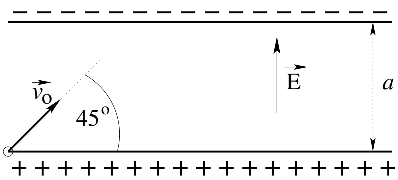
Tutorial series # 2 : Field and Potential I ———————— *February 2024*

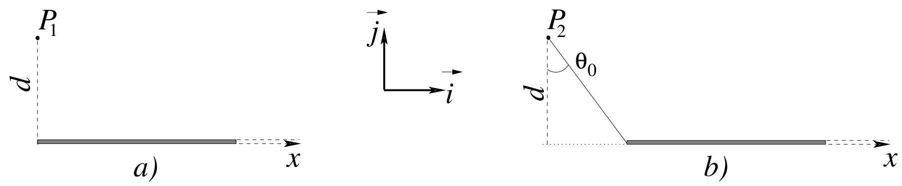
Exercise 1 : An electron is projected at an angle of 45° and an initial velocity v_0 from the left edge of the bottom plate of a parallel-plate arrangement as shown if the figure at the right. The plates are separated by $a = 2$ cm and are very long. Between them there is a uniform electric field $E = 10^3$ N/C. What is the maximum value $v_{0\max}$ that v_0 must not exceed to prevent the electron from hitting the top plate ? The mass and charge of the electron are $m_e = 9.109 \times 10^{-31}$ kg, and $q_e = -1.6 \times 10^{-19}$ C respectively. *Rép.* : 3.75×10^6m/s.



Exercise 2 : Two point charges equal to q are located at the points $(0; +a)$ and $(0; -a)$ of a rectangular axis system Oxy. a) Find the field $E(x)$ at the point $(x; 0)$. b) Show that for distant points, i.e. for $x \gg a$, $E(x)$ reduces to a simple form, an interpretation of which will be given. c) Graphically represent the shape of $E(x)$.
Rép. : a) $2kqx/(x^2 + a^2)^{3/2}$; b) $E(x) = k\,2q/x^2$.

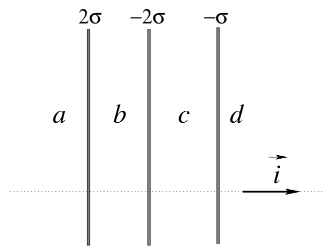
Exercise 3 : A ring (حلقة) of radius R and center O carries a positive charge Q uniformly distributed around its circumference. a) Determine the electric field $\vec{E}$ that the ring creates at a point M located at position x on its axis $x'Ox$. b) Calculate the potential at M. Take $V = 0$ for $|x| \to \infty$. c) Find the expression for $\vec{E}$ from the relation $\vec{E} = -\overrightarrow{grad}V$. d) Show that for points $x \gg R$, the ring behaves as if all its charge were concentrated at its center. *Rép.* : a) $\vec{E} = (kQx/(x^2 + R^2)^{3/2}\vec{i}$; b) $V = \frac{kQ}{(x^2+R^2)^{1/2}}$; d) For $x \gg R$, we have $x^2 + R^2 \approx x^2 \implies \vec{E} = (kQ/x^2)\vec{i}$ et $V = kQ/x$, the ring behaves as if all its charge were a point charge concentrated at its center.

Exercise 4 : A rectilinear, non-conducting wire of semi-infinite length is charged with a constant line density $\lambda > 0$. 1) Find, in the orthonormal basis $(\vec{i}, \vec{j})$, the field $\vec{E}_1$ at point P_1 (figure (a) below). What angle does this field make with the wire, i.e. the x-axis ? Does this angle depend on the distance d ?



2) Find the field $\vec{E}_2$ at the point P_2 defined by angle θ_0 and distance d (figure (b) above). Check that for $\theta_0 = 0$, we recover the result of question 1.
Rép. : 1-) $\vec{E}_1 = (k\lambda/d)(-\vec{i} + \vec{j})$, $\vec{E}_1$ est diricted at 45° above tne negative direction x axis ; 2-) $\vec{E}_2 = (k\lambda/d)(-\vec{i})$; 3-) $\vec{E}(M) = (k\lambda/d)(-\cos\theta_0\vec{i} + (1 - \sin\theta_0)\vec{j})$

Exercise 5 : a) Using Gauss's theorem, show that the field created by an infinite plane, carrying a uniform surface charge density σ, at a point located outside the plane at whatever distance is $\vec{E} = (\sigma/2\epsilon_0)\,\vec{n}$, where $\vec{n}$ is a unit vector normal to the plane, oriented outwards. b) The figure opposite shows three infinite parallel planes uniform densities 2σ, -2σ et $-\sigma$. Use the result of question a) to find, in terms of σ, ϵ_0 and the unit vector $\vec{i}$, the electric field in the regions a, b, c, and d. *Rép.* : a) $\vec{E} = (\sigma/2\epsilon_0)\,\vec{i}$; b) $\vec{E} = (5\sigma/2\epsilon_0)\,\vec{i}$; c) $\vec{E} = (\sigma/2\epsilon_0)\,\vec{i}$; d) $\vec{E} = (-\sigma/2\epsilon_0)\,\vec{i}$



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