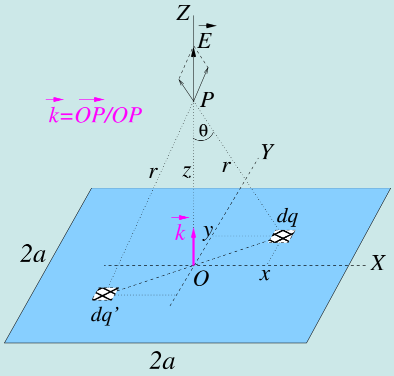
University of Batna 2 – Mostefa Ben Boulaïd

FACULTY OF MATHEMATICS
AND INFORMATICS
Departement Commun Base - Mathematics and Informatics

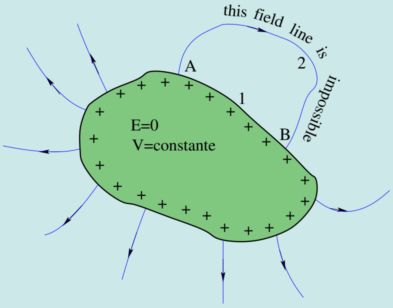
Course of Phy



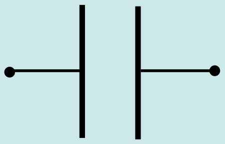
$$\vec{E} = \frac{\sigma}{\epsilon_0} \tan^{-1}\left(\frac{a^2}{z\sqrt{z^2+2a^2}}\right)\vec{k}.$$

If the size of the sheet became infinitely large, we would have to return to the case of the infinite plane :

$$\lim_{a\to\infty} \vec{E} = \frac{\sigma}{2\epsilon_0}\vec{k}.$$



Above, in blue, we see a uniformly charged square sheet (plate). At a point P on its axis (of symmetry), z-coordinate z, it creates the field $\vec{E}$ whose expression is written as shown on its right. Further to the right, in green, we have a conductor at equilibrium ; its excess charges are necessarily distributed over its outer surface. A field line emerging from the conductor cannot return to the conductor.
As for the pictures displayed below, the left shows the symbol used to represent a capacitor in an electrical diagram, the middle picture shows different types of commercially available capacitors, and the one on the right is a typical thunderstorm flash resulting from an electrostatic discharge between the clouds and the ground.



Prof. M. A. Belkhir 2023-2024

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Chapitre 1

Electrostatic interaction

1.1 Learning objectives

At the end of this chapter, the student will :
- understand the concept of electric point charge
- understand the quantization of electric charge carried by an object
- understand the principle of conservation of electric charge
- know how the SI unit of electric charge
- distinguish between a conductor and an insulator
- know how to charge an object
- describe Coulomb's law
- be able to solve problems involving Coulomb's law.

1.2 What is electrostatics ?

It is the study of the effects produced by *stationary, i.e. static, electrical charges. We'll see that electrostatic effects originate from the forces that electric charges exert on each other. Such forces are described by Coulomb's law. Electrostatic charges can set other electric charges in motion. The study of this motion is, of course, part of electrostatics.*

1.3 Lexique English-Arabic

Electricity = كهرباء

Electric, electrical =كهربائي

Electical charge = شحنة كهربائية

Electrization = تكهرب

Electrization by contact = تكهرب بالتماس

Electrization by friction = تكهرب بالإحكاك

Electrization by influence = تكهرب بالحث

Electron = إلكترون *; proton* = بروتون *; neutron* = نوترون

Free electron = إلكترون حر

Electroscope = كاشف كهربائي

Electronegative = كهرسالب

Electrostatic (adjective) = كهرساكن

Electrostatic interaction التفاعل الكهروستاتيكي

Electrostatics (noun) = كهرباء ساكنة أو كهرستتيكية

1.4 Concept of electric charge

In nature, every object (solid, liquid or gas) is made of atoms. An atom is the smallest part of a simple body (simple here means made of only one type of atom) that can combine chemically with another atom. In a piece of iron, for example, the smallest part is an iron atom.
An atom is made up of a set of electrons orbiting a core called the nucleus (النواة). The nucleus is made up of neutrons (their number is N_n) and protons (their number is N_p).
An atom is characterized by its atomic number Z and its mass number A, where $Z = N_p$ and $A = N_p + N_n$. The difference $A - Z$ gives the number of neutrons. For an atom with symbol X, this data is summarized in the following notation : ${}^{Z}_{A}\mathrm{X}$. For example, lithium (Li) has an atomic number of 3 and a mass number of 7, symbolized by ${}^{3}_{7}\mathrm{Li}$.

