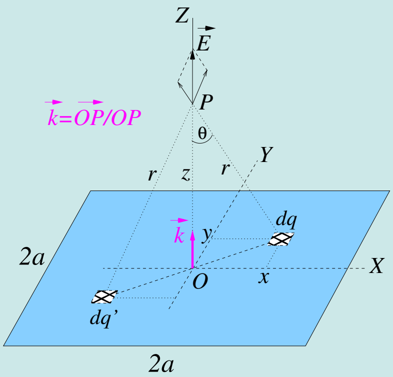
University of Batna 2 – Mostefa Ben Boulaïd

FACULTY OF MATHEMATICS
AND INFORMATICS
Departement Commun Base - Mathematics and Informatics

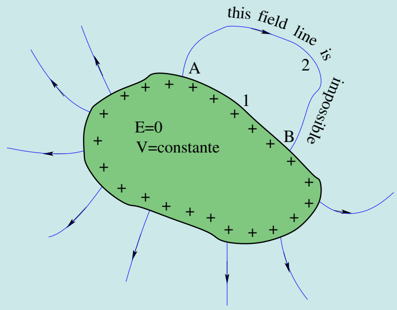
Course of Phy



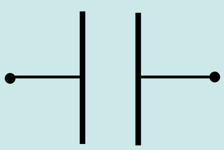
$$\vec{E} = \frac{\sigma}{\epsilon_0} \tan^{-1}\left(\frac{a^2}{z\sqrt{z^2+2a^2}}\right)\vec{k}.$$

If the size of the sheet became infinitely large, we would have to return to the case of the infinite plane :

$$\lim_{a\to\infty} \vec{E} = \frac{\sigma}{2\epsilon_0}\vec{k}.$$



Above, in blue, we see a uniformly charged square sheet (plate). At a point P on its axis (of symmetry), z-coordinate z, it creates the field $\vec{E}$ whose expression is written as shown on its right. Further to the right, in green, we have a conductor at equilibrium ; its excess charges are necessarily distributed over its outer surface. A field line emerging from the conductor cannot return to the conductor.
As for the pictures displayed below, the left shows the symbol used to represent a capacitor in an electrical diagram, the middle picture shows different types of commercially available capacitors, and the one on the right is a typical thunderstorm flash resulting from an electrostatic discharge between the clouds and the ground.



Prof. M. A. Belkhir 2023-2024

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Chapitre 1

Conductors in electrostatic equilibrium

1.1 Definition of an electrostatic equilibrium

We mentioned in Chapter 1 that a conductor is a body containing free electrons that move easily. If excess charge is brought to a conductor, it will interact with the charges (protons and electrons) in the conductor. This electrostatic interaction results in a rapid redistribution of electrons within the conductor, leading to a state where the charges stop moving and enter a state of equilibrium, or *electrostatic equilibrium.* An electrostatic equilibrium gives rise to a number of properties that we develop below.

1) At electrostatic equilibrium, the fiel is zero ($\vec{E} = \vec{0}$) at any point inside the conductor.

Knowing that charges are in equilibrium ($\vec{F} = q\vec{E} = \vec{0}$) inside the conductor, we deduce that $\vec{E} = \vec{0}$. *Notice* : If the conductor is placed in an external field, to reach a state of equilibrium the charges inside rearrange (redistribute) themselves to create a field that exactly compensates for the external field at every point inside the conductor.

2) A conductor in electrostatic equilibrium constitutes an equipotential volume.

Since the field derives from a potential ($\vec{E} = -\vec{\nabla}V$), a zero field inside the conductor implies that the associated potential is constant at all points inside the conductor : ($V = \text{constant}$).

3) Field lines running from or to the surface of a conductor in equilibrium are perpendicular to the surface.

According to the above, the field inside a conductor (charged or not) is zero. But this is not necessarily the case on the outside, especially if the conductor is charged. Since the potential is continuous across a charged surface, the potential at the surface will have the same value as that of an interior point infinitely close to the surface. We deduce that

4) the surface of the conductor has the same potential as the conductor, and is therefore an equipotential surface.

It follows that

5) It is impossible for a field line that emerges from a conductor to return to the conductor. Conversely, it is impossible for a field line that arrives at a conductor to be part of that conductor.

