Tutorial n°5

Limited Development

Exercice 1

Use the Taylor's formula to give the limited development of order 4 at $x_0 = 0$ for

$$f : x \mapsto \ln(1+x)$$

Exercice 2

Give the limited development of :

1. $g(x) = x^2 ln(x)$ of order 3 at $x_0 = 1$
2. $f(x) = ln(\cosh(x))$ of order 4 at $x_0 = 0$
3. $h(x) = \dfrac{ln(1+x)}{\sin(x)}$ of order 3 at $x_0 = 0$

Exercice 3

Compute the following limits using limited development:

$$\lim_{x\to 0} \frac{\sinh(x)}{\sin(x)}, \qquad \lim_{x\to 0} \frac{\cos(x)\sqrt{1+x}-1}{x}$$

Limited Development of Functions

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\sin(x) = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \cdots + (-1)^{n-1} \frac{1\times 3\times 5\times \cdots \times (2n-3)}{2^n n!} x^n + o(x^n)$$

$$\cosh(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1})$$

$$\sinh(x) = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2})$$