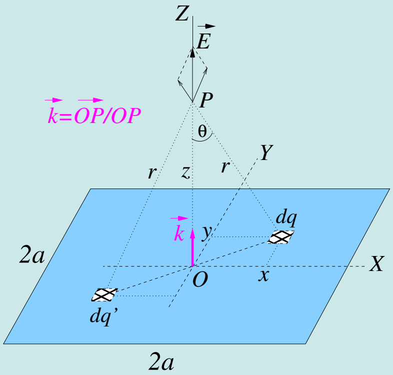
# University of Batna 2 – Mostefa Ben Boulaïd

Faculty of Mathematics and Informatics

Departement Commun Base - Mathematics and Informatics

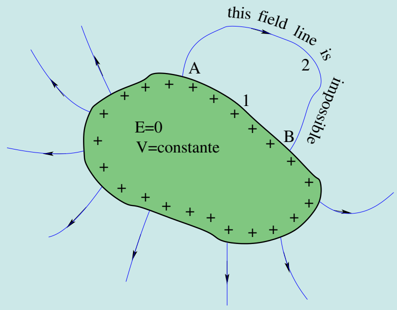

# Course of Phy



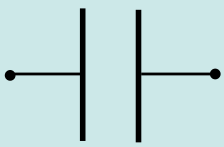

$$\vec{E} = \frac{\sigma}{\epsilon_0} \tan^{-1}\left(\frac{a^2}{z\sqrt{z^2+2a^2}}\right)\vec{k}.$$

If the size of the sheet became infinitely large, we would have to return to the case of the infinite plane :

$$\lim_{a\to\infty} \vec{E} = \frac{\sigma}{2\epsilon_0}\vec{k}.$$




*Above, in blue, we see a uniformly charged square sheet (plate). At a point $P$ on its axis (of symmetry), z-coordinate $z$, it creates the field $\vec{E}$ whose expression is written as shown on its right. Further to the right, in green, we have a conductor at equilibrium ; its excess charges are necessarily distributed over its outer surface. A field line emerging from the conductor cannot return to the conductor.*
*As for the pictures displayed below, the left shows the symbol used to represent a capacitor in an electrical diagram, the middle picture shows different types of commercially available capacitors, and the one on the right is a typical thunderstorm flash resulting from an electrostatic discharge between the clouds and the ground.*




Prof. M. A. Belkhir 2023-2024

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# Chapitre 1

# Field and potential in electrostatics

## 1.1 Learning objectives

At the end of this chapter, the student will :
- understand the concept of electric charge
- distinguish between a conductor and an insulator
- know how to charge an object
- describe Coulomb's law
- be able to solve problems involving Coulomb's law.

## 1.2 Lexicon English-arabic

Electric ( or electrostatic) field = حقل كهربائي (أي كهروساكن)

Potential energy = الطاقة الكامنة

Electric (electrostatic) potential energy = الكمون الكهربائي (أي كهروساكن)

Electric field (potential) created by a discrete set of point charges = قل (كمون) كهربائي ناتج عن عدة شحنات نقطية متفرقة

Electric field (potential) created by a continuous distribution of charges = (كمون) كهربائي ناتج عن توزيع مستمر للشحنة

Field lines = خطوط الحقل

Equipotential surfaces = سطوح متساوية الكمون

Electric field flux = تدفق الحقل الكهربائي

Gauss's theorem = نظرية غوص

Electric dipole = (ثنائي القطب الكهربائي)

Electric dipole moment = العزم الكهربائي لثنائي القطب

## 1.3 Concept of electric field

Let's consider a point charge $q'$ located at distance $r$ from another point charge $q$. Coulomb law (see chapter 1) teaches us that there is a mutual electric interaction, attractive ou repulsive, between $q$ and $q'$. The force that $q'$ feels from $q$ is :

$$\vec{F} = k\frac{q'q}{r^2}\vec{u}, \tag{1.1}$$

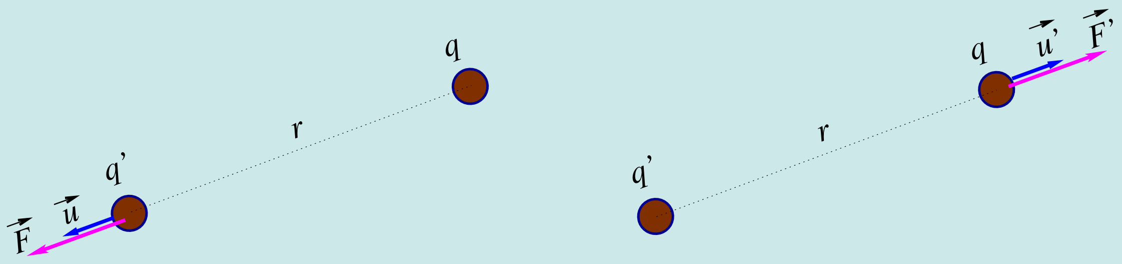



FIGURE 1.1 – Electrostatic interaction between $q$ and $q'$. On the figure, the force is supposed to be repulsive.

where $\vec{u}$ is the unit vector oriented, by definition, from the charge that acts to the charge that suffers, i.e. here from $q$ to $q'$. Conversely, the force exerted on $q$ by $q'$ is written as :

$$\vec{F'} = k\frac{q'q}{r^2}\vec{u'}, \tag{1.2}$$

where $\vec{u'}$ is the unit vector oriented $q'$ towards $q$.

Equation (1.1) can be rewritten as :

$$\vec{F} = q'\left(k\frac{q}{r^2}\vec{u}\right) \tag{1.3}$$

In this form, we see that $\vec{F}$ is a product of two quantities :
i) the charge $q'$, whose value is independent of the presence or absence of $q'$, and
ii) the quantity $\left(k\frac{q}{r^2}\vec{u}\right)$ whose expression is independent of the presence or absence of $q'$. This second quantity depends solely on $q$, $r$ and $\vec{u}$. It is a quantity that can be calculated for any point in space around the electrostatic charge $q$. In other words, because of its presence, *the point charge $q$ affects space by creating an electrostatic field,*

$$\vec{E} = k\frac{q}{r^2}\vec{u}, \tag{1.4}$$

*so that any other charge $q'$ around it feels a force.*

$$\vec{F} = q'\vec{E}. \tag{1.5}$$

Of course, the reasoning with $q$ is valid for any other point charge.