Electrons are negatively charged particles, protons positively charged particles and neutrons are particles electrically neutral.

A proton carries the positive charge $q_p = 1.602times10^{-19}\,\mathrm{C}$, an electron the negative charge $q_e = -1.602times10^{-19}\,\mathrm{C}$. In absolute value, protons and electrons carry the same charge $e = 1.602 \times 10^{-19}\,\mathrm{C}$. This charge is called "elementary charge" because it's the smallest observable and measurable charge that exists in nature.[1].

In a neutral atom, there are Z electrons (charge $-Ze$) and Z protons (charge $+Ze$). An atom that loses electrons ends up with more protons and becomes a positive ion (e.g. Cu^{++}), while an atom that gains electrons ends up with more electrons and becomes a negative ion (e.g. O^{--}).
Similarly, in a neutral object (عديمة الشحنة) there are as many electrons as protons. When an object prensents a lack or an excess (زيادة) of electrons, it is no longer neutral ; it is said to be electrically charged (مكهرب ، مشحون كهربائيا). Its charge is positive in the case of a lack of electrons and negative in the case of an excess.

Synonyms : Electrically charged = electrified = carries an electric charge. When the 'electric' context is clear (السياق واضح), we can simply say that an object is charged or carries a charge without needing to add the adverb 'electrically' or the adjective 'electric'.

1.5 Quantization of electric charge

Since, as mentioned above, electrification is nothing other than a lack or excess of electrons, the electric charge carried by an object is always an integral multiple of the elementary charge e. In other words, any observable electric charge Q,[2] is necessarily a multiple of e :

$$Q = \pm ne \tag{1.1}$$

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[1] It should be noted, however, that particle physics has demonstrated the existence of smaller charges, i.e. **quarks** with charges of $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$. So far, however, no one has been able to isolate a quark, so that we continue to regard the elamentary charge e as the smallest measurable charge

[2] The letters q and Q are, in general, the symbols used to denote electrical charges carried by objects.

where n is a positive integer or zero. The equation (1.1) is simply the expression of the quantization of electric charge.

1.6 SI unit of electric charge

The SI unit (Système International) of electric charge is the coulomb (symbol C). Typical charges carried by rubbed objects are in the microcoulomb, nanocoulomb or picocoulomb range. The coulomb therefore represents a very large quantity of charge. It is often more practical to use submultiples :
microcoulomb, $1\,\mu\mathrm{C} = 10^{-6}\,\mathrm{C}$;
nanocoulomb, $1\,\mathrm{nC} = 10^{-9}\,\mathrm{C}$;
and picocoulomb, $1\,\mathrm{pC} = 10^{-12}\,\mathrm{C}$.

Exercise : Electron and proton have charges of $q_e = -1.602\times10^{-19}$ C and $q_p = +1.602\times10^{-19}$ C respectively. a) Calculate the number of électrons a neutral object must gain to become $-1\,\mathrm{C}$ charged. b) Calculate the relative change in mass if initially it was 1 gram. The mass of the electron is $m_e = 9.109 times 10^{-31}$ kg.

Solution : a) To find the number of electrons, we divide $-1\,\mathrm{C}$ by the charge of one electron, i.e. -1.602×10^{-19} C. We find 6.242×10^{18}. In other words, it takes 6.242 billion billion electrons to produce a charge of -1 C. b) If m_i and m_f are the initial and final masses of the object, the relative variation in mass is given by $(m_f - m_i)/m_i$. Here $m_i = 1$ g, $m_f = m_i + m_{\mathrm{g}}$ (m_{g}= total mass of electrons gained by the object) $\Longrightarrow$ $(m_f - m_i) = m_{\mathrm{g}}$, which leads to a relative mass variation of 5.686×10^{-9}. This fraction is so small (نسبة ضئيلة للغاية) that we ignore (لا نأخذ بعين الاعتبار) the mass variation of an object between its neutral and electrified states.