To demonstrate this, consider two points A and B on the surface of a conductor in electrostatic equilibrium, and assume that a field line runs from A to B. Since the surface is equipotential, we must have :

$$V_A = V_B \implies V_A - V_B = 0. \tag{1.1}$$

But, using the equation $dV = -\vec{E} \cdot \vec{d\ell}$ (see section **??**, chapter 2), we must have at the same time :

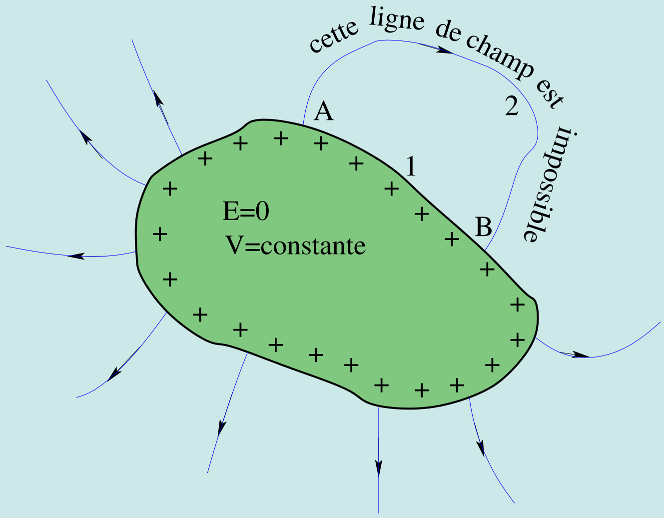

FIGURE 1.1 – The circulation of $\vec{E}$ from A to B is impossible along path 2 if A and B are two points on the surface of a conductor in electrostatic equilibrium.

$$\int_{V_A}^{V_B} dV = -\int_A^B \vec{E} \cdot \vec{d\ell} \implies V_A - V_B = \int_A^B \vec{E} \cdot \vec{d\ell}. \tag{1.2}$$

Since $\vec{E}$ is conservative, the integral $\int_A^B \vec{E} \cdot \vec{d\ell}$ does not depend on the path taken from A to B, especially if $\vec{d\ell}$ is taken on a field line. But we know that at any point on a field line, $\vec{E}$ is tangent to the line, i.e., parallel to the element $\vec{d\ell}$ surrounding the point under consideration. Therefore, the scalar product $\vec{E} \cdot \vec{d\ell}$ is necessarily $\neq 0$ (> 0 if we choose $\vec{d\ell}$ in the same direction as $\vec{E}$), so we have :

$$\int_A^B \vec{E} \cdot \vec{d\ell} \neq 0. \tag{1.3}$$

Equality (1.1) and inequality (1.3) show us that equality $V_A - V_B = \int_A^B \vec{\cdot}\vec{d\ell}$ cannot take place for points A and B on the surface of a conductor in electrostatic equilibrium. In other words,

6) Excess charges carried by a conductor in equilibrium are necessarily distributed over the surface of the conductor.

Consider a closed surface inside the conductor. According to property 1, since the closed surface is inside the conductor, the field is zero at all points on this surface. Gauss's theorem implies that the total charge contained in this surface is zero (there are as many + charges as − charges). As shown in figure 1.2(see dotted lines), we can choose this closedsurface so that it is just inside the conductor. Since $vecE = vec0$ at all points on this surface (property 1), we conclude that there is no *net* charge inside the conductor. If a conductor in a conductor in equilibrium carries excess charges, these are necessarily distributed over the surface of the conductor.

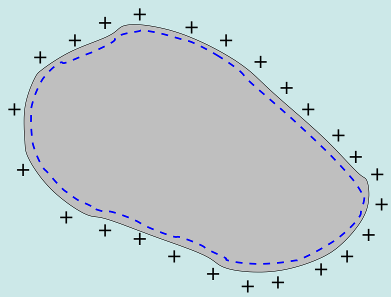

FIGURE 1.2 – Excess charges are distributed over the surface.

1.1.1 Case of a hollow conductor

1- Figure 1.3 shows an empty cavity dug into a conductor. According to the above (property 2), at equilibrium all points inside the conductor are at the same potential. For points A and B on the Figure 1.3, we then have : $V_A = V_B$. Knowing that $V_A - V_B = \int_A^B \vec{E} \cdot \vec{d\ell}$, it follows $\int_A^B \vec{E} \cdot \vec{d\ell} = 0$. Since the integral does not depend on the path taken to get from A to B, it must give 0 for any path, especially for a path through the cavity (Figure 1.3). We conclude that

7) The field inside the cavity is also zero, which implies that the potential inside the cavity is constant and equal by continuity to the potential of the conductor ; the points of the conductor and the cavity are at the same potential.

