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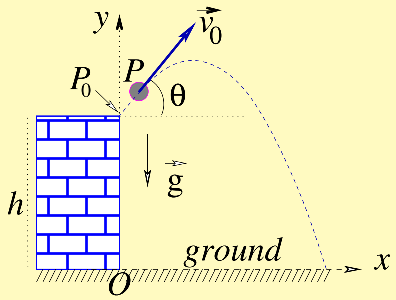
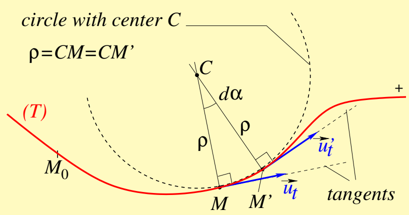


The equation of the trajectory of P is : $y = -\frac{g}{2v_0^2\cos^2\theta}x^2 + \tan\theta\, x + h$. The speed of the ball as it hits the ground is $v = \sqrt{v_0^2 + 2gh}$.
In terms of $\dot{x}$, $\dot{y}$, $\ddot{x}$, and $\ddot{y}$, the radius of curvature of M (right figure) reads : $\rho = \frac{(\dot{x}^2+\dot{y}^2)^{3/2}}{|\dot{x}\ddot{y}-\dot{x}\ddot{y}|}$.



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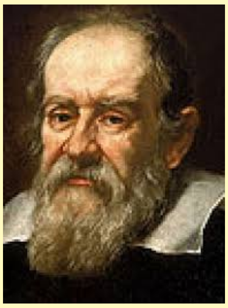
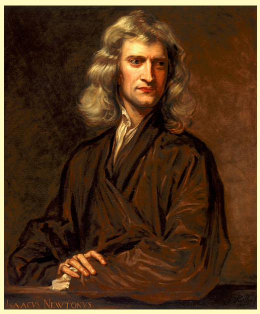


Galileo (left portrait) writes : "Aristotle declares that a 100-pound iron ball has already descended 100 cubits when a 1-pound ball has traveled only one cubit. I affirm that the two balls arrive together."

In 1687, Newton (right portrait) published the mathematical principles of natural philosophy (*Philosophiae naturalis principia mathematica*). In it, he described his discoveries on universal gravitation and the three famous laws, known as Newton's Laws. These laws describe the physical phenomena of inertia and the forces exerted on objects.



Above, the blue figure (left) shows the trajectory of a small ball P launched from the top of a building (height h) with a velocity $vecv_0$ making an angle θ with the horizontal. The expressions to its right give the equation of the ball's trajectory and its velocity when it hits the ground. The figure on the far right defines the radius of curvature at a point on the trajectory. When M' tends towards M (trajectory (T) in red), the normals to the tangents at M and M' meet at a point C called the center of curvature. The lengths CM and CM' are then equal to a quantity ρ called radius *of curvature. A circle with center C and radius ρ will necessarily pass through M and M'. As for the two portrait photos below, they are of Galileo (left) and Newton (right), two great scientists who left their mark on the history of science and contributed greatly to mechanics.*

Prof. M. A. Belkhir, année 2023-2024

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Chapitre 3

Dynamics of a point

Learning Objectives

By the end of this chapter, you will be able to :
- Understand and explain the concept of force ;
- Understand and explain Newton's laws ;
- Understand what is inertial (and non inertial) frame of reference ;
- Understand what is static and dynamic frictional forces ;
- Solve problems dealing with dynamics of a point.

3.1 What is dynamics in the context of this course ?

Dynamics is the study of motion in relation to the forces that cause it. It differs from kinematics by the fact that it is not concerned with the causes of motion.

