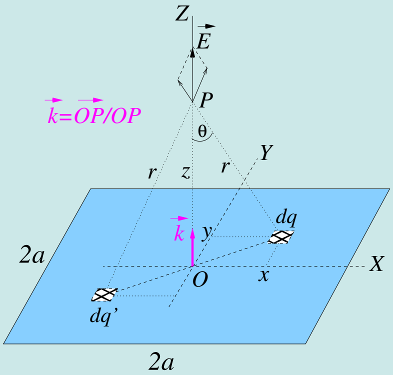
University of Batna 2 – Mostefa Ben Boulaïd

Faculty of Mathematics and Informatics

Departement Commun Base - Mathematics and Informatics

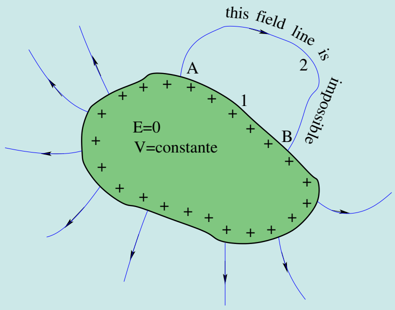
Course of Phy



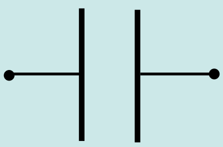
$$\vec{E} = \frac{\sigma}{\epsilon_0}\tan^{-1}\left(\frac{a^2}{z\sqrt{z^2+2a^2}}\right)\vec{k}.$$

If the size of the sheet became infinitely large, we would have to return to the case of the infinite plane:

$$\lim_{a\to\infty}\vec{E} = \frac{\sigma}{2\epsilon_0}\vec{k}.$$



Above, in blue, we see a uniformly charged square sheet (plate). At a point P on its axis (of symmetry), z-coordinate z, it creates the field $\vec{E}$ whose expression is written as shown on its right. Further to the right, in green, we have a conductor at equilibrium; its excess charges are necessarily distributed over its outer surface. A field line emerging from the conductor cannot return to the conductor.
As for the pictures displayed below, the left shows the symbol used to represent a capacitor in an electrical diagram, the middle picture shows different types of commercially available capacitors, and the one on the right is a typical thunderstorm flash resulting from an electrostatic discharge between the clouds and the ground.



Prof. M. A. Belkhir 2023-2024

Contents

Chapter 4

Current and resistance

In previous chapters, we have studied the effects of electrostatic charges, i.e. those due to stationary charges. We're now going to turn our attention to one of the most important properties associated with moving charges, namely electric current. The notions of resistance and electric circuits will be discussed in relation to electric current.

4.1 Electric current and resistance : the electrique conduction

4.1.1 Breakdown of an electrostatic equilibrium: notion of electric current

Consider two conductors A_1 and A_2 in electostatic equilibrium, isolated from each other and carrying charges Q_1 and Q_2. The previous equilibrium will be broken if the two conductors are brought into conductors in contact. As long as A_1 and A_2 are in contact, they form a single conductor $A_1 + A_2$. The total charge $Q_1 + Q_2$ will then spread over the whole $A_1 + A_2$ set, establishing a new state of equilibrium. The new state of equilibrium takes some time to be reached, during which time charges have moved from one conductor to the other. An electric current is said to have flowed between A_1 and A_2. The electric current flowing through a conductor is defined as the quantity (or rate) of charges per unit of time passing through a right cross-section of the conductor. By underlining the 'a' (undefined article), we mean that the section is taken at an arbitrary point on the conductor. If dQ is the charge that has passed through a given right section of the conductor over time dt, we have :

$$I = \frac{dQ}{dt}. \tag{4.1}$$

From previous equation, we deduce :

$$dQ = Idt \Longrightarrow Q = \int Idt. \tag{4.2}$$

If I is constant $(I(S_1) = I(S_2) = ...)$, i.e. if the current doesn't vary even if the conductor doesn't have the same right cross-section everywhere, then :

$$Q = I\int dt = It \Longrightarrow I = \frac{Q}{t}. \tag{4.3}$$

The SI unit of electric current is the ampere (symbol A). From the preceding equations, we see that

$$1\ A = 1\ \text{C/s}. \tag{4.4}$$

Note : One ampere therefore corresponds to a charge of 1 coulomb passing through a given right cross-section of a conductor in one second. In a metal conductor where the current is carried by electrons, a current of 1

A corresponds to $1\,\mathrm{C}/(1.6\times10^{-19}\,\mathrm{C/electron})\approx 2.5\times10^{18}$ electrons. In other words, 2.5 billions of billions of electrons pass through a section of a conductor in one second. In electronics, currents are often expressed in milliampere (mA) or even microampere (μA) : 1 mA= 10^{-3} A and 1 μA= 10^{-6} A.