2- If we consider a closed surface S as indicated by the dotted line in figure 1.3, the flow through S is

$$\Phi = \oint\!\!\oint_S \vec{E} \cdot \vec{dS}$$

where the integral is performed on the closed surface S. Since $\vec{E} = \vec{0}$, it follows that the flux is zero and that, after Gauss's theorem, the surface S contains no net charge. Knowing that there are no charges in the mass of the conductor or in the cavity, we conclude that there are none on the cavity surface S_{int} either.

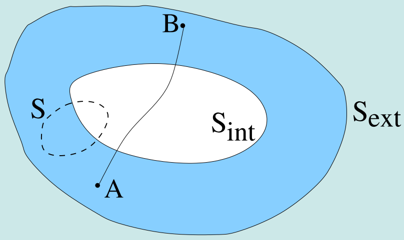

FIGURE 1.3 – Uncompensated charges are distributed over the external surface.

8) At equilibrium, excess charges in a hollow conductor can only be placed on its outer surface.

1.2 Field created by a conductor in equilibrium in its immediate vicinity : Coulomb's theorem

Let's consider a point M outside and infinitely close to the surface S of a conductor and let dS_{ext} be a surface element surrounding M and parallel to S. The field $\vec{E}$ at M is normal to S.

Now let's add a surface S_{int} inside the conductor and a lateral surface S_{lat} to form a closed surface. In Figure 1.4, we've constructed a cylindrical closed surface, but the surface doesn't need to be cylindrical (we can choose a non-circular cross-section). Let's write the flow through the closed surface. A priori, there are three contributions.

1) The flux through S_{int} is zero because the field there is zero.

2) The flux through S_{lat} is zero because the field is zero in its inner part (that is inside the conductor) and because the field is perpendicular to it ($\vec{E}$ perpendicular to $d\vec{S}_{\text{lat}}$) in its outer part.

3) The flux through S_{ext} is $d\phi = \vec{E} \cdot d\vec{S}_{\text{ext}} = EdS_{\text{ext}}$ because $\vec{E} \parallel d\vec{S}_{\text{ext}}$. If σ is the surface charge density on the conductor surface S, the charge inside the closed surface is σdS_{ext} and Gauss's theorem gives :

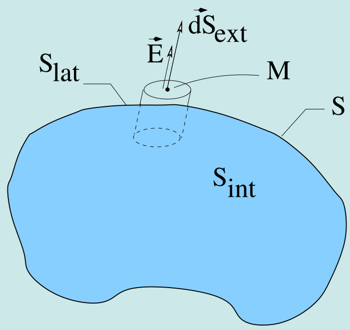

FIGURE 1.4 – Champ au voisinage d'un conducteur.

$$EdS_{\text{ext}} = \frac{\sigma dS_{\text{ext}}}{\epsilon_0}, \tag{1.4}$$

or, after simplifying by dS_{ext} :

$$E = \frac{\sigma}{\epsilon_0}. \tag{1.5}$$

The result (1.5) is known as Coulomb's theorem, which states that *the electric field in the immediate vicinity of a conductor in equilibrium is perpendicular to the surface of the conductor*. If *vecn* is a unit vector directed outwards normally to S, we have : $\vec{E} = \sigma/\epsilon_0\,\vec{n}$.

Depending on whether the density is positive or negative, $\vec{E}$ is directed respectively to the outside or the inside of the conductor. When the field strength in the vicinity of a conductor exceeds a certain value, a spark is observed : the medium surrounding the conductor becomes electrically conductive. This limiting field, of the order of 3×10^6 V/ m, is called *disruptive field.* If the conductor is in air, air molecules ionize when the field reaches or exceeds the disruptive field. In other words, air becomes conductive and a spark is observed.

