University of Batna 2-Institute of Industrial Hygiene and Safety

L1-Math1
2023/2024

Tutorial n°1

Logic and Mathematical Proof

Exercice 1

1/ Use quantifiers to state the following propositions :

1. 102 is a multiple of 3.
2. For any real numbers, if the cubes of two numbers are equal, then the numbers are equal.
3. For every natural number there exists a greater natural number.

2/ Determine whether the following formulas are true or false; and give thier negation :

P_1. $\forall x \in \mathbb{R}, (x+1)^2 > 0$

P_2. $\exists x \in \mathbb{R}, x^2 = -1$

P_3. $\exists! x \in \mathbb{R},\ 2x^2 + 3x = 0$

P_4. $\forall (x,y,z) \in \mathbb{R}^3,\ [(xz = yz)] \Longrightarrow x = y$

3/ Write the contrapositive of the following implication :

1. n is prime $\Longrightarrow (n = 2)$ or (n is odd).
2. $\forall n \geqslant 2,\ (n^2 - 1)$ is not divisible by 8, then n is even.

Exercice 2

Simplify the following statements :

- $[P \Longrightarrow (Q \Longrightarrow R)]$, $[(P \wedge Q) \Longrightarrow R]$. What can you deduce from this?

Prove the following equivalence using the truth table:

- $\overline{(P \Longrightarrow Q)} \Longleftrightarrow (P \wedge \bar{Q})$

Exercice 3

Prove that:

If x is a real number, then $|x+3| - x > 2$

Exercice 4

Prove by contradiction the following proposition :

Let n be an integer. If $n^2 + 5$ is odd, then n is even.

Exercice 5

Prove by induction that :

for every natural number n, $n^3 + 2n$ is divisible by 3.