The previous current will cease as soon as the new electrostatic equilibrium is reached. This is because, at equilibrium, the conductor $A_1 + A_2$ is equipotential and the charges are not subject to any field (force) that might cause them to flow. In this chapter, we'll mostly be talking about direct currents. To obtain a direct or permanent current (i.e. one that flows without stopping), we need to ensure that there exists between A_1 and A_2 ("always in contact") a potential difference. In other words, maintain the system in a state of *permanent disequilibrium*, which can be achieved by connecting A_1 and A_2 to terminals of a generator, such as a battery. The battery doesn't create charges, but circulates them between the two conductors, through it, thanks to the potential difference it maintains between them. The above definition of current does not

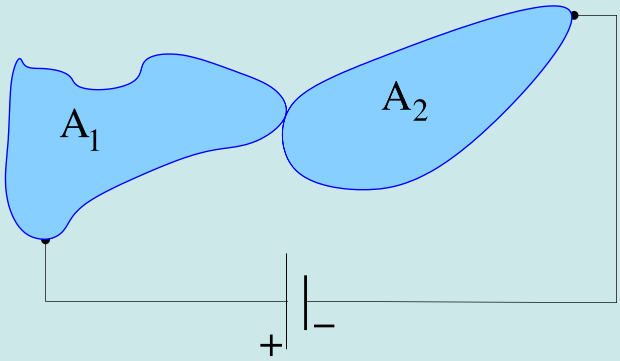


Figure 4.1: Maintain a potential difference between A_1 and A_2.

specify its direction of flow. In metals, the charge carriers are electrons, whereas in electrolytes they are either positive ions or negative ions, or both. In a given electric field, positive charges move in the direction of the field and negative charges in the opposite direction. In most situations, the effects of a flow of positive charges moving in one direction are equivalent to those of a flow of negative charges in the opposite direction. For reasons of historical consistency, we adopt the following convention suivante :

By convention, electric current is assumed to be the result of a displacement of positive charges.

The direction of current flow is indicated by an arrow. ($\longrightarrow$). When current is the result of a displacement of negative charges, such as electrons in metals, they actually move in the opposite direction to the conventional one.
An interesting consequence of the conventional direction is that current flows in the direction of decreasing potentials.[1]

In the presence of a generator, the current flows from pole + to the pole − outside the generator, but from the pole − to the pole + inside the generator.

4.2 Electrical current density

The electric current I measures the flow of charges through the entire cross-section of the conductor. The physical quantity that gives the current per unit area of the cross-section is the *current density*.

Consider a point M inside the conductor. Let S be a section of the conductor containing M and perpendicular to the direction of I. Let dS be an element of S (S also designates the surface of the section) and surrounding

[1]This follows from the fact that, as shown in Chapter 3, the electric field is oriented in the direction of decreasing potentials.

M. If dI is the current flowing through dS, then the current density is given by :

$$j = \frac{dI}{dS}, \tag{4.5}$$

from which we derive:

$$I = \int_S j dS. \tag{4.6}$$

Note that when the density j is constant, the equation (4.6) gives:

$$I = j \int_S dS = jS, \tag{4.7}$$

or

$$j = \frac{I}{S}. \tag{4.8}$$

Since the density j has a direction and a sense, it can be represented by a vector and the equation (4.6) is more generally written in vector form as :

$$I = \int_S \vec{j} \cdot d\vec{S}. \tag{4.9}$$

In the latter expression, the current I is given by the flux of the density $withj$ through the surface S. According to the study we carried out on the notion of flux when introducing the Gauss's theorem, the relation 4.9 can be generalized to a surface of any shape, provided it rests on the same solid angle. The SI unit of current density is $\mathrm{A/m^2}$. A density of 1 $\mathrm{A/m^2}$ can simply mean a current of 1 A flowing through a cross-section of 1 $\mathrm{m^2}$. But it could also mean a current of 2 A flowing through a cross-section of 2 $\mathrm{m^2}$ or a current of 0.1 mA flowing through a cross-section of 1 $\mathrm{cm^2}$.