1.3 Capacitor and capacitance

A capacitor is a device used to store electrical charge and electrical energy. A capacitor is any set of two conductors, one of which carries the positive charge Q, known as the capacitor charge, and the other the charge $-Q$, and where the facing surfaces of the two conductors are close to each other and separated by an insulator (vacuum, air, paper, etc.). The two conductors are called capacitor plates.
If V_+ and V_- are their respective potentials, we define the capacitance of the capacitor as :

$$C = \frac{Q}{V_+ - V_-}. \tag{1.6}$$

Capacitance is a **positive** quantity. It gives the amount of charge that a capacitor can store per unit of potential difference between its plates. In practice, we calculate the difference in potential between plates A and B without worrying about their sign. Then we take the absolute value $V = |V_B - V_A|$; the equation (1.6) then becomes :

$$C = \frac{Q}{V}. \tag{1.7}$$

To give equal and opposite charges to the two plates of a capacitor, we connect them for a short time to the terminals of a battery, Figure 1.5 a. A battery is a device with the property of maintaining a constant potential difference between its terminals. At equilibrium, the potential of each armature is the same as that of the terminal to which it is connected. The potential difference between the plates is therefore the same as that between the battery terminals. When the battery is disconnected, the charges remain on the plates due to their mutual attraction. Figure 1.5 b shows the electrical circuit diagram associated with Figure 1.5 a. Cells and batteries [1] (called *generators*) are symbolized by a large bar representing the positive terminal and a smaller, thicker bar representing the negative terminal. Figures 1.6 and 1.7 represent respectively the symbol of a capacitor in an electric circuit and an example of a capacitor sold on the market.

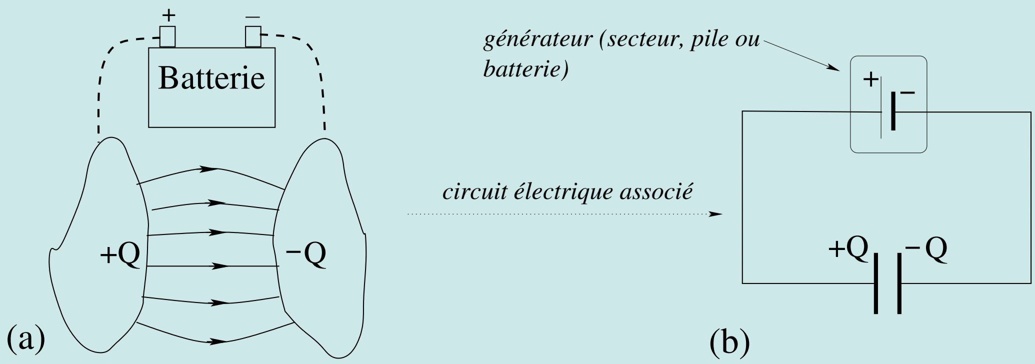

FIGURE 1.5 – Charging the plates with a battery.

1.4 Calculating the capacitance of typical capacitors

1.4.0.1 Parallel-plate capacitor

First, let's recall that the field created by an infinite plane charged with a uniform charge density σ is written as $\vec{E} = (\sigma/2\epsilon_0)\,\vec{n}$ where $\vec{n}$ is, by definition, a unit vector normal to the plane and directed away from the

1. A cell is a single device that converts chemical energy into electrical energy, while a battery is a collection of cells that provide a steady source of electrical energy.

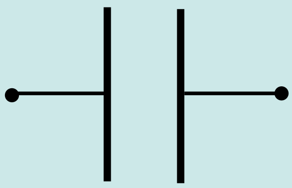

FIGURE 1.6 – Capacitor symbol in a standard electrical circuit diagram.

FIGURE 1.7 – Example of a capacitor sold on the market.

plane, see Chapter 2.

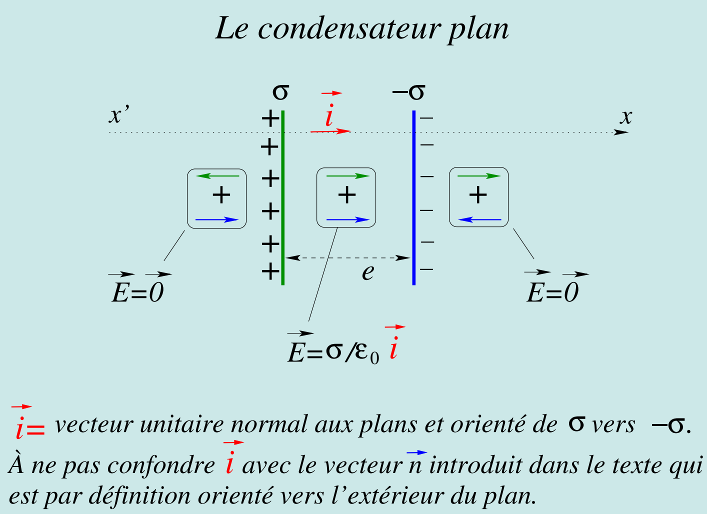

FIGURE 1.8 – Capacitance of a parallel-plate capacitor.