Exercise : If a body gives out 109 electrons every second, how much time is required to get a total charge of 1 C from it ? (a) 190.19 years ; (b) 150.12 years ; (c) 198.19 years ; (d) 188.21 years. Ans. : (c).

1.7 Conservation de la charge

In an isolated system, charge is neither created nor destroyed ; it can only be transferred from one element of the system to another element of the system. For a system composed of two elements, if a charge appears on one element of the system, a charge of the same value but opposite sign appears at the same time on the other element.

Example : A glass rod when rubbed with silk cloth, acquires a charge of $1.6\times10^{-11}\,\mathrm{C}$, then the charge on silk cloth will be :
A) $= 3.2\times10^{-11}\,\mathrm{C}$; B) $= -3.2\times10^{-11}\,\mathrm{C}$; C) $= -1.6\times10^{-11}\,\mathrm{C}$; D) $= 1.6\times10^{-11}\,\mathrm{C}$.
Answer : When glass rod is rubbed with silk, electrons move from rod to silk. Since silk gets electrons it becomes negatively charged and the number of electrons gained by silk is same as that lost by rod. Hence magnitude of charge on silk is same as that on rod.
Hence charge on silk $= -1.6\times10^{-11}\,\mathrm{C}$, the correct answer is C).

1.8 Coulomb Law

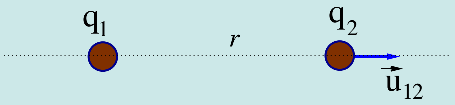
When a charge is carried by a point mass, we call it point charge (شحنة نقطية). The term charged particle is sometimes used instead of point charge.

Consider two point charges q_1 and q_2 separated by a distance r. Experiment shows that each acts on the other with a force
i) directed along the straight line joining q_1 and q_2,
ii) proportional to the product q_1q_2, répulsive if q_1 and q_2 are of the same sign , attractive if q_1 and q_2 are of opposite signs.

iii) inversely proportional to the square of the distance between q_1 and q_2.

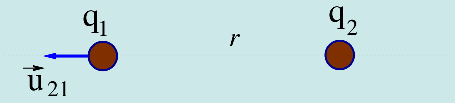
If we denote $\vec{F}_{1/2}$ the force with which q_1 acts on q_2, then

$$\vec{F}_{1/2} = k\frac{q_1 q_2}{r^2}\vec{u}_{12} \tag{1.2}$$



where $\vec{u}_{12}$ is, by definition, a unit vector along the line joining the two charges and oriented towards the charge which exerts the force $\vec{F}_{1/2}$ (تمارس القوة) to the charge that undergoes the force (يخضع للقوة) Conversely, the force exerted by q_2 on q_1 is written as :

$$\vec{F}_{2/1} = k\frac{q_1 q_2}{r^2}\vec{u}_{21} \tag{1.3}$$

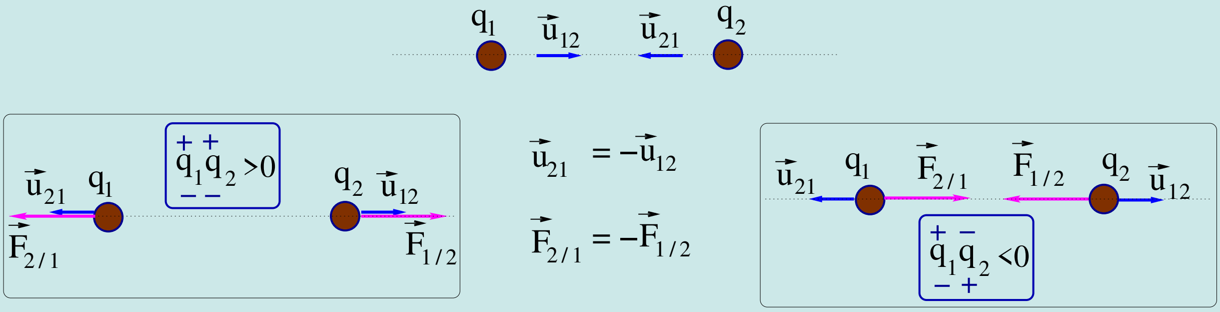


where the unit vector $\vec{u}_{21}$ is, this time, oriented from q_2 (the charge that exerts the force) to q_1 (the charge that undergoes the force).