3.2 Concept of force

Let's consider an object on a horizontal table. Case 1 : To cause the object to move in a given direction, it is necessary to apply a mechanical action on the object, that is, push or pull it in that direction. This action is called *a force*.
Case 2 : However, an object may be set in motion without pushing or pulling. Example 1 : object released at rest and undergoes free fall, here the force comes from Earth attraction. Example 2 : if the object is magnetic, it can be set in motion using a magnet without contact, here the force comes from the magnet.
In case 1, contact is necessary for the force to be exerted, this is a contact force. In the second case, contact is not necessary for the force to be exerted, the force is said to be a non-contact force or force at distance.
To sum up, in the absence of contact, the contact force doesn't exist because it comes from contact. On the other hand, a non-contact force between objects can apply without necessarily being in contact. Here "without necessairily" means that *force at a distance* can be exerted even if the two objects are touching.

To designate a force, we use the letters of the alphabet. The letter F is most often used. The letters P and T are also used for, respectively, the weight of an object and the force of tension in a wire. The action of one force gives the object an acceleration $\vec{a}_1$ in the same direction and sense as the applied force. The action of a second force in a different direction imparts to the object an acceleration $\vec{a}_2$ in this new direction. The simultaneous action of the two preceding forces imparts to the object an acceleration $\vec{a}$ given by the sum $\vec{a}_1 + \vec{a}_2$. This means that force, like acceleration, is a vector quantity and, as such, obeys the laws of vector calculus.

Drawing a diagram showing all the forces and their characteristics
Depending on the data at your disposal, this diagram can be qualitative (ignoring values) or quantitative (taking values into account).
On a diagram, forces can be drawn qualitatively if you don't know the quantities, or if they are of no importance to the problem under study. Sometimes, however, forces need to be represented quantitatively. In such cases, you need to define a scale that relates the force's value in newtons (N) to its length in centimeters (cm).

Example : If the scale is such that 1 cm represents 5 N, then a force of 12 N will be represented by a vector of length $\dfrac{12 \text{ N}}{5 \text{ N/cm}} = 2.4$ cm.

Question : A block at rest on the Earth's surface is subjected to two forces : its weight and the reaction force of the surface. For each force, say whether it is a contact force or a force at a distance.

Answer : Weight results from the gravitational attraction exerted by the Earth on the block. This attraction exists whether or not the block is in contact with the Earth's surface - it's a force at a distance. The reaction of the surface exists only if there is contact between the block and the Earth's surface, it's a contact force.

3.3 Newton's laws

3.3.1 Newton's first law also known as Principle of inertia

Principe d'inertie
All bodies are against changing their state of rest or motion and this opposition is called *inertia*.
Newton's first law states that any body at rest or moving in uniform motion (i.e. in moving a straight line with contant speed), continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

In other words, *an object undergoing no force or subjected to forces of zero resultant is either at rest or animated by a uniform rectilinear motion.*

All the above can be summarized using the following relationships :

$$\sum \vec{F}_{ext} = \vec{0} \Longleftrightarrow \text{la vitesse } \vec{v} \text{ est constante (nulle ou non)}$$

$$\sum \vec{F}_{ext} \neq \vec{0} \Longleftrightarrow \vec{v} \text{ n'est pas constante}$$

Example 1 : Apply the principle of inertia to answer the question below. The small black balls in the figure below represent the positions of a material object at equal time intervals. In each of the situations (1), (2), (3) and (4), say whether the forces applied to the object compensate or do not compensate.

Solution :
Situation (1) : the object travels equal distances over equal intervals of time. We deduce that the velocity does not change, but is constant. Therefore, according to the principle of inertia, the sum of the forces applied to the object is equal to 0, i.e. the forces cancel each other out.
Situation (2) : the distances covered during equal time intervals are not equal. We deduce that the velocity varies and that, consequently, the forces applied to the object do not compensate.
Situation (3) : same response as for the situation (2).
Situation (4) : the object maintains the same position at all times, in other words, it is at rest (stationary). This means that the forces applied to the object cancel each other out.