4.2.1 Relationship between current density and drift velocity

We mentioned earlier that the physical reason for current is charge displacement. Due to thermal agitation, the electrons of an insulated metallic conductor move in the same way as the molecules of a gas enclosed in a container. As is often the case, we assume that the conductor is a metal wire. Movement in the wire gives rise to no resultant displacement (no net current), as the number of electrons crossing a right cross-section of wire from left to right is equal to the number of electrons crossing it from right to left. If we connect the ends of the wire to a battery, figure (**??**), we create a constant potential difference across the wire, which gives rise to an electric field E in the wire. A force $-e\vec{E}$ is then exerted on the electrons in the wire, which

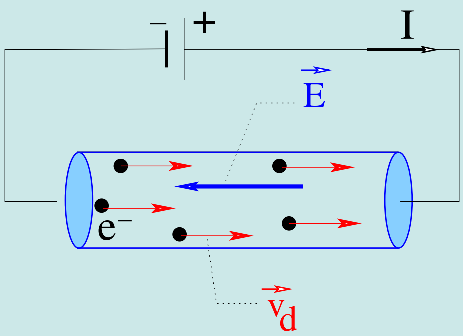


Figure 4.2: Relationship between density $\vec{j}$ and drift velocity $\vec{v_d}$.

are accelerated in the opposite direction to the field. But the electrons constantly collide with the atoms

[2] in the metal. Each time they collide, they lose some of their kinetic energy, which is transformed into the vibrational energy of the atoms. A regime in which the kinetic energy gained (supplied by the field $\vec{E}$) is entirely converted into vibrational energy is rapidly reached. The electrons then possess an overall velocity along the wire referred to as *drift velocity* and noted v_d. This is illustrated in figure 4.2. If there are n electrons per unit volume of the wire, and if ℓ is the length of the wire, the absolute value of the total charge in the wire is :

$$Q = ne \times \ell S, \tag{4.10}$$

where S is the riht cross-section of the wire. This charge crosses the wire in a time

$$t = \frac{\ell}{v_d}. \tag{4.11}$$

The corresponding electric current is

$$I = \frac{Q}{t} = \frac{ne\ell S}{\ell} v_d = neSv_d. \tag{4.12}$$

The ratio I/S (see equation (4.8)) gives the current density:

$$j = nev_d. \tag{4.13}$$

In vector form, we have (the direction of the vector being important, the $-$ sign of the electron's charge must be restored):

$$\vec{j} = n(-e)\vec{v}_d. \tag{4.14}$$

The following example gives an estimate of the drift speed in metals. Consider a copper wire of cross-section 5 cm^2 and carrying a current of 10 A. Recall that if M is the atomic molar mass of an element, the number of atoms in M grams of that element is equal to the Avogadro's number N_A, $N_A = 6.022 \times 10^{23}\,\mathrm{mol}^{-1}$. In a mass of 1 gram, there are N_A/M atoms. In m grams, the number of atoms N is then given by :

$$N = \frac{m}{M} N_A. \tag{4.15}$$

If V is the volume bounding the mass m, the density is $\rho = m/V$. The atomic density, i.e. the number of atoms of atoms per unit volume is written:

$$n' = \frac{N}{V}. \tag{4.16}$$

Taking into account $N = (m/M)N_A$ and $V = m/\rho$, we obtain:

$$n' = \frac{\rho N_A}{M}. \tag{4.17}$$

We'll assume that the current flows from the copper's conduction electrons and that each copper atom provides one conduction electron. The electron density n is then given by :

$$(n \text{ électrons/unité de volume}) = (n' \text{ atomes/unité de volume}) \times (1 \text{ électron/atome}). \tag{4.18}$$

For copper, $\rho = 8.9 \times 10^3\,\mathrm{kg/m^3}$ and $M = 63.5 \times 10^{-3}\,\mathrm{kg/mol}$. With these values, we find:

$$n = 8.5 \times 10^{28} \text{ électrons/atome}. \tag{4.19}$$

[2] A more advanced study shows that in reality, collisions are with crystalline defects (lacunae, impurities, antisites, etc.)