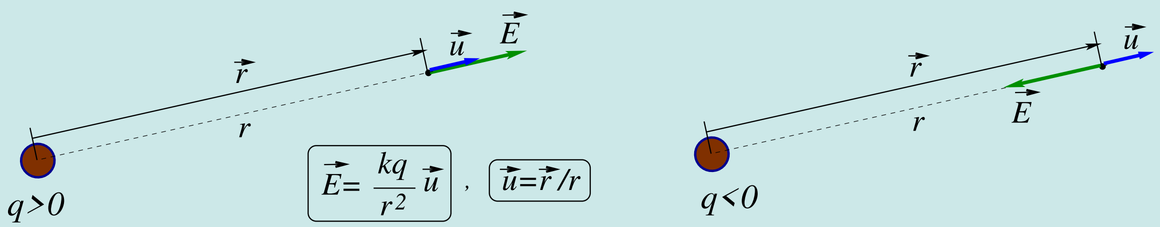



FIGURE 1.2 – Field created by a single point charge

By definition, the unit vector $\vec{u}$ is carried by the vector $\vec{r}$ joining the "source" charge $q$ to the "point under consideration", and has the same direction. Note that "source charge" here means the charge from which the field originates, and "point under consideration" is the point where the field is expressed.

$$\vec{u} = \frac{\vec{r}}{r}. \tag{1.6}$$

The field $\vec{E}$ is a vector. It has the same direction as $\vec{u}$ if $q$ is positive, and the opposite direction if $q$ is negative. Its modulus is proportional to $|q|$ and inversely proportional to the square of the distance $r$.

Given (**??**), the equation (1.4) can take the equivalent form :

$$\vec{E} = kq\frac{\vec{r}}{r^3}. \tag{1.7}$$

From equation (1.5) , we derive :

$$\vec{E} = \frac{\vec{F}}{q'} \tag{1.8}$$

This last equation shows that the SI unit of electric field is newton per coulomb (N/C). It also allows us to interpret the electric field at a point as the force that would be felt by a 1 C point charge placed at that point. We'll see in section 1.9 that the electric field can also be expressed, alternatively, in V/m, an equivalent SI unit.

## 1.4 Electrostatic field created by a discrete set of point charges

Consider a discrete set of point charges $q_1$, $q_2$, $q_3$, ...located respectively at non-zero distances $r_1$, $r_2$, $r_3$, ...from a point in the space surrounding them.
At this point, these charges create the fields $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, ...respectively.
The total electrostatic field is the vector sum of all these fields :

$$\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ... \tag{1.9}$$

The field created by each point charge is calculated independently of the presence of other charges. For example :

$$\vec{E}_1 = \frac{kq_1}{r_1^2}\,\vec{u}_1 \tag{1.10}$$

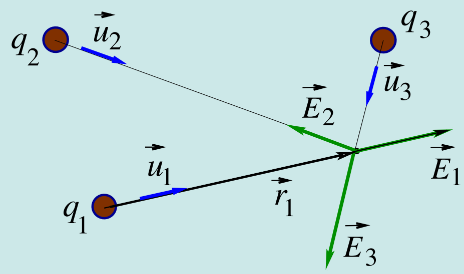
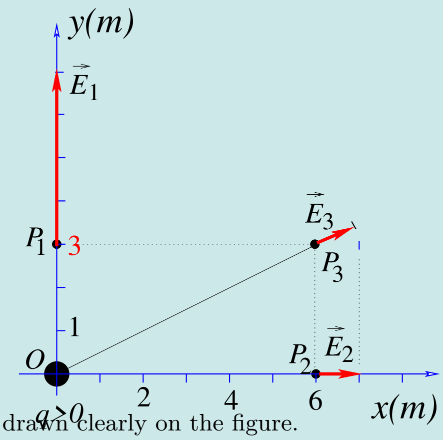



FIGURE 1.3 – Field created by a discrete set of point charges

*Note* : A point charge creates a field at all points in space around it, but not at the point where it is located. It doesn't feel its own field ; it can only feel the field created by other charges. In other words, if for example we have a set of 3 charges $q_1$, $q_2$, and $q_3$, and we want to calculate the electric field at the point where $q_2$ is located, this field is equal to the sum of the fields created by $q_1$ and $q_3$ : $\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_3$.

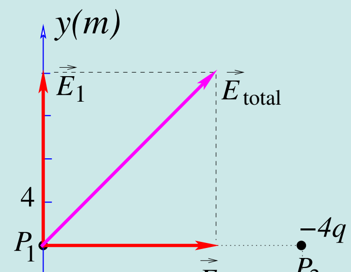
### Exemple 1 :

A point charge $q$ is placed at the origin $O$ of a rectangular system of axes $xy$. The axes $x$ and $y$ are graduated in meters. 1) Calculate the modulus $E_1$ of the electrostatic field created by $q$ at point $P_1(0\,;\,3)$. Take $k = 9times10^9$ SI units and $q = 1$ nC. Draw the electrostatic field vector $\vec{E}_1$ (scale : 1N/C will be represented by 4 cm on the drawing)[1]. 2) Express the modulus $E_2$ of the electrostatic field created by $q$ at the point $P_2(6;\,0)$ as a function of $E_1$ then draw $\vec{E}_2$. 3) Express the modulus $E_3$ of the electrostatic field created by $q$ at point $P_3(6;\,3)$ as a function of $E_1$, then draw $\vec{E}_3$. 4) Calculate the total field at point $P_1$ if a second charge equal to $-4q$ is placed at point $P3$. Give its modulus and the angle it makes to the $+y$ axis.