From the definition of the unit vectors $\vec{u}_{21}$ and $\vec{u}_{12}$, it's clear that $\vec{u}_{21} = -\vec{u}_{12}$ and, consequently,

$$\vec{F}_{2/1} = -\vec{F}_{1/2} \tag{1.4}$$

The equality (1.4) says that Coulomb's force obeys the principle of action and reaction (Newton's third law).



Although the vectors $\vec{u}_{12}$ and $\vec{u}_{21}$ are, by definition, completely defined (modulus, direction and sense), the same cannot be said of the forces $\vec{F}_{1/2}$ and $\vec{F}_{2/1}$. Their direction is determined by the sign of the product of the charges $q_1 q_2$. They are attractive if q_1 and q_2 have opposite signs, and repulsive if q_1 and q_2 have the same sign. The figure above summarizes the various situations depending on the sign of each of the charges. The proportionality constant k is a positive constant, sometimes called the Coulomb constant. In the SI system, this constant is written as

$$k = \frac{1}{4\pi\epsilon_0}. \tag{1.5}$$

The constant epsilon$_0$ is called the dielectric permittivity of (or dielectric constant) of vacuum and is $8.854187817... \times 10^{-12}\,\mathrm{C}^2\ \mathrm{N}^{-1}\,\mathrm{m}^{-2}$. The equation (1.5) then gives : $k = 8.987551787... \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$, but for most numerical applications, we take

$$k \approx 9 \times 10^9\ \mathrm{N\,m^2\,C^{-2}}.$$

Now that we know the value of k, we can see from equation (**??**) that if $q_1 = q_2 = 1\,\mathrm{C}$ and $r = 1\,\mathrm{m}$, then the two charges will repel each other with a force of $9 \times 10^9\,\mathrm{N}$. This force is equivalent to the weight of a 900 000 000 kg mass ! Clearly, the coulomb, as mentioned above, is too large to express the usual static charge quantities, hence the usefulness of coulomb submultiples.

Remarque :

If $\vec{r}_{12}$ *denotes the vector joigning* q_1 *to* q_2*, then* $\vec{u}_{12} = \vec{r}_{12}/r_{12} = \vec{r}_{12}/r$ *and equation* (1.2) *takes also the form*

$$\vec{F}_{1/2} = k\frac{q_1 q_2}{r^3}\,\vec{r}_{12}. \tag{1.6}$$

Similarly, if $\vec{r}_{21}$ *denotes the vector joigning* q_2 *to* q_1*, then* $\vec{u}_{21} = \vec{r}_{21}/r_{12} = \vec{r}_{21}/r$ *and equation* (1.3) *takes also the form*

$$\vec{F}_{2/1} = k\frac{q_2 q_1}{r^3}\,\vec{r}_{21}. \tag{1.7}$$

1.8.1 Multiple charges : principle of superposition

Coulomb's law gives the force created by one charge on another. When there are several charges q_1, q_2, q_3, ..., q_{N} *acting on a charge* Q*, the total force on* Q *is obtained by making the vector sum of the forces exerted on it by* q_1, q_2, ...

$$\vec{F}_Q = \vec{F}_{1/Q} + \vec{F}_{2/Q} + ... + \vec{F}_{\mathrm{N}/Q} = \sum_{i}^{N} \vec{F}_{i/Q}. \tag{1.8}$$

The force $\vec{F}_{i/Q}$ *exerted by* q_i *on* Q *is calculated according to Coulomb's law, independently of the presence of other charges :*

$$\vec{F}_{i/Q} = k\frac{q_i Q}{r^2{}_{iQ}}\vec{u}_{iQ},$$

where r_{iQ} *denotes the distance from* q_i *to* Q*, and* $\vec{u}_{iQ}$ *is a unit vector running in the direction* $q_i \to Q$.