Let's consider two parallel infinite planes separated by a distance e, one charged with a positive density σ, the other with $-\sigma$, $\sigma =$ constant. In Figure 1.8, the fields are expressed as a function of $\vec{i}$, where $\vec{i}$ is a unit vector normal to the plane and oriented from σ to $-\sigma$. Note that $\vec{i}$ does not have the same definition as $\vec{n}$ (recall that $\vec{n}$ comes from the expression of the fielf created by a plane $\vec{E} = (\sigma/\epsilon_0)\,\vec{n}$ where $\vec{n}$ is a unit vector normal to the plane and directed outwards).
The principle of superposition (sum of the fields created separately by the two planes) gives $\vec{E} = \vec{0}$ in the region outside the two planes and $\vec{E} = (\sigma/\epsilon_0)\,\vec{i}$ between the two planes. To calculate the potential difference between the two planes (the two armatures), we use the relation $dV = -\vec{E}\cdot\vec{dr} = -(\sigma/\epsilon_0)\vec{i}\cdot\vec{dr}$. Since $\vec{i}\cdot\vec{dr} = \vec{i}\cdot(dx\vec{i} + dy\vec{j} + dz\vec{k}) = dx$, it comes :

$$\int_{V_\sigma}^{V_{-\sigma}} = -\int_0^e (\sigma/\epsilon_0)\,dx, \text{ soit } V_\sigma - V_{-\sigma} = \frac{\sigma e}{\epsilon_0}. \tag{1.8}$$

In reality, a planar capacitor is made up of two parallel plates of finite dimensions. They therefore have a finite area A and respectively carry the charges $\sigma A = Q$ and $-\sigma A = -Q$.

Edge effects mean that the field created by these two plates is not strictly constant between them, and is not strictly zero on the outside. But when the distance e between the plates is small compared to their dimensions, we can, with a good approximation, calculate the field as if the plates were of infinite dimensions. The previous results can then be applied. As a function of Q, the equation (1.8) is written as :

$$V_\sigma - V_{-\sigma} = Q\frac{e}{A\,\epsilon_0}. \tag{1.9}$$

The capacitance C of a parallel-plate capacitor is then

$$C = \frac{Q}{V_\sigma - V_{-\sigma}} = \frac{A\,\epsilon_0}{e}. \tag{1.10}$$

1.4.1 Spherical capacitor

Let's consider a capacitor made up of two concentric spherical plates 1 and 2, with radii R_1 and R_2 respectively and separated by a vacuum ($R_1 < R_2$). The armatures carry the charges $Q_1 = +Q$ and $Q_2 = -Q$. The electric field is radial due to symmetry.

According to Gauss's theorem, the field at a point between the plates at distance r ($R_1 < r < R_2$) from the center is as follows :

$$\vec{E} = \frac{+Q}{4\pi\epsilon_0 r^2}\vec{u_r}, \tag{1.11}$$

where $\vec{u_r}$ is a unit vector directed radially outwards. The potential difference between the plates is given by the flow of $-\vec{E}$ between the two pltes.

If we choose a path running from plate 1 (potential V_+) to plate 2 (potential V_-), we have :

$$\int_{V_+}^{V_-} dV = \int_{R_1}^{R_2} -\vec{E}\cdot\vec{d\ell}, \tag{1.12}$$

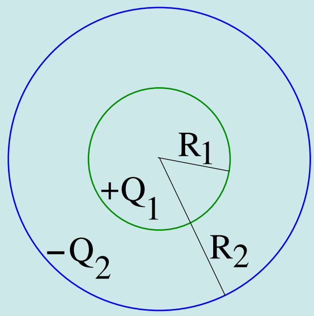

FIGURE 1.9 – Capacitance of a spherical capacitor

or,

$$V_- - V_+ = -\int_{R_1}^{R_2} \frac{+Q}{4\pi\epsilon_0 r^2}\vec{u_r}\cdot\vec{d\ell}. \tag{1.13}$$