1. To represent a vector, we must choose a scale such that all the vectors can be drawn clearly on the figure.





1) Calculons $E_1$ :

$$E_1 = \frac{kq}{OP_1^2} = \frac{9 \times 10^9 \times 10^{-9}}{3^2} = 1\,\mathrm{N/C}. \qquad (1.11)$$

The vector $\vec{E}_1$ (see figure above) is carried by the line joining $q$ to $P_1$, i.e. by the $y$ axis. It is oriented in the direction of $\overrightarrow{OP_1}$ because $q > 0$ : $\vec{E}_1 = E_1\vec{j}$. Since it is 1,N/C, its length on the drawing will be 4 cm, as required by the statement.

2) Let's express $E_2$ as a function of $E_1$ : Before we start, let's notice that $OP_2 = 2OP_1$.

$$E_2 = \frac{kq}{OP_2^2} = \frac{kq}{(2OP_1)^2} = \frac{1}{4}\frac{kq}{OP_1^2} = \frac{1}{4}E_1. \qquad (1.12)$$

The vector $\vec{E}_2$ is oriented along $\overrightarrow{OP_2}$. Its length on the drawing will be $4\,\mathrm{cm}/4 = 1$ cm.

3) Let's express $E_3$ as a function of $E_1$ : $OP_3^2 = OP_1^2 + OP_2^2 = OP_1^2 + 4OP_1^2 = 5OP_1^2$.

$$E_3 = \frac{kq}{OP_3^2} = \frac{kq}{5OP_1^2} = E_1/5 = 0.2\,E_1. \qquad (1.13)$$

The vector $\vec{E}_3$ is oriented along $\overrightarrow{OP_3}$. Its length on the drawing will be $4\,\mathrm{cm}/5 = 0.8$ cm.

4) The field created at $P_1$ by the charge $-4q$ placed at $P_3$ is : $\vec{E}_{-4q} = k(-4q)(\overrightarrow{P_3P_1}/P_3P_1^3) = -4kq(-6\vec{i}/6^3) = (kq/9)(\vec{i}) = 1\,\mathrm{N/C}\,\vec{i}$. The total field at $P_1$ is : $\vec{E}_{\text{total}} = \vec{E}_1 + \vec{E}_{-4q} = 1\,\mathrm{N/C}\,\vec{i} + 1\,\mathrm{N/C}\,\vec{j} = 1\,\mathrm{N/C}\,(\vec{i} + \vec{j})$. It makes 45° with $+y$, its modulus is $1\,\mathrm{N/C}\,\|(\vec{i} + \vec{j})\| = \sqrt{2}\,\mathrm{N/C}$.

## 1.5 Electrostatic field created by a continuous distribution of charges

Sometimes, charges are distributed in such a way that they are very close or even stick together. These are referred to as continuous charge distributions or systems, as opposed to the discrete set discussed above.

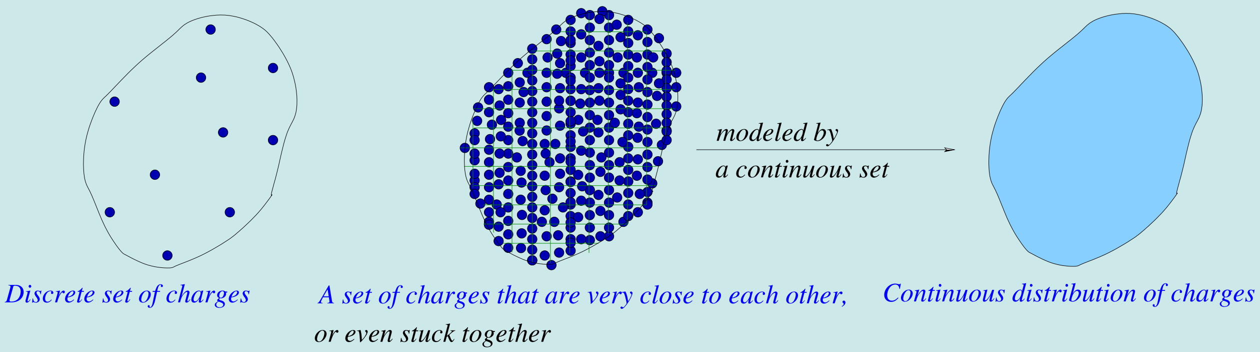



FIGURE 1.4 – Continuous distribution of charges

To calculate the field created by a continuous system, we divide the system into small elements $dq$ that can be assimilated to point charges. Each element $dq$ creates the elementary field at a point $P$ in the surrounding space, in accordance with equation (1.4) (see figure below) :

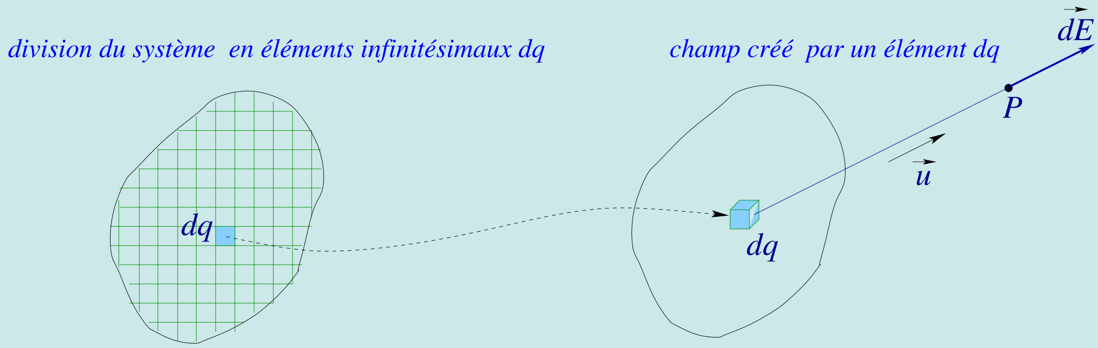



FIGURE 1.5 – Field created by a continuous distribution of charges

$$d\vec{E} = k\frac{dq}{r^2}\vec{u}, \tag{1.14}$$

The total field is obtained by adding together the fields created at $P$ by all $dq$ elements. Mathematically, this operation is expressed using the integral :

$$\vec{E} = k\int_{distribution}\frac{dq}{r^2}\vec{u}. \tag{1.15}$$

To evaluate this integral, it is necessary to write all quantities that vary as a function of a single variable. We can have a continuous distribution on a wire (Linear distribution), a plate (surface distribution) or a bulky object (Linear distribution).