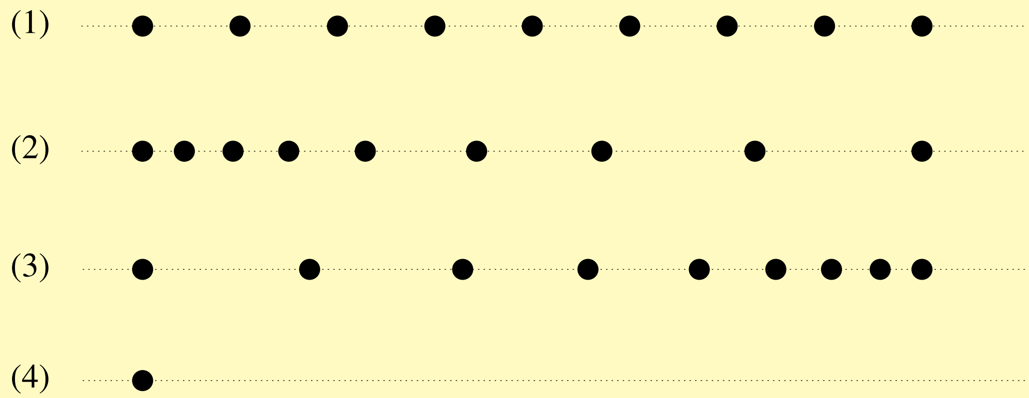


FIGURE 3.1 – Whether or not forces applied to a system compensate depending on the motion

Contrapositive of the principle of inertia
The contrapositive of the principle of inertia states that if an object is neither at rest nor in rectilinear and uniform motion, then we can deduce that the sum of the external forces acting upon it is not zero (do not compensate). In other words, if the sum of external forces acting upon an object is not zero, then the object is neither at rest nor in uniform rectilinear motion, but is accelerating.

3.3.2 Newton's second law also known as *Fundamental principle of dynamics (FPD)*

Newton's second law of motion states that the acceleration of an object is directly proportional to the resultant force $\vec{F}$ applied to it.

$$\boxed{\vec{F} = m\,\vec{a}}$$

It might become much clearer if we write the previous formula in the equivalent form : $\vec{a} = \vec{F}/m$. In this form, the law can be interpreted as follows : When a resultant force F is applied to an object of mass m, the object acquires an acceleration a, directed in the direction of the force and whose value is F divided by m.

Notice : By resultant force, we mean the vector sum of all forces applied to the object : $\vec{F} = \sum \vec{F}_{ext}$.

Example 2 :
The two masses $m_1 = 100$ kg and $m_2 = 50$ kg in the figure below are connected by means of a rope, inextensible and of negligible mass, which passes through a pulley of negligible mass and no friction. The two masses rest frictionless on inclined planes as shown in the figure. Take $\mathrm{g} = 9.8\ \mathrm{m/s^2}$.
a) In which direction (left or right) will the system slide ? b) Calculate the acceleration of the two blocks. c) What is the tension in the rope ?

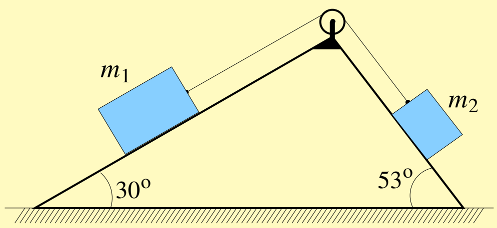


FIGURE 3.2 – Example 2

Solution : a) The system (the 2 masses + pulley + rope) is subjected to the 4 forces $m_1\,\vec{\mathrm{g}}$ and $m_2\,\vec{\mathrm{g}}$ (weight of the two masses) and $\vec{R}_1$ and $\vec{R}_2$ (normal reactions of the planes on the masses).
The string tensions T_1 and T_2 are forces internal to the system under consideration ; they are not part of the external forces. Since the string is inextensible and of negligible mass, the tension T is the same at all its points, so $T_1 = T_2 = T$.

Decomposing the weight in directions parallel and perpendicular to the plane, we can see from the drawing that R_1 and $m_1\,\mathrm{g}\cos(30)$ cancel each other out, as do R_2 and $m_2\,\mathrm{g}\cos(53)$. The two remaining external forces are $m_1\,\mathrm{g}\sin(30)$ and $m_2\,\mathrm{g}\sin(53)$.