1.9 Conductive and insulating materials

From an electrical point of view, most matrices (مواد) are either conductors (ناقلة) (metals, alloys, the human body, water, ...) or insulators (عازلة) (glass, sulfur, plastic, amber, rubber, ...). A conductor is a body in which electric charges can move, whereas in an insulator electric charges cannot circulate. As a result, when we charge a conductor, the charges spread (توزع) all over the body, even far from where they were deposited. But when you put charges on an insulator, the charges stay where they were initially placed (تبقى حيث تم إنتاجها).

1.9.1 How to charge an object ?

To electrify (charge) an object, electrons are removed or added. In practice, electrification is the result of transferring electrons from one object to another. There are three main ways of electrifying or electrically charging an object.

1) Charging by friction : *When two bodies are rubbed together, electrons are transferred from one to the other. The body that gains electrons ends up with more electrons than protons and becomes negatively charged, and the body that loses electrons ends up with more protons than electrons and becomes positively charged.*

But how do you know which object is gaining or losing electrons ?

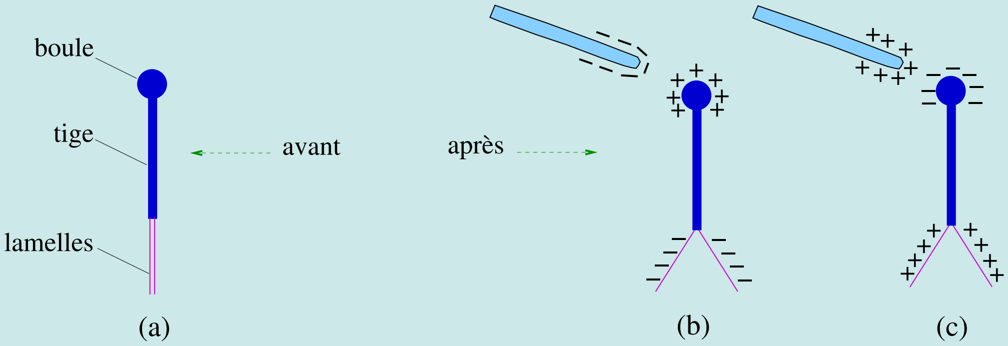
Based on experience, we arrange the objects in such an order that when two of them are rubbed together, the one before the other on the list (known as the "triboelectric list") becomes positively electrified. A non-exhaustive triboelectric list is shown below :

Dry human skin - Leather - Rabbit fur - Glass - Quartz - Human hair - Nylon - Wool - Cat fur - Silk - Aluminum - Paper - Cotton - Steel - Wood - Amber - Copper - Silver - Gold - Platinum - Polystyrene - Cellophane - PVC - Silicone - Teflon - Silicone rubber*.*

Examples :
i) Glass comes before wool on the previous list. If you rub them together, the glass will become positively charged. But if the glass is rubbed with rabbit fur, it will become negatively charged because the rabbit fur precedes the glass on the list.
ii) Similarly, if a PVC (polyvinyl chloride) rod is rubbed with wool, electrons will flow from the wool to the rod, which becomes negatively charged.

2) Charging by contact : *If a charged object touches a neutral conductor, part of the object's charge will be transferred from the object to the conductor. The charged object can be a conductor or an insulator, but the object to be charged must be a conductor to allow the charge to be transferred to it.*

3) Charging by influence : *Let's take an object charged placed close to a neutral conductor, but without touching it. Depending on whether the object is negatively or positively charged, the electrons in the conductor will either move as far away from the object as possible, or come as close as possible to the object and gather on the side of the conductor near the object. Electrification by influence can be illustrated by experimenting with the electroscope (المكشاف الكهربائي). The electroscope is a conductive assembly consisting of a vertical metal rod (قضيب معدني) at the end of which hang two parallel light lamellae and a ball attached to the top of the rod (figure a).*
The electroscope is neutral overall. A charged rod is brought close to the ball of the electroscope. We systematically observe a repulsion of the lamellae. Here's the explanation. If the rod is negatively charged, it repels free electrons from the electroscope. These electrons end up in excess in the strips, which become negatively charged and repel each other (figure b). If the rod is positively charged, it attracts free electrons from the electroscope. These electrons find themselves in defect in the lamellae, which become positively charged and repel each other (figure c).