**Line (or linear) distribution**
For a line distribution $L$ (for example a charged wire), we can define a linear charge density

$$\lambda = \frac{dq}{dl}$$

where $dl$ (which can be called $dx$, $dy$, $dz$ if, for example, the wire is arranged along one of the axes of an $xyz$ system.) is a small element of wire length. The unit of $ambda$ is $C$/m (or $C$m$^{-1}$). With this definition, the equation (1.15) for a linear distribution is written as :

$$\vec{E} = k\int_L\frac{\lambda dl}{r^2}\vec{u}. \tag{1.16}$$

**Surface distribution**
For a surface distribution $S$ (e.g. a charged disk), we can define a surface charge density

$$\sigma = \frac{dq}{ds}$$

where $ds$ is an élément of the surface. The unit of $\sigma$ is, of course, C/m$^2$ (or $C$m$^{-2}$). With this definition, the equation (1.15) for a surface distribution is written as :

$$\vec{E} = k\iint_S\frac{\sigma ds}{r^2}\vec{u}. \tag{1.17}$$

**Volume distribution**
For a volume distribution $V$ (e.g. a charged solid body), we can define a volume charge density

$$\rho = \frac{dq}{dv}$$

where $dv$ is an element of volume $V$. The unit of $\rho$ is C/m$^3$ (or $C$m$^{-3}$). Equation (1.15) becomes :

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \iiint_V \frac{\rho dv}{r^2}\vec{u}. \tag{1.18}$$

**Example 2** : Calculate the field created at distance $d$ by an infinitely long straight wire charged with negative uniform linear density $-\lambda$. What would have changed if we had taken a uniform linear density $\lambda > 0$ ?

*Solution :*

Let's take a point $P$ at distance $a$ from the wire. from the wire. Let's call $O$ the orthogonal project of $P$ onto the wire. For simplicity's sake, we'll work with respect to the basis $(O; \vec{n},\vec{t})$ where $\vec{n} = \overrightarrow{OP}/OP$ and $\vec{t}$ the unit vector directly perpendicular to $\vec{n}$, i.e. carried by the wire (figure $b$) opposite). To calculate the field, نقوم بتقسيم السلك إلى عناصر صغيرة بما يكفي بحيث يمكن اعتبارها كتل نقطية the wire is broken down into $dl$ elements small enough to be considered as point masses, as shown in figure $a$) opposite. Each $dl$ can be identified by its distance $l$ from $O$ (see figure $b$)). Since the wire is charged with a constant density $-\lambda$, each $dl$ carries the point charge $dq = -\lambda dl$ and creates in $P$ the field

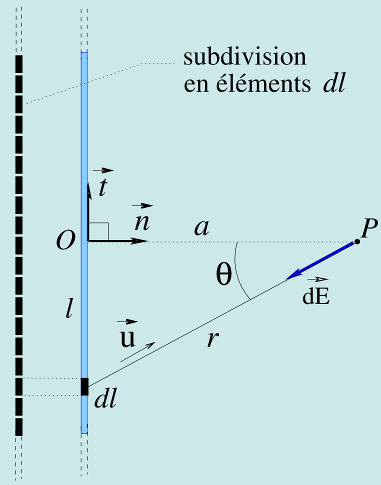
$$d\vec{E} = \frac{-k\lambda dl}{r^2}\vec{u}. \tag{1.19}$$

In the basis $(\vec{n},\vec{t})$, $\vec{u} = \cos\theta\,\vec{n} - \sin\theta\,\vec{t}$ and consequently :

$$d\vec{E} = \frac{-k\lambda dl}{r^2}(\cos\theta\,\vec{n} - \sin\theta\,\vec{t}) \tag{1.20}$$

The total field is obtained by summing the contributions of all $dl$ elements, i.e. by integrating $d\vec{E}$ over the entire wire :

$$\vec{E} = \int_{\text{fil}} \frac{-k\lambda dl}{r^2}(\cos\theta\,\vec{n} - \sin\theta\,\vec{t}) \tag{1.21}$$




a) b)

As we move along the different $dl$, $l$ varies, generating variations in $\theta$ and $r$. To perform the integral, we need to express these three variables as functions of a single variable. The simplest choice is to write everything as a function of $\theta$. Note that $\theta$ is referenced to the $OP$ axis. From $\tan\theta = l/a$ we derive $l = d\tan\theta$ and $dl = ad\theta/\cos^2\theta$ (i). On the other hand, knowing that $\cos\theta = a/r$ we deduce $r^2 = a^2/\cos^2\theta$ (ii). From (i) and (ii) we obtain $dl/r^2 = d\theta/a$ and by substitution in equation (1.21), we arrive at :

$$\vec{E} = \frac{-k\lambda}{a}\int_{\text{fil}} (\cos\theta\,\vec{n} - \sin\theta\,\vec{t})\,d\theta \tag{1.22}$$

$$= \frac{-k\lambda}{a}\int_{\text{fil}} \cos\theta\,d\theta\,\vec{n} + \frac{-k\lambda}{a}\int_{\text{fil}} -\sin\theta\,d\theta\,\vec{t}. \tag{1.23}$$

The integration variable is $\theta$. To describe the whole wire, we need to integrate from $\theta = -\pi/2$ to $\theta = +\pi/2$. The first integral gives $\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = [\sin\theta]_{-\pi/2}^{\pi/2} = 2$ ; whereas the second is $\int_{-\pi/2}^{\pi/2} -\sin\theta\,d\theta = [\cos\theta]_{-\pi/2}^{\pi/2} = 0$. Finally

$$\vec{E} = \frac{-2k\lambda}{a}\,\vec{n} \tag{1.24}$$

We obtain a total field perpendicular to the wire. The parallel component (component along $\vec{t}$) is zero, as we would expect from the symmetry of the system. Because of its infinite length, the wire is symmetrical with respect to $O$. It's easy to see that two symmetrical $dl$ elements produce a resultant field along $\vec{n}$. The same is true for any pair of symmetrical elements, leading to the conclusion that the whole wire produces a total field along $\vec{n}$.