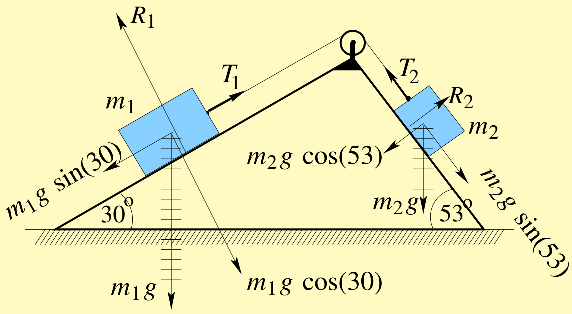


FIGURE 3.3 – Example 2 : solution

a) The system will move to the side of the strongest force. $m_1\,\mathrm{g}\sin(30) = 100 \times 9.8 \times 0.5 = 490$ N and $m_2\,\mathrm{g}\sin(53) = 50 \times 9.8 \times 0.8 = 392$ N. We conclude that the system will move to the left (on the side of m_1).
b) The acceleration of the system is given by $a = \vec{F}_{\text{total}}/m_{\text{total}} = (490 - 392)/(100 + 50) = 98/150 = 0.65\ \mathrm{m/s^2}$.
c) The FPD applied to m_1 is written as : $m_1\,\mathrm{g}\sin(30) - T = m_1 a \implies T =_1\ \mathrm{g}\sin(30) - m_1 a = m_1(\,\mathrm{g}\sin(30) - a) = 100(4.9 - 0.65) = 425$ N.

3.3.3 Newton's third law also known as Principle of action and reaction

For every action, there is an equal and opposite reaction. More explicitly, whenever an object A exerts on an object B, a force $\vec{F}_{A/B}$, object B also exerts an equal and opposite force $\vec{F}_{B/A}$ on A :

$$\boxed{\vec{F}_{B/A} = -\vec{F}_{A/B}}$$

.

Remark : Note that the two forces are not applied to the same object. Here are some examples of *action-reaction* pairs :
1- When I push on the desktop (I apply an ACTION), I feel a force against my hand, i.e., the desktop is pushing back on my hand with as much force as I apply to it (the desktop applies back a REACTION). If this wasn't happening, my hand would accelerate through the desktop !
2- When you walk, you push the ground backwards (ACTION) and the ground pushes you forwards (REACTION).
3- When you swim, you push the water backwards (ACTION) and the water pushes you forwards (REACTION).
4- In the same way, a car wouldn't run without these two forces. The wheels push the road backwards (ACTION), and the road pushes the wheels forwards (REACTION).
5- In all the above examples, the ACTION-REACTION pair arises from contact between the two objects involved : hand-desk, feet-ground, wheels-road, etc. Action-Reaction forces can also exist between objects that are not in contact. This is the case between objects having mass (see next subsection) and between electrically charged objects, as we'll see in the second semester.

3.3.4 Newton's Law of Universal Gravitation

Let's explain the title. Here Universal = All and Gravitation = attraction. In other words, all objects that have mass attract each other.
More quantitatively, two masses m_A and m_B attract each other with opposing forces of equal magnitude, in accordance with Newton's third law. This magnitude is proportional to the product of the two masses, and inversely proportional to the square of the distance separating them. The line of action of these two forces is the straight line passing through the centers of gravity of the centers of the two

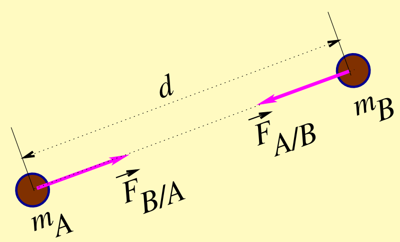


FIGURE 3.4 – Universal attraction

masses.

$$\vec{F}_{B/A} = -\vec{F}_{A/B}$$

.
Their magnitude is :

$$F_{B/A} = F_{A/B} = G\frac{m_A m_B}{d^2}$$

where G is called the universal gravitational constant and is equal to $6.674 \times 10^{-11}\,\mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{kg}^{-2}$ and d is the distance separating the two masses.

