Tutorial n°2

Sets, Relations and Functions

Exercise 1

1/ We consider the following sets :

$$A = \left\{ x \in \mathbb{Z},\ |x-1| < \frac{3}{2} \right\}, \quad B = \{3,4\}, \quad C = \left\{ x \in \mathbb{N},\ \frac{2x+3}{2} \leqslant 4 \right\},$$
$$D = \{0,1,2,5\}, \quad E = \{1,2,3,4\}.$$

1. Describe the sets A and C using Roster method.
2. Determine which of these sets are equal or subsets of which other of these sets.
3. Determine the cardinality of A and B, then conclude the cardinality of $A \times B$ and $\mathcal{P}(A)$.
4. Find $A \cap B$, $A \cup B$, $C \setminus E$, $\complement_D(A)$, $A \times B$ and $\mathcal{P}(A)$.

2/ Let $A =]-\infty, 1[\cup]2, +\infty[$, $B =]-\infty, 1[$ and $C =]2, +\infty[$.

Find $\complement_{\mathbb{R}}(A)$ and $\complement_{\mathbb{R}}(B) \cap \complement_{\mathbb{R}}(C)$. What can you conclude?

Exercise 2

Let $A, B, C \in \mathcal{P}(E)$ and $f : E \to F$ be a function, prove the following

1/ $A \subseteq B \implies f(A) \subseteq f(B)$

2/ $\left\{ \begin{array}{l} A \subseteq B \\ \wedge \\ B \cap C = \varnothing \end{array} \right. \implies A \cap C = \varnothing$ (proof by contradiction)

Exercise 3

Let $\mathcal{R}$ be the relation defined on $\mathbb{Z}$ by : $\forall n, m \in \mathbb{Z},\ n\mathcal{R}m \iff \exists k \in \mathbb{Z}, n - m = 3k$

1. Determine whether $\mathcal{R}$ is reflexive? symmetric? antisymmetric? transitive?
   What can you conclude ?
2. Find the equivalence classes $\mathcal{C}(2)$ and $\mathcal{C}(5)$.

Exercise 4

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^2 - 4x + 5$

1/ Find $f^{-1}(\{5\})$

2/ Is f injective ?

3/ Prove that $\forall x \in \mathbb{R},\ f(x) \geqslant 1$

4/ Is f surjective ?

5/ Let $g : \big]-\infty, 2\big] \to \big[1, +\infty\big[$ be a function defined by $g(x) = f(x)$

- Prove that g is bijective, and find g^{-1}.