With a linear density $\lambda > 0$, we obtain the same result as above, but replace $-\lambda$ by $\lambda$. In the figure, you just need to reverse the direction of the field vectors.

## 1.6 Lines of field

To visualize the electrostatic field (or any other vector field) at all points in space, we use *field lines*.

A field line is defined as a curve which is tangent at each of its points to the electric field vector. The field lines are oriented in the direction of the electrostatic field.

The figure below shows the field lines for the infinitely long, uniformly charged straight wire. The calculation of the field has already been done in example 2 of section **??** where we found that the field at any point outside the wire points perpendicularly towards the wire. Based on the definition, it is easy to deduce that the field lines are straight lines oriented perpendicularly towards the wire. What we see in FIGURE **??**-(a) is valid in any plane containing the wire.

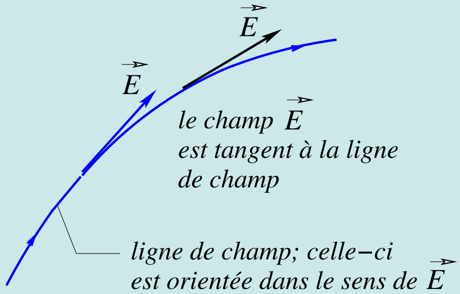



FIGURE 1.6 – Ligne de champ

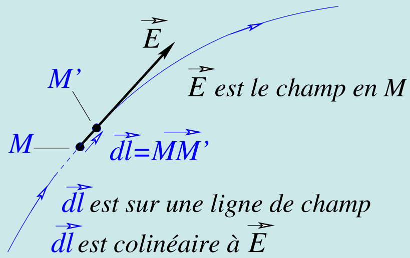
### 1.6.1 Equation of a field line

Consider an infinitesimal element $\vec{dl}$ taken on a field line. On the FIGURE **??** b, $\vec{dl}$ is identified with the vector $\overrightarrow{MM'}$, $M$ and $M'$ being two infinitely close points on the line.
The field $\vec{E}$ at $M$ is parallel (because tangent at $M$) to $\vec{dl}$, which can be expressed by the equation :

$$\vec{E} \times \vec{dl} = \vec{0}. \tag{1.25}$$

This is vectorial equation of a field line. Let's write it in different coordinate systems :

1. Cartesian coordinates :
$$\frac{dx}{E_x} = \frac{dy}{E_y} = \frac{dz}{E_z} \tag{1.26}$$
2. Cylindrical polar coordinates :
$$\frac{d\rho}{E_\rho} = \frac{\rho d\theta}{E_\theta} = \frac{dz}{E_z} \tag{1.27}$$
3. Spherical polar coordinates :
$$\frac{dr}{E_r} = \frac{r d\theta}{E_\theta} = \frac{r \sin\theta d\phi}{E_\phi} \tag{1.28}$$
4. Two-dimensional case : in 2 dimensions, the equation (1.26) reduces to :
$$\frac{dx}{E_x} = \frac{dy}{E_y} \tag{1.29}$$




(b)

FIGURE 1.7 – Equat-Ligne de champ

Similarly, in plane polar coordinates, the equation (1.27) reduces to :

$$\frac{d\rho}{E_\rho} = \frac{\rho d\theta}{E_\theta} \tag{1.30}$$

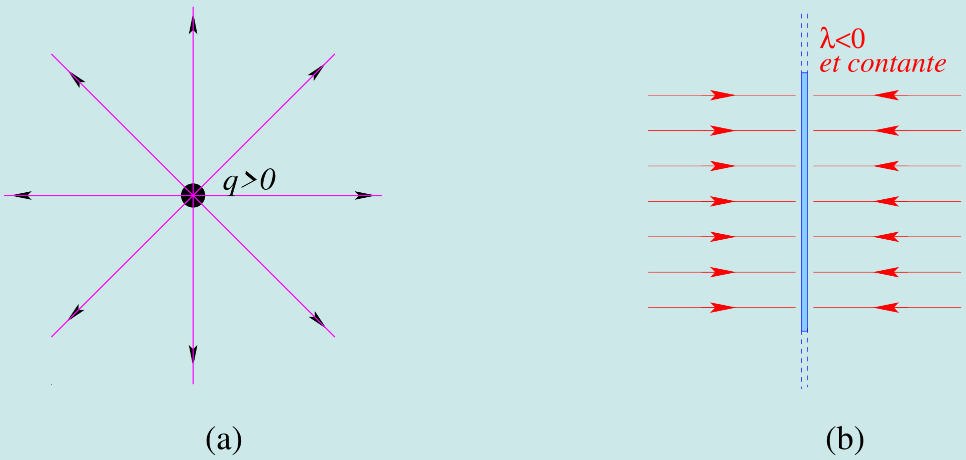



FIGURE 1.8 – (a) : Field lines of a charge $q > 0$, b) : Field lines of an infinitely long, straight wire charged with constant density $\lambda < 0$ ; (c) : An element taken on a field line at a point $M$. The vector $\vec{d\ell}$ is collinear with the electric field $\vec{E}$ at $M$.

Field lines have the following properties :

1- Field lines always start at positive charges and end at negative charges.

2- The number of field lines starting from or ending at a charge is proportional to the size of the charge.

3- Field intensity is proportional to the density of the lines, i.e. the number of lines crossing a unit area normal to the field.

4- Field lines never intersect. The reason is that at a given point in space, the field can only have one direction.

5- We'll also see that field lines are perpendicular to every equipotential surface they intercept, including conductor surfaces.

### 1.6.2 Gauss's theorem

Gauss's theorem can be stated as follows : *the flux of the electric field through any closed surface $S$ is $q_{\text{int}}/\epsilon_0$, $q_{\text{int}}$ being the algebraic sum of all charges contained within the volume bounded by $S$.*

$$\oiint_S \vec{E} \cdot \vec{dS} = \frac{q_{\text{int}}}{\epsilon_0}. \tag{1.31}$$

Notice :
- The closed surface $S$ is sometimes referred to as a Gaussian surface and sometimes denoted $S_G$.
- $S$ can contain a discrete and/or continuous system of charges.
- $\vec{dS}$ is a surface-vector element of $S$ (شعاع السطح العنصري), directed perpendicularly outward to the surface element $dS$.