Vector $\vec{u}_r$ being unitary, $\vec{u}_r \cdot \vec{d\ell}$ gives the projection of $\vec{d\ell}$ on $\vec{u}_r$. Since $\vec{u}_r$ is directed along the radius vector $\vec{r}$, it's natural to write $\vec{u}_r \cdot \vec{d\ell} = dr$. Equation (1.13) then becomes :

$$V_- - V_+ = -\int_{R_1}^{R_2} \frac{+Q}{4\pi\epsilon_0 r^2}dr = -\frac{Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{R_1}^{R_2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \tag{1.14}$$

We take out the capacity (see definition of equation (**??**)),

$$C = \frac{Q}{V_+ - V_-} = 4\pi\epsilon_0\frac{R_1R_2}{R_2 - R_1}. \tag{1.15}$$

Remark 1 : If e denotes the separation between the two spheres, we have : $R_2 - R_1 = e$. If this separation is small, the radii of the spheres are practically equal and then :

$$C = 4\pi\epsilon_0\frac{R_1^2}{e} = \frac{\epsilon_0\,S}{e}, \tag{1.16}$$

where S is the surface area of the spheres. The result is the same as for the planar (parallel-plate) capacitor. When the spheres are only slightly separated, the spherical capacitor has the same capacitance as a planar capacitor whose plate area is equal to that of the spheres.

Remark 2 : As $R_2 \to \infty$, we get from equation (1.15), $C = 4\pi\epsilon_0 R_1$ A single isolated sphere is therefore equivalent to a spherical capacitor whose outer shell has an infinitely large radius.

1.4.1.1 Cylindrical capacitor

A cylindrical capacitor consists of two concentric, conducting cylinders of length ℓ very large compared to the radii. The inner cylinder of radius R_1 carries the charge $Q_1 = +Q$. The outer cylinder is a shell of inner radius R_2 and carries the charge $Q_2 = -Q$. Let's calculate its capacitance. The electric field is radial here also. At a point between the plates located at distance r from the axis, Gauss's theorem gives :

$$E = \frac{Q}{2\pi\epsilon_0 \, r\ell}. \tag{1.17}$$

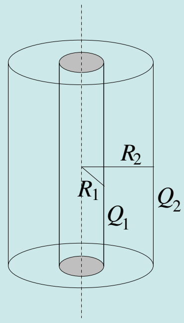

FIGURE 1.10 – Capacitance of a cylindrical capacitor.

As in the case of the spherical capacitor, the potential difference between the plates is as follows :

$$V_2 - V_1 = -\int_{R_1}^{R_2} \frac{+Q}{2\pi\epsilon_0 r\ell} dr = -\frac{Q}{2\pi\epsilon_0 \ell} \ln \frac{R_2}{R_1}. \tag{1.18}$$

We get

$$C = \frac{Q}{V_1 - V_2} = \frac{2\pi\epsilon_0 \ell}{\ln\,(R_2/R_1)}. \tag{1.19}$$

Remark : If e denotes the separation between the two cylinders, we have $R_2 = R_1 + e = R_1(1 + e/R_1)$, $R_2/R_1 = 1 + e/R_1$ et ln $(R_2/R_1) = \ln(1 + e/R_1)$. When $e \ll R_1$ (cylinders slightly separated), we get : $e/R_1 \ll 1$ and therefore $\ln(1 + e/R_1) \approx e/R_1$. The capacitance of a cylindrical capacitor takes then the form :

$$C = \frac{2\pi\epsilon_0 \ell}{e/R_1} = \frac{\epsilon_0 2\pi R_1 \ell}{e} = \frac{\epsilon_0 \, S}{e}, \tag{1.20}$$

where S is the lateral surface area of the cylinders. Here again, we find the result of the parallel-plate capacitor. When the cylinders are slightly separated, the cylindrical capacitor has the same capacity as a parallel-plate capacitor whose plate area is equal to that of the lateral surface of the cylinders.

1.5 Combination of capacitors

A capacitor is characterized not only by its capacitance, but also by the maximum potential difference that can be applied to it without damaging it, i.e. without causing the insulation breakdown between the plates, in which case the capacitor is no longer usable.

We can combine a set of several capacitors in various ways and determine the *equivalent capacitance* of the combination, i.e. the capacitance of the single capacitor equivalent to the set. The two most common combinations are described below : parallel combination and series combination.