To round off what you've already learned, I'll give you two Internet addresses where you can watch two videos showing some interesting electrostatic phenomena :
https ://www.youtube.com/watch ?v=3BnX230Yfvo ; https ://www.youtube.com/watch ?v=gz1NSzdqtm0

1.10 Questions and exercises

1- *The nucleus of an atom is made up of electrically neutral neutrons and positively charged protons. Why doesn't the nucleus not fly apart despite the very strong electrostatic repulsion between the protons ?*
The nucleus doesn't fly apart because there is a force preventing it from doing so. This force is neither gravitational nor electrical. It's called the nuclear force, or strong interaction. It represents one of the four fundamental forces of nature, and is responsible for the cohesion of the nucleus in atoms.

2- *Suggest a way of positively electrifying a metal sphere from a negatively charged plastic rod.*
The sphere is brought close to a negatively charged rod. The electrons in the sphere will be pushed as far away from the rod as possible.
ii) Connect the sphere to earth via a conducting wire, and the sphere's electrons will be pushed away to earth. iii) Remove the connection to earth, and the sphere is left with a deficit of electrons. The result is a positively charged sphere.

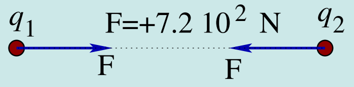
3- *The image on the cover page (صفحة الغلاف), bottom right, shows an example of electrostatic discharge that occurs in stormy weather between a cloud and the ground. Document and then explain this natural phenomenon, called lightning (برق), from the point of view of electrostatics.*

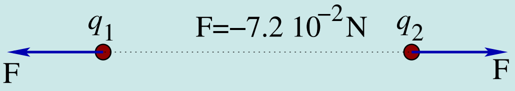
4- *List the different ways in which an object can be electrified.*
There are three ways of electrifying an object : by friction (or rubbing), by contact (or conduction) and by influence (or induction).
Friction electrification involves rubbing one material against another, creating a displacement of electrons from one material to the other. This method is best suited to electrifying insulators. Electrification by influence or induction (suitable for electrifying metals). Contact or conduction electrification (also suitable for electrifying metals).

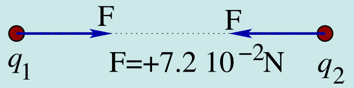
Here are a few experiments that each of you can perform.
i) A balloon rubbed with a piece of wool deflects a thin stream of water flowing from the tap.
ii) A plastic object (ruler, lighter, pen, etc.) rubbed against the scalp attracts small pieces of paper. If the load is substantial, they also deflect a thin stream of water.

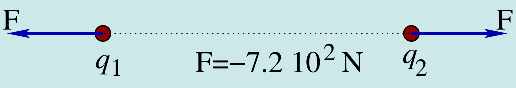
For numerical applications, we'll take $k = 9 \times 10^9$ N$\cdot$ m$^2\cdot$C^{-2}.

Example 1 : *Two point charges* $q_1 = 1\,\mu\mathrm{C}$ *and* $q_2 = -2\,\mu\mathrm{C}$ *are at a distance* $d = 50\,\mathrm{cm}$ *from each other. What forces (magnitude, direction and sense) act on these charges ? Select one of the answers below :*

A)


C)


B)


D)


Answer : The forces are attractive because the charges have opposite signs. The modulus is $k|q_1q_2|/d^2 = 9 \times 10^9 \times |(1 \times 10^{-6})(-2 \times 10^{-6})|/(0.5)^2 = 7.2 \times 10^{-2}\,\mathrm{N}$. *So, the correct answer is B).*

Example 2 : *Two identical copper spheres, each with mass* $1\,\mathrm{kg}$, *are separated by 1 m. a) How many electrons does each sphere contain ? b) How many electrons would have to be extracted from each sphere to achieve a repulsion of* 10^4 *N between the two spheres ? c) What fraction of the total number of electrons contained in each sphere represents the answer to question b) ?*

Gives : In 63.5 grams of copper, there are N_A *atoms* (N_A *= Avogadro's number =* 6.02×10^{23}) *and there are 29 electrons in each copper atom.*

Answer : a) 2.75×10^{26} *electrons ; b)* 6.59×10^{15} *electrons ; c)* 2.39×10^{-11}.