### 1.6.3 Application of Gauss's theorem for electric field calculation

The application of Gauss's theorem is very useful for calculating the electrostatic field created by charge distributions with a high degree of symmetry. In this case, a judicious choice of a Gauss surface $S_G$ will make it easy to calculate the integral (1.31). Such a choice may be guided by the following observations :

1. If, at all points on a surface $S$, the field $\vec{E}$ maintains a constant modulus and is directed perpendicular to $S$, then $E \cdot \vec{dS} = EdS\cos(0\,\text{ou}\,\pi) = \pm E\,dS$, + if $\vec{E}$ parallel to $\vec{dS}$ and − if $\vec{E}$ antiparallel to $\vec{dS}$.

$$\oiint_S \vec{E}\cdot\vec{dS} = \pm\oiint_S EdS = \pm E\oiint_S dS = \pm ES. \qquad (1.32)$$

2. If, at all points, the field is directed parallel to $S$, then $\vec{E}$ is perpendicular to $\vec{dS}$ and the integral gives zero.
3. If, at all points on a surface $S$, the field $\vec{E}$ is zero, then the integral gives zero.

This is what we'll show in the various examples below.

**Worked example 3 :**

Using Gauss's theorem, let's find the result of the field created at distance $a$ by an infinite rectilinear wire carrying a constant, nonnegative linear charge -*lambda* C/m, (example 2, section 1.5).

**Solution :**

Since Gauss's theorem is valid regardless of the shape of the closed surface, the choice of the Gaussian surface will be the one that offers the greatest ease of calculation. In general, this choice is dictated by the symmetry of the of the charge distribution. In the case of the infinite wire uniformly charged wire, the field is directed perpendicular to the wire and has the same modulus at all points located at the distance $a$ from the wire.

The appropriate choice of Gauss surface Gauss surface is a cylinder of radius $a$ and length $L$. The closed surface is the sum of the lateral surface $S_1$ and the two base surfaces $S_2$ and $S_3$ : $S = S_1 + S_2 + S_3$. The flux through $S$ is

$$\Phi = \int_{(S)} \vec{E}\cdot\vec{dS} = \int_{(S_1)} \vec{E}_1\cdot d\vec{S}_1 + \int_{(S_2)} \vec{E}_2\cdot d\vec{S}_2 + \int_{(S_3)} \vec{E}_3\cdot d\vec{S}_3, \qquad (1.33)$$

where $\vec{E}_i$ is the value of $\vec{E}$ at any point on $dS_i$. For any $\vec{dS}$ element taken on $S_2$ or $S_3$, $\vec{E}$ and *vecdS* are perpendicular to each other, and therefore no flux passes through $S_2$ and $S_3$. So,

$$\Phi = \int_{S_1} \vec{E}_1\cdot d\vec{S}_1. \qquad (1.34)$$

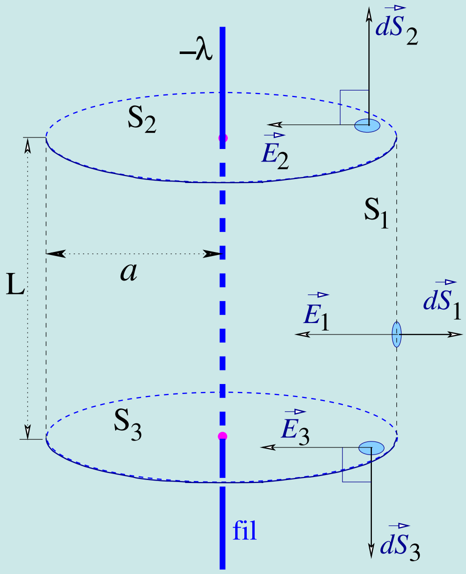



FIGURE 1.9 – Gauss's theorem applied to the calculation of the field created at distance $a$ by an infinite rectilinear wire carrying a negative constant linear charge $-\lambda$ C/m

On $S_1$, $\vec{E}$ is antiparallel to $\vec{dS}_1$ so that $\vec{E}\cdot d\vec{S}_1 = -EdS_1$. Knowing, moreover, that $\vec{E}$ keeps a constant modulus at all points of $S_1$, the integral (1.34) gives

$$\Phi = \int_{S_1} -EdS_1 = -ES_1 = -E2\pi aL$$

By simply observing the figure, we can see that the charge contained in the cylinder is $-\lambda L$. Applying Gauss's theorem, we have :

$$-E2\pi aL = \frac{-\lambda L}{\epsilon_0}, \text{ d'où l'on tire } E = -\frac{\lambda}{2\pi\epsilon_0 a} = -\frac{2k\lambda}{a}. \qquad (1.35)$$

which is indeed the result obtained by direct calculation in Example 2. It's much simpler with Gauss's theorem. It's worth making the following point. Gauss's theorem states that external charges do not contribute to the flux of the electric field, but they do contribute to the total field $\vec{E}$. Here, the external charges are in the part of the wire outside the cylinder.

## 1.7 Le potentiel électrique

We saw earlier that when a charge $q'$ is at a point where there is an electric field $\vec{E}$, it feels a force $\vec{F} = q'\vec{E}$. The electric force $\vec{F}$ is a conservative force (قوة محافظة) and, as such, is derived from potential energy. In other words, the charge $q'$ also has a potential energy $E_p$ such that :

$$\boxed{\vec{F} = -\overrightarrow{grad}E_p} \equiv \frac{\partial E_p}{\partial x}\,\vec{i} + \frac{\partial E_p}{\partial y}\,\vec{j} + \frac{\partial E_p}{\partial z}\,\vec{k} \tag{1.36}$$

When $\vec{E}$ is produced by a point charge $q$ located at distance $r$ from $q'$, then $\vec{E} = (kq/r^2)\vec{u}_r$ and equation (1.36) becomes :

$$q'\frac{kq}{r^2}\vec{u}_r = -\overrightarrow{grad}E_p \tag{1.37}$$

Since the term on the left depends only on $r$, so does the term on the right, and therefore $\overrightarrow{grad}E_p = (dEp/dr)\vec{u}_r$ ; the equation (1.37) is then written :

$$q'\frac{kq}{r^2} = -\frac{dEp}{dr} \tag{1.38}$$

from which we derive

$$E_p = -\int q'\frac{kq}{r^2}dr = q'\frac{kq}{r} + C \tag{1.39}$$

Potential energy is defined to within a constant $C$. When there are no charges at infinity, the potential cancels when $r \to \infty$ and in this case $C = 0$, which gives :

$$E_p = q'\frac{kq}{r} \tag{1.40}$$

Unless otherwise stated, this is the case for the remainder of this chapter.

