mathcal{R}$, for all $a,b \in A$.
   In other words, $\forall a \in A, \forall b \in A,\ (a,b) \in \mathcal{R} \implies (b,a) \in \mathcal{R}$
   Examples : Consider the above example :
   $\mathcal{R}_1$ is symmetric, because in each case (b,a) belongs to the relation whenever (a,b) does.
   $\mathcal{R}_2$ is not symmetric, because there exists $(3,4) \in \mathcal{R}_2$ but $(4,3) \notin \mathcal{R}_2$.

3. **Antisymmetric :** A relation $\mathcal{R}$ on a set A is called *antisymmetric* if $(a,b) \in \mathcal{R}$ and $(b,a) \in \mathcal{R}$ then $a = b$, for all $a,b \in A$.
   In other words, $\forall a \in A, \forall b \in A,\ (a,b) \in \mathcal{R} \wedge (b,a) \in \mathcal{R} \implies a = b$
   Examples : Consider the following relations on $\{1,2,3,4\}$
   $\mathcal{R}_3 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$
   $\mathcal{R}_4 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$
   $\mathcal{R}_3$ is antisymmetric, because there is no pair of elements a and b with $a \neq b$ such that both (a,b) and (b,a) belong to the relation.
   $\mathcal{R}_4$ is not antisymmetric, because there is $(1,2) \in \mathcal{R}_4$ and $(2,1) \in \mathcal{R}_4$ but $1 \neq 2$.

4. **Transitive :** A relation $\mathcal{R}$ on a set A is called *transitive* if whenever $(a,b) \in \mathcal{R}$ and $(b,c) \in \mathcal{R}$, then $(a,c) \in \mathcal{R}$, for all $a,b,c \in A$.
   In other words, $\forall a,b,c \in A,\ (a,b) \in \mathcal{R} \wedge (b,c) \in \mathcal{R} \implies (a,c) \in \mathcal{R}$
   Examples : $\mathcal{R}_3$ is transitive because if (a,b) and (b,c) belong to $\mathcal{R}_3$, then (a,c) also does.
   $\mathcal{R}_1$ is not transitive because $(4,1)$ and $(1,2)$ belong to $\mathcal{R}_1$, but $(4,2)$ does not.

Exercise : Determine whether the relation $\mathcal{R}$ is reflexive, symmetric, antisymmetric, and/or transitive.

$$\forall x,y \in \mathbb{R}, \quad x\mathcal{R}y \iff x^2 - y^2 = x - y$$

2.2 Equivalence Relations

A relation $\mathcal{R}$ is said to be an equivalence relation if it is simultaneously reflexive, symmetric, and transitive on A.

Equivalence Classes :

Let $\mathcal{R}$ be an equivalence relation on A. Let $a \in A$,
the equivalence class of a denoted by $C(a)$ is defined as the set of all those point of A which are related to a under the relation $\mathcal{R}$. ie $C(a) = \{x \in A \,|\, x\mathcal{R}a\}$

- Let $\mathcal{R}$ be an equivalence relation on A and $a, b \in A$, where $a\mathcal{R}b$, then a and b have the same equivalence class.

Example :

Let $A = \{1,2,3,4,5\}$, and $\mathcal{R}$ is an equivalence relation on A defined as $\mathcal{R} = \{(a,b) \,|\, a+b \text{ is even}\}$.
Thus $C(1) = \{1,3,5\}$ and $C(2) = \{2,4\}$

2.3 Partial Order Relation

A relation $\mathcal{R}$ is said to be a partial order relation if it is simultaneously reflexive, antisymmetric, and transitive on A. set on which there is a partial order relation defined is called a *poset*.

Example :

- $\leqslant$ is a partial order relation on $\mathbb{N}, \mathbb{Z}$ and $\mathbb{R}$.
- The relation $\mathcal{R} = \{(a,b) \,|\, a \subseteq b\}$ define on the power set of $A = \{1,2,3\}$ is a partial order relation.

Remark 1 *The word "partial" in partial order indecates that not every pairs of elements in a set is comparable. (two elements a and b of poset A are called comparable if $a\mathcal{R}b$ or $b\mathcal{R}a$).*

Example : In the poset, $(\mathcal{P}(A), \subseteq)$, where $A = \{1,2,3\}$
$\{1,3\} \nsubseteq \{2\}$ and $\{2\} \nsubseteq \{1,3\}$. Therefore, $\{1,3\}$ and $\{2\}$ are not comparable. On the other hand, $\{1\} \subseteq \{1,3\}$ are comparable.