1.5.1 The parallel combination of capacitors

Consider n capacitors of capacitance C_i, $(i = 1,...,n)$. A parallel combination is obtained by grouping them as shown in Figure a. The left-hand ends of all the capacitors are connected to point the + terminal of a battery (point A) and the right-hand ends are connected to the − terminal (point B). As you can see, all capacitors are subject to the same potential difference $V = V_A - V_B$.

As a result of this potential difference, capacitor C_1 carries the charge Q_1, capacitor C_2 carries the charge Q_2, capacitor C_3 carries the charge Q_3, and so on ...

These charges are given by : $Q_1 = C_1 V,\ Q_2 = C_2 V,\ Q_3 = C_3 V, ..., Q_n = C_n V$. The total electric charge of the combination is : $Q = \sum_i Q_i = (C_1 + C_2 + ... + C_n)V$. We deduce the capacitance of the combination :

$$C = \frac{Q}{V} \rightarrow \boxed{C = C_1 + C_2 + ... + C_n.} \tag{1.21}$$

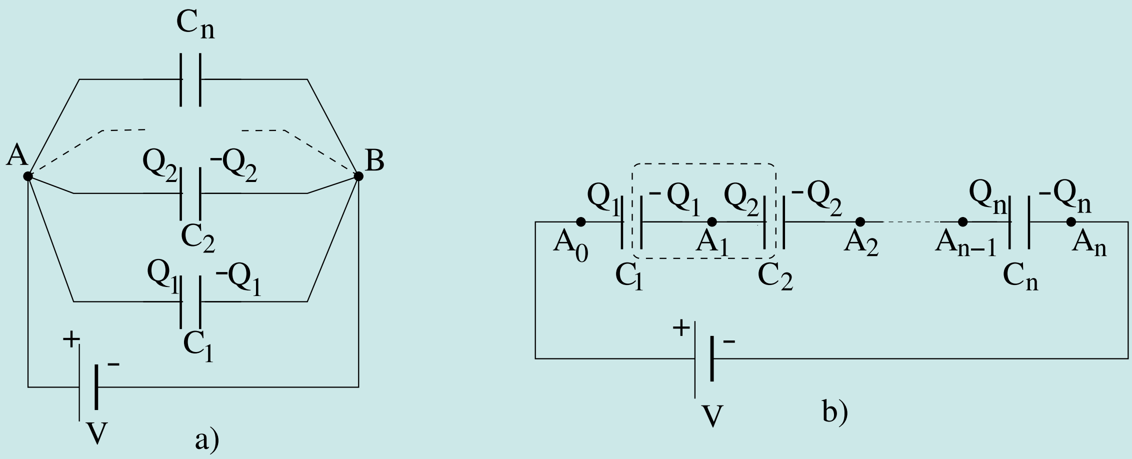

FIGURE 1.11 – Combination of capacitors a) parallel, (b) series.

A parallel combination of capacitors with capacitances C_1, C_2, C_3, ...is equivalent to a single capacitor with capacitance C equal to the sum of the individual capacitances. We deduce that the capacitance C of the group is always greater than the largest individual capacitance.
If you need a higher capacitance than those available, you need to combine them in parallel.
Example : Find the equivalent capacitance of the combination below :

1.5.2 The series combination of capacitors

The combination of n capacitors in series is shown in Figure b. The right end of one capacitor is connected to the left end of the next one. The whole system is put under the potential difference V. This potential difference will generate on each capacitor the charge $Q_1 = C_1\, V_1$, $Q_2 = C_2\, V_2$, ...
If the capacitors are initially neutral, the framed piece (the piece around A_1) will remain globally neutral even after the potential difference has been applied. It follows that

$$Q_2 + (-Q_1) = 0 \implies Q_1 = Q_2. \tag{1.22}$$

This result is obviously valid between capacitors 2 and 3, between 3 and 4, and so on, leading to :

$$Q_1 = Q_2 = Q_3 = \ldots = Q_n. \tag{1.23}$$

When initially neutral capacitors are combined in a series, they all acquire the same charge. Calling this charge Q and using the relation $Q_i = C_i V_i$, $i = 1,2,\ldots$, the potential difference at the ends of each capacitor is :

$$V_1 = Q/C_1, \quad V_2 = Q/C_2, \ldots, \quad V_n = Q/C_n.$$

The total potential difference is :

$$V = V_1 + V_2 + \ldots + V_n = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}\right).$$

We deduce the capacitance

$$C = \frac{Q}{V} \to \boxed{C = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}\right)^{-1}, \text{ ou bien } \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}.} \tag{1.24}$$

The reciprocal of the capacitance C of a set of capacitors combined in series is equal to the sum of the reciprocals of the individual capacitances. Such a set is equivalent to a single capacitor of capacitance C. The capacitance C of the series combination is always smaller than to the smallest of the individual capacitances.