Using equation (4.8), the current density is :

$$j = \frac{10\text{A}}{5 \times 10^{-6}\text{m}^2} = 2 \times 10^{+6}\text{A/m}^2. \tag{4.20}$$

The drift velocity is obtained by applying the equation (4.13) :

$$v_d = \frac{j}{ne} = 1.5 \times 10^{-4}\,\text{m/s}. \tag{4.21}$$

This value shows that in conductors, the entire electron gas moves at an extremely low speed, less than 1 m per hour. The question arises as to why, with such a low speed, the lamps light up almost instantly after the switch is turned off. The answer is as follows. As soon as you turn on the switch, a disturbance is created at one end of the wire. This disturbance propagates through the wire via the electric field at a very high speed (the speed of light). A mechanical analogy is to consider a tube full of marbles. If an additional marble is introduced at one end of the tube, a marble will almost instantaneously exit the tube from the other end.

4.3 Resistance, resistivity and conductivity

We have seen that to produce an electric current in a wire, it is necessary to maintain a potential difference between its ends. The existence of a potential difference implies the existence of an associated electric field along the wire. This field which is responsible for the circulation of current, i.e. the drift of the charges. There are several indications that charge drift encounters some kind of force of opposition or resistance in the material. For example, if this field (i.e. the potential difference) is removed, the current is then cancelled out. To maintain the current, we apply a potential difference to overcome this resistance.
It is also known from experience that same potential difference applied to the ends of similar wires made of different materials produces *different* electric currents. This means that not all materials conduct electricity in the same way. We express this difference difference by the notion of electrical *resistance*.
The resistance is often referred to as R and is defined as the ratio of the potential difference V between the ends of the wire to the electric current I flowing through it:

$$R = \frac{V}{I}. \tag{4.22}$$

For the same potential difference, materials with a higher electrical resistance will produce a lower current.
The SI unit of resistance is the ohm (symbol Ω). From the equation (4.22), we see that $1\ \Omega = 1$ V/A.
The resistance of a given material depends on its geometric shape, size, temperature,..., and the electrical properties of the medium. It can also depend on V or I.
When R depends on neither I nor V (i.e. R is constant), the relationship (4.22) is often written in linear form :

$$V = RI, \tag{4.23}$$

and is called Ohm's law. In this case, the temperature is supposed not to vary when measuring V and I. The conductors that obey Ohm's law are called ohmic. Some materials, especially semiconductors, are not ohmic, they don't obey Ohm's law. Figure (4.3) shows a type of resistor sold on the market. The material making up the electrical resistor itself is found inside the insulating coating that encases the resistor. In this example, we have a so-called carbon layer resistor where the inside is filled with a carbon composition. Other types of resistor, which will not be presented here, also exist. The choice of type depends on the intended use of the resistor.
The coating shows a set of rings of different colors. Each color corresponds to a number (figure 4.3). The correspondence between the numbers and the colors of the rings constitutes what is known as the color code, and makes it possible to determine value of a resistor in units of Ω. To read this value, the resistor must first