In equation (1.8), we define the electric field at a point as the force felt by a unit charge placed at that point. Similarly, we introduce the concept of *electric potential*, denoted $V$, at a point as the potential energy of a unit charge placed at that point, i.e. the potential energy per unit charge. We therefore write :

$$\boxed{V = \frac{E_p}{q'}} \to E_p = q'V \tag{1.41}$$

Potential is a scalar (non-vector) quantity. From the previous equation, we see that the SI unit of $V$ is the joule per coulomb (J/C). This unit is called the volt (V), in tribute to the Italian scientist Alessandro Volta (1745-1827) for his great work on electricity : $1\,\mathrm{V} = 1\,\mathrm{J/C}$.

From equations (1.40) and (1.41), we derive the expression for the potential created by a point charge $q$ at distance $r$ :

$$\boxed{V = k\frac{q}{r}} \tag{1.42}$$

In the case where $V$ is created by a set of charges $q_1$, $q_2$, $q_3$, ..., it is written as :

$$\boxed{V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + k\frac{q_3}{r_3} + \ldots = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right)} \tag{1.43}$$

## 1.8 Potential created by a continuous distribution of charges

An elementary charge $dq$ of the distribution creates at distance $r$ the elementary potential $dV = kdq/r$ and $V_{total}$ is obtained by integration.

a) For a volume distribution of charge density $\rho$ (in C/m$^3$), an element of volume $dv$ taken from the distribution carries the charge $dq = \rho\, dv$ and we have :

$$V_{\text{total}} = \iiint_{\text{volume}} k\frac{\rho dv}{r}\,(+constante). \tag{1.44}$$

b) For a surface distribution of charge density $\sigma$ (in C/m$^2$), a $ds$ surface element taken from the distribution carries the charge $dq = \sigma\, ds$ and we have :

$$V_{\text{total}} = \iint_{\text{surface}} k\frac{\sigma ds}{r}\,(+constante). \tag{1.45}$$

c) For a linear distribution of charge density $\lambda$ (in C/m), a $dl$ linear element taken from the distribution carries the charge $dq = \lambda\, dl$ and we have :

$$V_{\text{total}} = \int_{\text{ligne}} k\frac{\lambda dl}{r}\,(+constante). \tag{1.46}$$

**Notice :** In the case where charges are distributed in a finite-dimensional volume, the potential tends towards 0 as we move away to infinity, setting the *constant* at 0. When the distribution is infinite, we cannot choose $V = 0$ at infinity. In this case, the *constant* is determined by choosing a reference potential $V = V_0$ at a position other than infinity. For an infinite charge distribution, it is generally not possible to directly calculate the potential created by it. The first step is to calculate the electric field, then use the relation $dV = -\vec{E}\cdot d\vec{r}$ to obtain the potential.

## 1.9 The Electric Field Derived from the Potential

Now that we know that $E_p = q'V$, the equation (1.36) is written :

$$q'\vec{E} = -\overrightarrow{grad}\, q'V, \tag{1.47}$$

or, simplifying by $q'$ ($q'$ is not affected by the gradient) :

$$\boxed{\vec{E} = -\overrightarrow{grad}\, V} \tag{1.48}$$

By multiplying the 2 members scalarly by $-d\vec{r}$, we have :

$$\overrightarrow{grad}\, V\cdot d\vec{r} = -\vec{E}\cdot d\vec{r}. \tag{1.49}$$

Developing it on an orthonormal basis $(\vec{i},\vec{j},\vec{k})$, the left-hand member gives : $\left(\frac{\partial V}{\partial x}\vec{i} + \frac{\partial V}{\partial y}\vec{j} + \frac{\partial V}{\partial z}\vec{k}\right)\cdot(dx\vec{i} + dy\vec{j} + dz\vec{k}) = \left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$. But $\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$ is nothing more than the total differential $dV$, which leads to :

$$\boxed{dV = -\vec{E}\cdot d\vec{r}}. \tag{1.50}$$

Equation (1.50) is an equivalent form to equation (1.48). *Know both forms by heart.* The equation (1.50) also shows that potential can be expressed in volts per meter ( V/m). This is a unit equivalent to N/C, but in practice V/m is more commonly used.