The same circuit may have a *mixed* combination, i.e. series and parallel. To obtain the equivalent capacitance of the *mixed* arrangement, we first process the single identifiable arrangements within the mixed arrangement, then replace each arrangement with its equivalent capacitance, and finally calculate the total equivalent capacitance of the initial combination.

But more complex set-ups may be encountered where the capacitor combination is neither in parallel nor in series. In this case, to find the equivalent capacitance, we need to look for the relationship $Q = C_{acuteequiv}V$ linking the voltage V across the grouping to the total charge Q of the grouping.

1.6 Electrostatic potential energy stored in a capacitor

Charging a capacitor involves transferring charges from the plate with the lower potential to the plate with the higher potential. More concretely, when a neutral capacitor is connected to the terminals of a battery, the battery transfers electrons from the plate connected to the + terminal of the battery to the plate connected to the − terminal. The plate connected to the + terminal becomes positively charged because it lacks electrons, and the plate connected to the − terminal, which has an excess of electrons, becomes negatively charged.The charging process therefore requires the expenditure of energy. At a certain point in the process, the charge transferred is q, ($+q$ on the + plate and $-q$ on the − plate) and so the potential difference across the capacitor is : $v = q/C$, C being the capacitance of the capacitor. The transfer of an additional charge dq requires work :

$$dW = dq\, v = \frac{q}{C}\, dq. \tag{1.25}$$

The total work required to increase the charge transferred from 0 to Q [2] is :

$$W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{2}\left[\frac{q^2}{C}\right]_0^Q = \frac{Q^2}{2C}. \tag{1.26}$$

This work is stored as electrical potential energy U_{el} in the capacitor, so :

$$U_{el} = \frac{Q^2}{2C}. \tag{1.27}$$

At the end of the process, the potential difference between the plates is $V = Q/C$ and the previous result can also be written as :

$$U_{el} = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV. \tag{1.28}$$

Energy density Sometimes, energy is considered to be stored in the electric field between the plates of the capacitor. In the particular case of the parallel-plate capacitor, the volume between the plates is $A\,e$. The electrical potential energy per unit volume, called the energy density and denoted u, is written as :

$$u = U_{el}/(\text{unité de volume}) = \frac{1}{2}CV^2/(A\,e). \tag{1.29}$$

Since the field between the plates is constant, we have : $V = Ee$, and hence

$$u = \frac{1}{2}CE^2\, e^2/(A\,e) = \frac{1}{2}CE^2\, e/A. \tag{1.30}$$

2. The maximum charge that can be transferred is determined by the product of the battery's electromotive force E_{em} and the capacitor's capacitance C : $Q_{\max} = CE$.

Knowing that for a parallel-plate capacitor (see equation (1.10)) $C = \epsilon_0 A/e$, it follows :

$$u = \frac{1}{2}\frac{\epsilon_0 A}{e}\frac{E^2 e}{A} = \frac{1}{2}\epsilon_0\, E^2. \tag{1.31}$$

The energy density is finally written as :

$$u = \frac{1}{2}\epsilon_0\, E^2. \tag{1.32}$$

The expression (1.32) does not refer to the capacitor, it only depends on the field E. Although it has been established for the particular case of the parallel-plate capacitor, it is valid at any point in space where there is an electric field $\vec{E}$.

1.7 Questions

A conducting sphere of radius 0.5 m is charged with billions of billions of electrons, and electrostatic equilibrium is established. The electric field at the surface of the sphere is 550 V/ m.

1) How are the excess electrons distributed on the sphere ? 2) Consider a test charge moving between two points inside the sphere. What is the work done on the charge by the electric field ?

Choose one answer :

☐ A- 1) The charge is uniformly distributed over the surface of the sphere.
    2) The work done by the electric field is 0 J.

☐ B- 1) The charge resides nowhere on the surface of the sphere.
    2) The work done by the electric field is 1 J.

☐ C- 1) The charge resides inside the sphere.
    2) The work done by the electric field is 8 J.

☐ D- 1) The charge resides somewhere on the surface of the sphere.
    2) The work done by the electric field is 5 J.