University of Batna 2-Institute of Industrial Hygiene and Safety | **L1-Math1 2023/2024**

Tutorial n°4

Elementary Function and Application

Exercice 1

We consider the following function :

$$f :]1, +\infty[\longrightarrow]-1, +\infty[$$
$$x \longmapsto f(x) = x\ln(x) - x$$

1) Show that f admits an inverse function f^{-1} defined on J to be determined.

2) Find $f^{-1}(0)$ and $\left(f^{-1}\right)'(0)$.

Exercice 2

1) Show that, for all $x \in \mathbb{R}$,

$$\cos(2x) = \frac{1-\tan^2(x)}{1+\tan^2(x)}.$$

2)Show that

$$\arccos\left(\frac{4}{5}\right) = 2\arctan\left(\frac{1}{3}\right).$$

Exercice 3

Let f be the function defined as

$$f(x) = \arcsin\left(\frac{1-x^2}{1+x^2}\right).$$

1) Show that f is defined and continuous on $\mathbb{R}$.

2) Show that f is differentiable on $\mathbb{R}^*$ and find the derivative of f on $\mathbb{R}^*$.

3) Evaluate $\lim_{x\to+\infty} f(x)$.

Exercice 4

1) Find

$$\cosh\left(\frac{1}{2}\ln(3)\right) \quad \text{and} \quad \sinh\left(\frac{1}{2}\ln(3)\right).$$

2) Using the formula : $\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$.

1. Solve the following equation:

$$2\cosh(x) + \sinh(x) = \sqrt{3}\cosh(5x)$$

2. Simplify the expression :

$$\cosh(2\,\mathrm{arsinh}(x)).$$

Exercice 5

1. By using the definitions of hyperbolic functions in terms of exponentials , prove that :

$$\mathrm{artanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \qquad x \in]-1, 1[$$

2. Solve the equation

$$\mathrm{artanh}(x) = \ln(3)$$