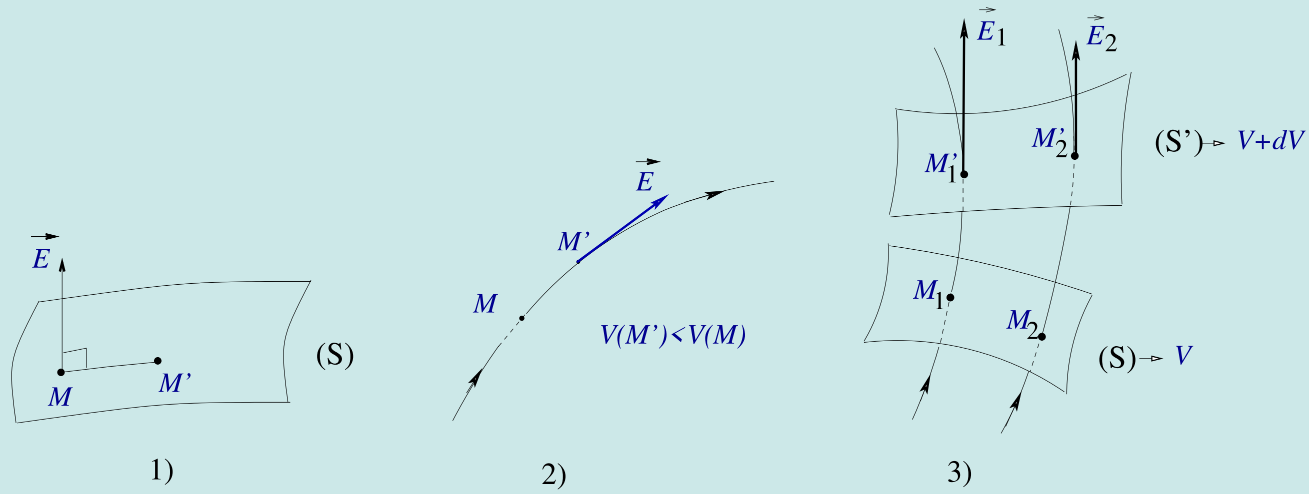



FIGURE 1.10 – 1) Field lines ⊥ at equipotential surfaces ; 2) Potential decreases along a field line ; 3) Equipotential surfaces are tighter in regions of intense field.

## 1.10 Equipotential surfaces and their relationship to field lines

An equipotential surface (or simply an equipotential) is a surface whose points are at the same potential. From the definition we deduce the following properties :
1) Field lines are always perpendicular to surfaces equipotential surfaces. Indeed, if we take two points $M$ and $M'$ infinitely close ($\overrightarrow{MM'} = d\vec{r}$) on the same equipotential surface (Figure 1 below), then $dV = V(M') - V(M) = 0$, it follows, from equation (1.50), that $\vec{E} \cdot \overrightarrow{MM'} = 0$ and therefore $\vec{E}$ perpendicular in $M$ to the surface. This result also means that no work is required to displace a charge on an equipotential.

2) The potential decreases along a field line. In other words field lines are oriented towards decreasing potentials. In fact (Figure 2 below), for a displacement of $d\vec{\ell} = \overrightarrow{MM'}$ ($M$ and $M'$ are on the same line of field), we have : $dV = -\vec{E} \cdot d\vec{\ell}$. For $M'$ infinitely close to $M$, $\overrightarrow{MM'}$ is parallel to $\vec{E}$, it follows that : $dV = V(M') - V(M) = -\|\vec{E}\| \times \|d\vec{\ell}\| < 0$. We then have $V(M) < V(M')$, which demonstrates our statement.

3) Equipotential surfaces are tighter in regions of high field strength than in regions of low field strength. To see this, let's consider equipotential surfaces $(S)$ of potential $V$ and $(S')$ of potential $V + dV$ (Figure 3 below). Going along a field line (i.e. perpendicularly) from a point $M_1$ of $(S)$ to a point $M_1'$ of $(S')$, we have : $dV = -\vec{E}_1 \cdot d\vec{\ell}_1 = - \mid \vec{E}_1 \mid , \mid d\vec{\ell}_1 \mid$. Going on another field line point $M_2$ of $(S)$ to a point $M_2'$ of $(S')$, we have : $dV = -\vec{E}_2 \cdot d\vec{\ell}_2 = - \mid \vec{E}_2 \mid \mid d\vec{\ell}_2 \mid$.If we assume that the field is stronger in the region of $M_1$ , i.e. if $\mid \vec{E}_1 \mid > \mid \vec{E}_2 \mid$ then $\mid d\vec{\ell}_1 \mid < \mid d\vec{\ell}_2 \mid$.

Figure 1.11 shows two examples of equipotential surfaces. In (a), equipotential surfaces of a point charge, which are of course spheres centered on the charge. In (b), equipotential equipotential surfaces associated with a constant field. At the same time, we've drawn a few field lines to show that they are indeed perpendicular to the equipotential equipotential surfaces.

## 1.11 Electrostatic dipole

We call *electrostatic dipole* a set of two equal and opposite charges $+q$ and $-q$, separated by a distance $d$ small compared to the distance at which the effects are studied. Dipole theory applies, among other things, to certain polar molecules in which the center of mass of the positive charges does not coincide with that of

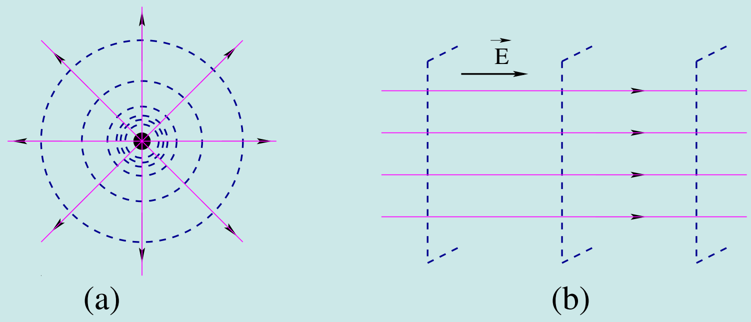



FIGURE 1.11 – Two examples of field lines (solid line) and equipotential surfaces (dotted line)

the negative charges. The HCl et $H_2O$ are examples of polar molecules, they form permanent dipoles. In a non-polar molecule, a dipole can be induced by the action of an external electric field (say how ?) ; the induced dipole disappears if the field is removed.

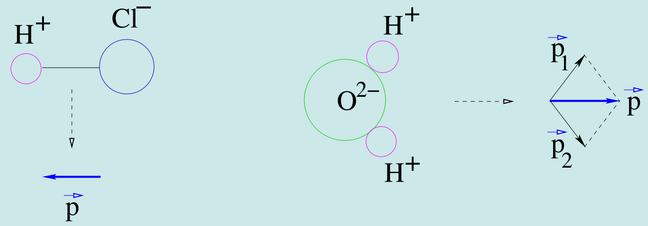



FIGURE 1.12 – Molecules HCl and $H_2O$ with their dipole ; water molecule is composed by two dipoles $\vec{p}_1$ and $\vec{p}_2$ : $\vec{p}_1 + \vec{p}_2 = \vec{p}$.