Elements of Poset

Let a poset $(A, \mathcal{R})$ where A is some arbitrary set and relation $\mathcal{R}$ is a partial order relation defined on a set A.

Minimal Element:

An element x of a set A is called a minimal element if there is no $y \in A$ such that, $y\mathcal{R}x$ and $y \neq x$.
In other words,

$$\forall y \in A, \ y\mathcal{R}x \implies y = x$$

- A poset can have more than one minimal element.

Maximal Element:
An element x of a set A is called a maximal element if there is no $y \in A$ such that, $x\mathcal{R}y$ and $x \neq y$. In other words,

$$\forall y \in A,\ x\mathcal{R}y \implies y = x$$

- A poset can have more than one maximal element.

Example 1 :
Consider the set $A = \{\{1,2,3\},\{4\},\{5,6\},\{1,2,3,5,6\}\}$, the relation defined on A is $\subseteq$, then

The minimal elements are $\{1,2,3\}$, $\{4\}$ and $\{5,6\}$. i.e., These do not contain proper subsets which are members of this collection.

The maximal elements are $\{4\}$ and $\{1,2,3,5,6\}$. i.e., these are not proper subsets of other sets which are members of this collection.

Example 2 :
Which elements of the poset $(\{2,4,5,10,12,20,25\},|)$ are maximal, and which are minimal?

The maximal elements are $12, 20$, and 25, and the minimal elements are 2 and 5.

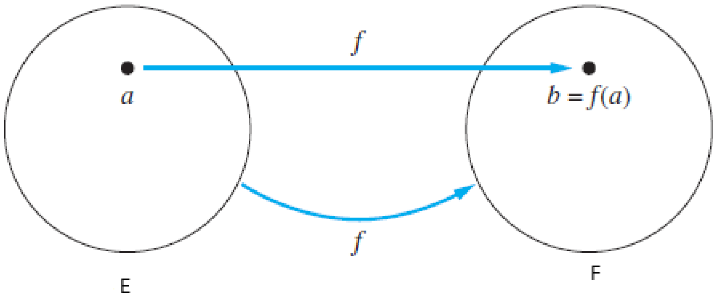
3 Functions

Definition 3.1

A function from a set E into a set F, denoted $f : E \longrightarrow F$ is a relation from E into F such that each element of E is related to exactly one element of the set F.

- Functions are sometimes also called Maps or Mappings.
- We say that E is the domain of f and F is the codomain of f.
- The range of f is the set of all images of elements of E.
- For a function, It is customary to use the $y = f(x)$ notation instead of the xfy notation.
- If f maps an element of the domain to zero elements or more than one element of the co-domain, then f is not well-defined.

Representation : Let $f : E \longrightarrow F$ be a function from E to F, we say that f maps E to F.

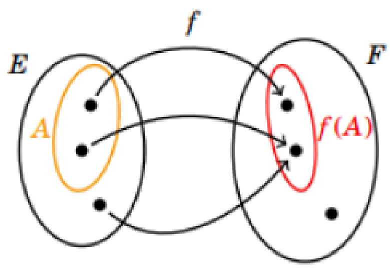


When $f(a) = b$, we say that b is the image of a and a is a preimage of b. The domain of f is given as

$$\mathcal{D}_f = \left\{x \mid \exists y \in F,\ y = f(x)\right\}$$

3.1 Direct Image and Inverse Image

Direct Image : Let f be a function from E to F and let A be a subset of E. The image of A under the function f is the subset of F that consists of the images of the elements of A. We denote the image of A by $f(A)$, and



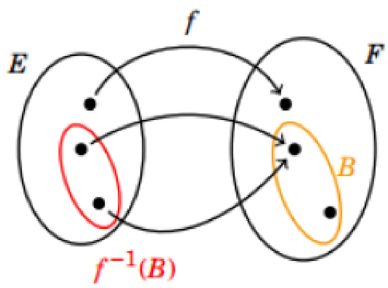
$$f(A) = \left\{f(x) \mid x \in A\right\}$$

Example : Let $E = \left\{a, b, c, d\right\}$, $F = \left\{\alpha, \beta, \gamma, \epsilon\right\}$; and define the function $f : E \longrightarrow F$, with $f(a) = f(b) = \beta$, $f(d) = \gamma$, $f(c) = \epsilon$, then
$f(\{b\}) = \{\beta\}$, $f(\{a, c\}) = \{\beta, \epsilon\}$, $f(\{a, c, d\}) = \{\beta, \epsilon, \gamma\}$.