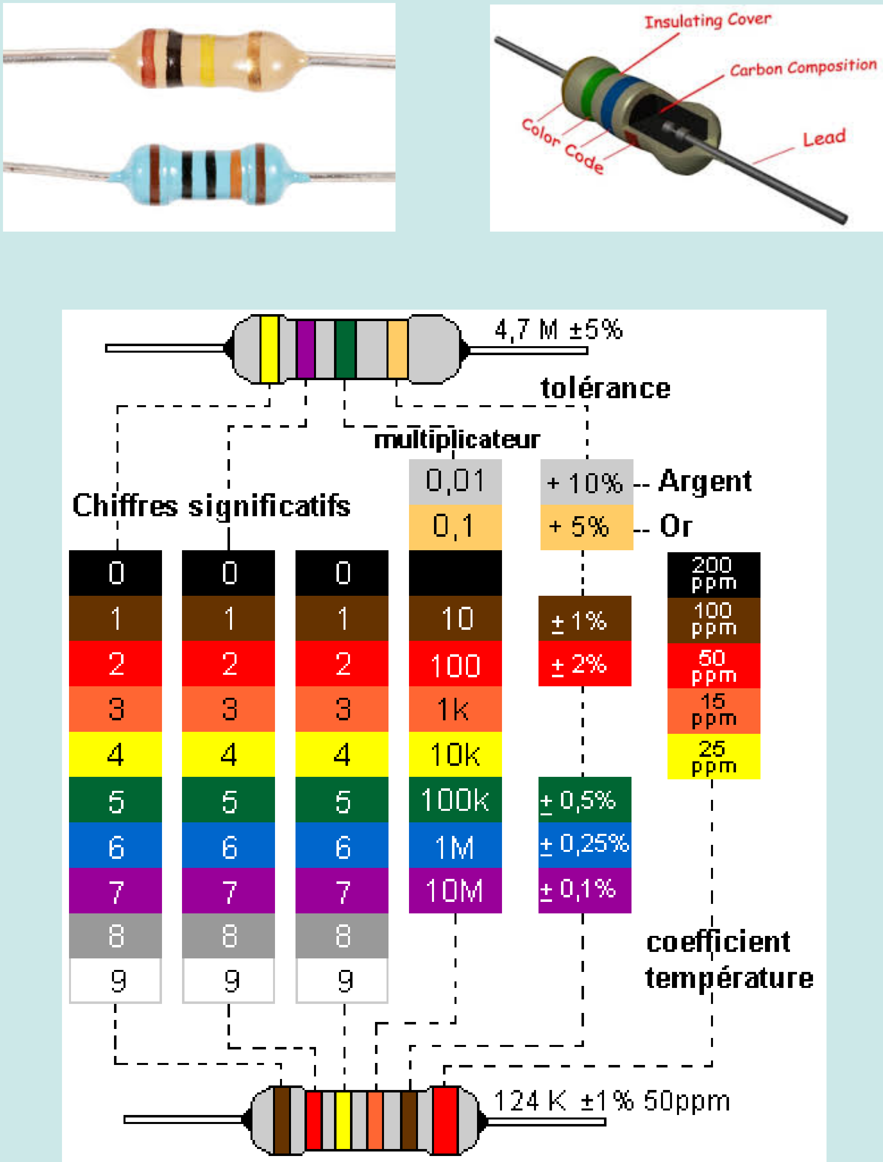


Figure 4.3: Example of a commercial resistor with a cross-sectional view of the inside of the coating on the right. In this example, the inside is filled with a carbon composition, making it a carbon-film resistor. Other types of resistor, which will not be presented here, also exist. The choice of type depends on the intended use of the resistor. The figure below explains the color code and shows how to read the value of a resistor.

be placed in the right direction. Usually, the resistor has a gold or silver ring, which should be placed to the right. In other cases, the larger ring is placed on the right.
In an electrical circuit, we symbolize the electrical resistance between two points by :

Figure 4.4: Symbol for resistance in an electrical diagram.

4.3.1 Définition de la résistivité

In subsection 4.2.1 page 35 we introduced the drift velocity $\vec{v}_d$ of electrons in a material. This velocity is proportional to the field $\vec{E}$ prevailing in the material. But since the density is $\vec{j} = n(-e)\vec{v}_d$, it follows that $\vec{j}$ is proportional to $\vec{E}$. We express this proportionality by :

$$\vec{E} = \rho\vec{j} \tag{4.24}$$

or

$$\vec{j} = \frac{1}{\rho}\vec{E}. \tag{4.25}$$

The constant ρ is called electrical **resistivity** of the material and its SI unit is the ohm-mètre $(\Omega \cdot \mathrm{m})$. It is a characteristic of the medium. The table below gives some typical values of resistivity at ordinary temperature (at about 20 °C).

Resistivity of some materials (at $\sim 20\,°$C)

Material	Resistivity $(\Omega \cdot \mathrm{m})$
Mica	2×10^{15}
Glass	10^{12} à 10^{13}
Silicon	2200
Germanium	0.45
Carbon (graphite)	3.5×10^{-5}
Aluminum	2.8×10^{-8}
Copper	1.7×10^{-8}
Silver	1.5×10^{-8}

We often prefer to write the equation (4.25) in the form

$$\vec{j} = \sigma\vec{E}. \tag{4.26}$$

The constant σ $(= 1/\rho)$ is called **electrical conductivity**. It is measured in $(\Omega \cdot \mathrm{m})^{-1}$. The siemens per meter (S/m) is also used as a unit of conductivity. [3].

Note that since V and I are macroscopic quantities, equation $V = RI$ is the macroscopic macroscopic form of Ohm's law when R is constant.

