Tutorial series # 3 : Field and Potential II ——————— March 2024

Exercise 1 : What is the potential created by a point charge of 5 nC at a distance of 3 cm ? 5 cm ? 10 cm ? 1 m ? 10 m ? 1 km ? Plot these results on a graph. *Ans.* : 1500 V, 900 V, 450 V, 45 V, 4.5 V. The graph will show that the absolute potential of a point charge decreases rapidly with distance.

Exercise 2 : A typical lightning bolt can carry up to 30 Cof charge across a potential difference of 100 million volts between the points of discharge. a) How much energy does this release ? b) If we could use this energy to light a 60 Wlamp, how long would it last ? Express the result in months.
Ans. : a) 3×10^9 J ; b) 5×10^7s, i.e., about 19 months.

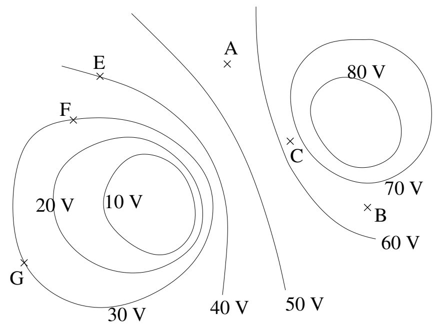
Exercise 3 : Two parallel infinite metal plates carry equal and opposite charges and are 5 cm apart. A charge of 8 µC placed between the plates is subjected to a force of $2.4 \times 10^{-2}\vec{i}$N, where $\vec{i}$ is the unit vector directed perpendicularly from the positive plate to the negative plate. Find the potential difference $V_+ - V_-$ between the plates.

Ans. : $dV = -\vec{E} \cdot d\vec{r} = -(\vec{F}/q) \cdot d\vec{r} = -(F/q)dr$, after integration we arrive at : $V_+ - V_- = 150$ V.

Exercise 4 : A point charge q is at position $(0; 3)$ relative to an orthornomal coordinate system $(O, \vec{i}, \vec{j})$. a) Determine the potential V at any point $(x, y) \neq (0, 3)$. b) Calculate the electric field $\vec{E}$ at the point $(4, 0)$. c) Find the result of b) from the potential V.

Exercise 5 : **This exercise will be solved in class.**
A disk of negligible thickness (قرص ذو سمك ضئيل) has radius a and carries a uniform charge density σ C/m^2. a) Show that the potential created by the disk at a point M located at position y on its axis (y can be positive or negative) is : $V = (\sigma/2\epsilon_0)\left((a^2 + y^2)^{1/2} - |y|\right)$. Take $V = 0$ when $y \to \pm\infty$. b) Deduce the electric field $\vec{E}$. c) Study the behavior of the disk with respect to a very distant point, i.e. for $y \gg a$. d) What is $\vec{E}$ for $R \to \infty$? Same question for $y \to 0$? Interpréter. *Gives :* $\int r dr/(r^2 + y^2)^{1/2} = (r^2 + y^2)^{1/2}$

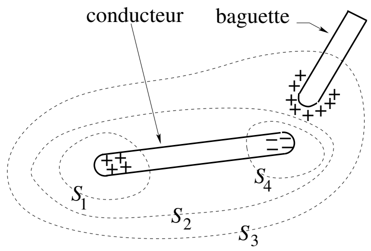
Exercise 6 : The figure opposite shows a set of equipotential lines obtained by intersecting equipotential surfaces with a plane.



1) Sketch with dotted lines and orientate the field line passing through A and the one passing through B.
2) Show the electric field $\vec{E}_C$ at point C (direction and sense). 3) Mark with a point D the region where the field is strongest. 4) If an electron moves from E to F, what will be the change in its kinetic energy ? 5) Same question when the electron moves from F to G.
Ans. : 1) Field line $\perp$ to equipotential line ; 2) Field vector tangent to field line and points from strongestto weakest potentials ; 3) Field is stronger is the region where equipotential lines are tighter ; 4) $\Delta E_{cin} = -\Delta E_{pot}$.

Exercise 7 : Four charges $+q$, $+2q$, $-2q$, $-q$ are placed in this order on the corners of a square of side a. Express in terms of q and a : 1) the total internal potential energy of the system formed by the four charges. 2) the potential energy of a fifth charge q placed at the center of the square.
Ans. : 1) $-kq^2(1 + 2\sqrt{2})/a$; 2) 0.

Exercise 8 : A charge is induced on an insulated, initially neutral conductor by approaching a rod carrying a positive charge $+q$ at its end. The negative charge induced by influence on the conductor is conductor is $-q/2$. 1) What is the positive charge induced on the conductor ? 2) What is the electric field flux through the closed surfaces S_1, S_2, S_3 and S_4 ?
Ans. : 1) $+q/2$; 2) $q/2\epsilon_0$, 0, q/ϵ_0 et $-q/2\epsilon_0$.



Exercise 9 : The space between two concentric spheres, of center O and radii R_1 and R_2, with $R_2 > R_1$, is filled with a charged non-conductive material with constant density $\rho > 0$.
1) Using Gauss's theorem, calculate the electrostatic field at distance r from the center for the regions : a) $r < R_1$, b) $R_1 < r < R_2$ and c) $r > R_2$.
2) Deduce the potential $V(r)$. Take $V(r) = 0$ for $r \to \infty$ and assume that the potential is continuous at points $r = R_1$ and $r = R_2$, i.e. at the points joining the different regions.

Ans. : 1) The symmetry of the problem leads to a radial field $\vec{E} = E\,\vec{e}_r$; a) $r < R_1 : E = 0$; b) $R_1 < r < R_2$: $E = \rho/3\epsilon_0(r^3 - R_1^3)/r$; c) $r > R_2 : E = (\rho/3\epsilon_0)(R_2^3 - R_1^3)/r^2$.

2) The potential $V(r)$ can be deduced from $\vec{E} = -\overrightarrow{grad}\,V$ which can also be written as $dV = -E\,dr$: a) $V = (\rho/2\epsilon_0)(R_2^2 - R_1^2)$; b) $V = (\rho/3\epsilon_0)(3R_2^2/2 - r^2/2 - R_1^3/r)$; c) $V = (\rho/3\epsilon_0)(R_2^3 - R_1^3)/r$.

— *End of the Tutorial serie*