Inverse Image : Let f be a function from E to F and let B be a subset of F. The inverse image of B under the function f is the subset of E that consists of the preimages of the elements of B. We denote the inverse image of B by $f^{-1}(B)$, and

$$f^{-1}(B) = \left\{x \in E \mid f(x) \in B\right\}$$

• $f^{-1}(B)$ is the part of E formed by the preimages of the elements of B. We can therefore write



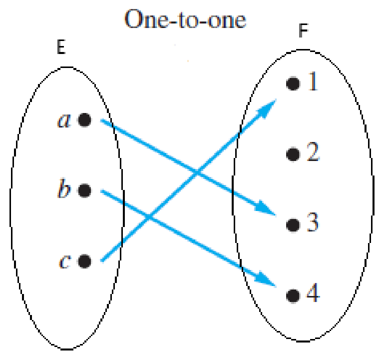
$$x \in f^{-1}(B) \iff f(x) \in B$$

Example Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ such that $f(x) = 4x + 5$ and $B = \left\{x \in \mathbb{R} \,|\, x > 0\right\}$

$$\begin{aligned} f^{-1}(B) &= \left\{x \in \mathbb{R} \,|\, f(x) > 0\right\} \\ &= \left\{x \in \mathbb{R} \,|\, 4x + 5 > 0\right\} \\ &= \left\{x \in \mathbb{R} \,|\, x > \frac{-4}{5}\right\} \end{aligned}$$

3.2 Properties of Functions

Let E and F two sets, and $f: E \longrightarrow F$ a function



Injective : f is injective (one-to-one) if every element in E is mapped to a unique element in F. More formally

$$\forall x_1, x_2 \in E, f(x_1) = f(x_2) \implies x_1 = x_2$$

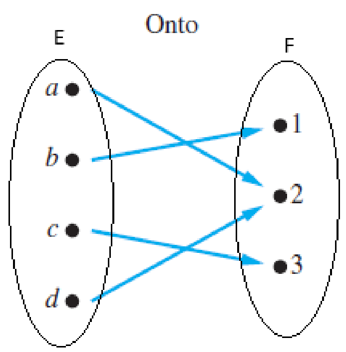
Example

Prove that $f: [0, +\infty[\longrightarrow \mathbb{R}$ defined by $f(x) = x^2$ is injective.

Proof: Suppose that $f(x_1) = f(x_2)$ for some $x_1, x_2 \in [0, +\infty[$, then

$$\begin{aligned} f(x_1) = f(x_2) &\iff x_1^2 = x_2^2 \\ &\iff |x_1| = |x_2| \quad (\text{note } x_1, x_2 \geqslant 0) \\ &\iff x_1 = x_2 \end{aligned}$$

Thus, f is injective. ■



Surjective : f is surjective (onto) if every element in F is mapped to by some element in E. More formally,

$$\forall y \in F, \exists x \in E, \; y = f(x)$$

Example

Prove that $f: \mathbb{R} \longrightarrow [0, +\infty[$ defined by $f(x) = x^2$ is surjective.

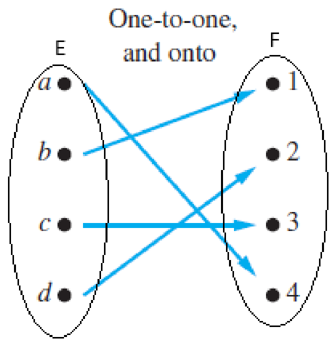
Proof: $\forall y \in [0,+\infty[, \exists x \in \mathbb{R}, \ y = f(x)$, then

$$\begin{aligned} y = f(x) &\iff y = x^2 \\ &\implies |x| = \sqrt{y} \\ &\implies (x = \sqrt{y}) \vee (x = -\sqrt{y}) \end{aligned}$$

Hence, $\forall y \in [0,+\infty[, \exists x = \sqrt{y} \in \mathbb{R}, \ y = f(x)$. Consequently, f is surjective. ■

Bijective : f is bijective if every element in F is mapped to by unique element in E. More formally

$$\forall y \in F, \exists! x \in E, \ y = f(x)$$



Example

Prove that $f : [0,+\infty[\longrightarrow [0,+\infty[$ defined by $f(x) = x^2$ is bijective.