In the same way, since both $\vec{E}$ and $\vec{j}$ are local quantities, the equation (4.24) (or (4.25) or (4.26)) is the local form of Ohm's law.

Let's write down the relationship between resistance R and resistivity e for a wire-shaped conductor with cross-section (circular or not) S and length ℓ, subjected to a potential difference between its ends. Assuming that the electric field in the wire is uniform, we have $V = E\ell$. By substitution in $j = I/S = E/\rho$, we find :

$$R = \frac{\rho\,\ell}{S}. \tag{4.27}$$

The resistance of a filamentary conductor is directly proportional to the length and inversely proportional to the the cross-sectional area.

[3] Siemens is the SI unit of electrical conductance, conductance being the inverse of resistance: $1\,\mathrm{S} = 1\ \Omega^{-1}$. The siemens was adopted as a unit derived from the International System in homage to the German inventor Werner von Siemens (1816-1892)

4.4 Temperature-dependence of resistivity

In general, the resistivity of a material depends on its temperature. The ρ resistivity of a metal at temperature T is given by the following empirical relationship :

$$\rho = \rho_0[1 + \alpha(T - T_0)], \tag{4.28}$$

where ρ_0 is the resistivity measured at a certain temperature of reference T_0 and α is the coefficient of temperature of the measured resistivity in $°C^{-1}$. Equation (5.26) is only valid in a limited temperature range.

4.5 Deviations from Ohm's law

For ohmic conductors, the current-voltage curve ($I = f(V)$) is a straight line over a wide temperature range. Some materials, such as semiconductors, deviate from Ohm's law. For example, the junction diode, made of semiconductor materials, has a characteristic ($I = f(V)$) that is not a straight line. However, at a given point (I,V), the equation $R = V/I$ can be used as a definition of resistance, even if Ohm's law is not satisfied.

4.6 Association of resistors

As we did with capacitances, we can associate resistors in various combinations. The most frequently encountered are combinations of resistors in series and in parallel. We'll look at these below, and find the equivalent resistance in each case.

4.6.1 Association of resistors in series

Let's start with the case of two resistors R_1 and R_2. When they are associated in series, as shown in figure 4.5, the current I is the same in both. Since the current flows through the resistors in the same direction, the field in the resistors is also in the same direction. It follows that the potential difference (given by $int - \vec{E} \cdot d\vec{r}$) at the terminals of the set is equal to the sum of the individual potential differences, that is :

$$V = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2)I. \tag{4.29}$$

The two resistors in series are equivalent to a single resistor R_{12} :

$$R_{1+2} = R_1 + R_2. \tag{4.30}$$

What if there are three resistors R_1, R_2 and R_3 in series? We replace R_1 and R_2 by their equivalent resistor

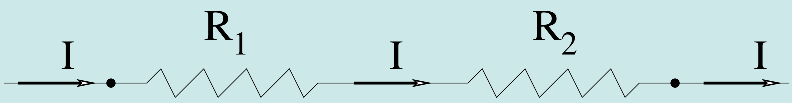


Figure 4.5: Association of resistors in series.

$R_{1+2} = R_1 + R_2$ and the problem is reduced to the previously studied case of two resistors in series: R_{1+2} and R_3. The equivalent resistance of the set of three resistors is therefore written as :

$$R_{1+2+3} = R_{1+2} + R_3 = R_1 + R_2 + R_3. \tag{4.31}$$

The same reasoning is valid for any number n of resistors, and the result is generalized as follows :

$$R_{\text{eq}} = R_1 + R_2 + \cdots + R_n. \tag{4.32}$$

The equivalent resistor of a combination of resistors in series is simply the sum of the individual resistors.

This result has the same form as that obtained for parallel capacitors (equation (3.21)). Note that the equivalent resistor of an association of resistors in series has a value that's always greater than the greatest of the individual resistors. Indeed, if the greatest resistance is R_g, then :

$$R_{\text{eq}} - R_g = R_1 + R_2 + \cdots + R_{g-1} + R_{g+1} \cdots + R_n. \tag{4.33}$$

Since the right-hand member is positive, we get :

$$R_{\text{eq}} - R_g > 0 \Longrightarrow \boxed{R_{\text{éq}} > R_g}. \tag{4.34}$$

4.6.2 Association of resistors in parallel

Figure 4.6 shows an association of three resistors in parallel. In such an association, the potential difference