Execise 1 : *The position vectors of point charges* q_1 *and* q_2 *are* $\vec{r}_1$ *and* $\vec{r}_2$ *respectively. The electrostatic force on*

q_1 due to q_2 in vector form is : a) $kq_1q_2(\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|^3$, b) $kq_1q_2(\vec{r}_2 - \vec{r}_1)/|\vec{r}_2 - \vec{r}_1|^3$, c) $kq_1q_2\vec{r}_1/|\vec{r}_1 - \vec{r}_2|^3$, d) $kq_1q_2\vec{r}_2/|\vec{r}_2 - \vec{r}_1|^3$.

SOLUTION : In accordance with Coulomb's law expressed as ((1.7)), we have : $\vec{r}_{21} = \vec{r}_1 - \vec{r}_2$, $r^3 = r_{21}^3 = |\vec{r}_1 - \vec{r}_2|^3$, so the correct answer is a).

Execise 2 : The electric force on a charge q_1 due to q_2 is $(4\vec{i} - 3\vec{j})$ N. The unit vector in the direction of electric force on q_2 due to q_1 will be : a) $(4/5)\,\vec{i} - (3/5)\,\vec{j}$), b) $-(4/5)\,\vec{i} + (3/5)\,\vec{j}$), c) $(4/5)\,\vec{i} + (3/5)\,\vec{j}$), d) $-(4/5)\,\vec{i} - (3/5)\,\vec{j}$).

Execise 3 : A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. The maximum force of repulsion between them will occur, when : a) $Q_1 = Q; Q_2 = Q$, b) $Q_1 = Q/3; Q_2 = 2Q/3$, c) $Q_1 = Q/2; Q_2 = Q/4$, d) $Q_1 = Q/2; Q_2 = Q/2$.

Execise 4 : Three charges Q_1 (positive), Q_2 (positive) and q are placed on a straight line such that q is somewhere in between Q_1 and Q_2. If this system of charges is in equilibrium, what should be the magnitude and sign of charge q ? a) $|q| = (Q_1 + Q_2)/2$, q negative ; b) $|q| = (Q_1 + Q_2)/2$, q positive ; c) $|q| = Q_1Q_2/(\sqrt{Q_1} + \sqrt{Q_2})^2$, q negative ; d) $|q| = Q_1Q_2/(\sqrt{Q_1} + \sqrt{Q_2})^2$, q positive.

SOLUTION : System in equilibrium means all the three charges are in equilibrium. For Q_1 to be in equilibrium, the two electrostatic forces it is subjected to by Q_2 and q must be equal and opposite. Since Q_1 and Q_2 are both positive, the force between them is repulsive. Therefore, the force between q and Q_1 must be attractive that means that q is negative. In terms of magnitudes, we then have : $kQ_1Q_2/R^2 = k|q|Q_1/x^2$ or $R/x = \sqrt{Q_2/|q|}$ (1) where R and x denote distances to Q_1 of Q_2 and q respectively. Similarly, net force on q is zero : $kQ_1|q|/x^2 = kQ_2|q|/(R - x)$ or $(R - x)/x = \sqrt{Q_2/Q_1}$ or even $R/x = (\sqrt{Q_1} + \sqrt{Q_2})/\sqrt{Q_1}$ (2). From (1) and (2), $\sqrt{Q_2}/\sqrt{|q|} = (\sqrt{Q_1} + \sqrt{Q_2})/\sqrt{Q_1}$ or $Q_2/|q| = (\sqrt{Q_1} + \sqrt{Q_2})^2/Q_1$. Therefore, the correct answer is $|q| = Q_1Q_2/\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2$, and q negative.