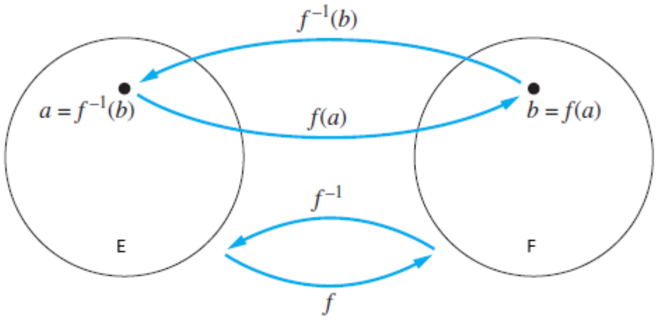
Proof: $\forall y \in [0,+\infty[, \exists! x \in [0,+\infty[, \ y = f(x)$, then

$$\begin{aligned} y = f(x) &\iff y = x^2 \\ &\implies x = \sqrt{y} \end{aligned}$$

Hence, $\forall y \in [0,+\infty[, \exists! x = \sqrt{y} \in [0,+\infty[, \ y = f(x)$. Consequently, f is bijective. ■

3.3 Inverse Functions

Let f be bijection from the set E to the set F. The inverse function of f is the function that assigns to an element b belonging to F the unique element a in E such that $f(a) = b$. The inverse function of f is denoted by $f^{-1} : F \longrightarrow E$. Hence, $f^{-1}(b) = a$ when $f(a) = b$.



Example

Let $f:\mathbb{R}\longrightarrow\mathbb{R}$ defined by $f(x)=\dfrac{4x+2}{5}$. suppose f is invertible so , find the inverse of f

$$\begin{aligned} f(x)=\frac{4x+2}{5} &\iff y=\frac{4x+2}{5} && \text{(Replace } f(x) \text{ with } y\text{)}\\ &\implies x=\frac{5y-2}{4} && \text{(Solve the equation for } x\text{)}\\ &\implies y=\frac{5x-2}{4} && \text{(Interchange } x \text{ and } y\text{)}\\ &\implies f^{-1}(x)=\frac{5x-2}{4} && \text{(Replace } y \text{ with } f^{-1}(x)\text{)} \end{aligned}$$

4 Exercises

Exercise 1

1/Describe the following sets using Roster method:

$$A=\{\frac{1}{n}\,|\,n\in\{3,4,5,6\}\} \qquad B=\{x\in\mathbb{Z}\,|\,x=x+1\}$$

2/ Let A and B be sets. Show that

$$a)\ A\cap(B\backslash A)=\varnothing \qquad b)\ A\cup(B\backslash A)=A\cup B \qquad c)\ \mathcal{P}(A)\subseteq\mathcal{P}(B)\iff A\subseteq B.$$

Exercice 2

Let $\mathcal{R}$ be a relation defined on $\mathbb{R}$ by : $\forall(x,y)\in\mathbb{R}^2,\ x\mathcal{R}y\iff\cos^2(x)+\sin^2(y)=1$.

1) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{R}$.

2) Find the equivalence class of 0.

Exercise 3

Let $\mathcal{R}$ be a relation defined on $\mathbb{N}$ by : $\forall n,m\in\mathbb{N},\ n\mathcal{R}m\iff m$ divise n.

Prove that $\mathcal{R}$ is a partial order relation on $\mathbb{N}$.

Exercise 4

For $f:\mathbb{R}\to\mathbb{R},\ x\longmapsto x^2+1$, find each of the following :

1/ $f^{-1}([-1,1])$ and $f^{-1}([0,10])$

2/] $f([-1,1])$ and $f([1,3])$

Exercise 5

Is the following functions injectives, surjectives, bijectives ?

1) $$\begin{aligned} f : [0,+\infty[&\longrightarrow [1,+\infty[\\ x &\longmapsto f(x) = 3x^2 + 4x + 1 . \end{aligned}$$

2) $$\begin{aligned} g : [-1,1] &\longrightarrow \mathbb{R} \\ x &\longmapsto g(x) = \sqrt{1-x^2} . \end{aligned}$$

Exercise 6

We consider the following function :

$$\begin{aligned} g :]-\infty,0[&\longrightarrow]1,+\infty[\\ x &\longmapsto g(x) = 1 + \frac{1}{x^2} . \end{aligned}$$

1. Show that g is bijective.
2. Find g^{-1}.