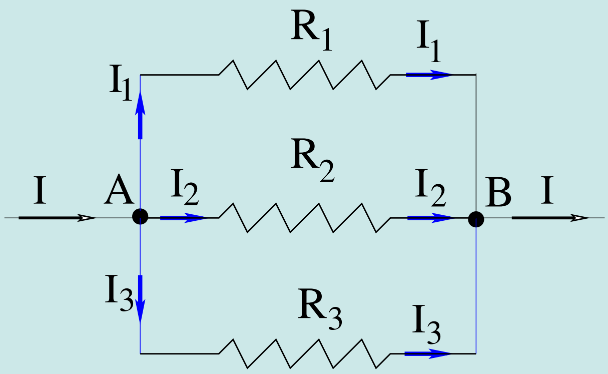


Figure 4.6: Association of resistors in parallel.

$V = V_A - V_B$ is the <u>same</u> across each resistor. The total current I flows into A and out of B. It is divided in the three resistors into I_1, I_2 and I_3, so that

$$I = I_1 + I_2 + I_3. \tag{4.35}$$

Using Ohm's law ("equation (4.22)"), the prevoius equation becomes :

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right). \tag{4.36}$$

If R_{eq} is the equivalent resistor between A and B, then we have

$$R_{\text{eq}} = \frac{V}{I},$$

and from equation (4.36), we derive the ratio V/I :

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}. \tag{4.37}$$

Extending this reasoning to n resistors in parallel, we find :

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right)^{-1}. \tag{4.38}$$

The equivalent resistor of an association of resistors in parallel is equal to the inverse of the sum of the inverses of the individual resistors.

Equivalently, this result is also written as :

$$\frac{1}{R_{\text{éq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}, \tag{4.39}$$

and is expressed as follows : *The equivalent resistor of an association of resistors in parallel is such that its inverse is equal to the sum of the inverses of the individual resistors.* Note the similarity of this result (for resistors **grouped in parallel**) to that obtained for capacitors **grouped in series**, equation (3.24).
In the form 4.39, we see that the equivalent resistor of an association of resistors in parallel always has a value lower than the smallest of the individual resistors. Indeed, if the smallest resistor is R_p, then :

$$\frac{1}{R_{\text{éq}}} - \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{p-1}} + \frac{1}{R_{p+1}} + \cdots + \frac{1}{R_n}. \tag{4.40}$$

Since the right-hand side is positive, we get :

$$\frac{1}{R_{\text{éq}}} - \frac{1}{R_p} > 0 \Longrightarrow \frac{1}{R_{\text{éq}}} > \frac{1}{R_p} \Longrightarrow \boxed{R_{\text{éq}} < R_p}. \tag{4.41}$$

4.7 Joule's law

To maintain a current in a given AB medium, energy must be expended. Let a constant current I flow through this medium under the influence of a constant potential difference $V = V_A - V_B$. If the current flows for time t, it is equivalent to a charge $q = It$. When q moves from A to B, its electrical potential energy decreases by

$$U = qV = It(V_A - V_B). \tag{4.42}$$

This relationship is valid regardless of the medium between A and B. This medium may be a resistor, a motor, an accumulator, etc. The lost potential energy is transformed into a form that depends on the medium. When the medium is an ohmic conductor with resistor R (4.7), energy is converted into thermal

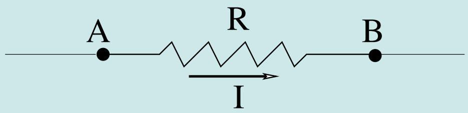


Figure 4.7: Joule's law.

energy. But, in this case we have : $V_A - V_B = RI$ hence

$$U = RI^2 t. \tag{4.43}$$

The rate at which the potential difference delivers energy to the medium is the power

$$P = dU/dt = (dq/dt)V = IV,$$

which gives in the case of a resistor

$$P = RI^2 = \frac{V^2}{R}. \tag{4.44}$$

The dependence of U (or P) on I^2 was first established by British physicist James Prescott Joule (1818-1889). For this reason, the equations (4.43) and (4.44) are known as Joule's Law. The loss of thermal energy in a resistor is known as the *Joule effect*. The SI unit of power is the *joule per second* (J/s), a unit called *watt* (W) in homage to James Watt, a Scottish engineer (1736-1819).