Example : What is the gravitational force does the Earth exert on me (mass $m = 80$ kg) me right now (mass $m = 80$ kg) ? We are at Fesdis, Batna 2 University, altitude : $Alt = 1050$ m. We give : Mass of Earth : $M_\mathrm{E} = 5.97 \times 10^{24}\,\mathrm{kg}$; radius of Earth : $R_\mathrm{E} = 6.37 \times 10^6\,\mathrm{m}$. Knowing that radius of Earth is, by definition, the distance from the center to sea level, the distance separating me from the center is $R_\mathrm{E} + Alt$. Therefore, the gravitational force we're looking for is

$$F_{M_\mathrm{E}/m} = G\frac{M_\mathrm{E} \times m}{[R_\mathrm{E} + Alt]^2} = 6.674 \times 10^{-11}\,\mathrm{N} \cdot \mathrm{m}^2 \cdot \mathrm{kg}^{-2}\frac{5.97 \times 10^{26}\ \mathrm{kg} \times 80\ \mathrm{kg}}{[6.37 \times 10^6\,\mathrm{m} + 1050\ \mathrm{m}]^2} = 785.3\ \mathrm{N}$$

3.4 Galilean reference frame or Inertial frame of reference

Although not explicitly mentioned, Newton's laws are only verified in certain special reference frames called *Galilean reference frames*, also known as *inertial frame reference.*
A frame of reference is said to be Galilean if the principle of inertia is verified in this frame of reference.
In mechanics, to study the motion of a body, it is necessary to specify the reference frame. In practice, the frame of reference is a solid, assumed to be stationary, to which a system of Oxyz axes is attached.

The principle of inertia implies that an object can only appear to be at rest or moving in a uniform rectilinear motion in a fixed frame of reference.
We deduce that a Galilean frame of reference is a fixed frame of reference or one animated by uniform rectilinear motion. In other words, a galilean frame of reference is any frame of reference that is not accelerated. An accelerated frame of reference is said to be a non Galilean frame of reference.
It follows that any frame of reference that is stationary or in uniform rectilinear motion relative to a Galilean frame of reference is also Galilean.

3.4.1 Important Galilean reference frames

1) **The heliocentric reference frame :** Also known as Kepler's reference frame, this considers the Sun as the solid of reference. Its origin is the Sun's center of mass. Each of the three axes of this frame of reference points to a distant star, assumed to be stationary relative to the Sun. The heliocentric reference frame is suitable for studying the motion of stars orbiting the Sun : planets, comets, etc.

Limitation : the Sun and the distant stars used to define the frame of reference are not stationary, even if their relative motion to each other is small. They are on the periphery of our Milky Way Galaxy, rotating on itself : they thus revolve around the galaxy's center at a speed of around 200 km/s. So, strictly speaking, the heliocentric frame of reference is not perfectly Galilean. However, since the time required to make a complete revolution is long (200 million years), we consider this frame of reference to be Galilean for time intervals that are small compared with the period of revolution.
2) The geocentric frame of reference is a frame of reference whose origin is the center of the Earth and whose three axes point towards distant stars, those used by the heliocentric frame of reference. Since the heliocentric frame of reference is considered to be galilean, we can ask ourselves under what conditions the

geocentric frame of reference is also galilean. We know that the Earth revolves around the Sun on an elliptical trajectory at a speed of 30 km/ s, making one complete revolution in one year. Because of this rotation (not uniform rectilinear motion), the geocentric frame of reference is accelerated relative to the heliocentric frame of reference, and is therefore not Galilean. However, on a sufficiently small portion of the ellipse, i.e. when the durations involved are much less than one year, the motion is rectilinear and uniform, and the geocentric frame of reference can then be considered as a Galilean frame of reference.

3) The laboratory reference frame is a reference frame originating at a point on the Earth's surface, and whose axes are linked to the Earth, which rotates with it around its own axis. Because of this rotation, the laboratory frame of reference is not a Galilean frame of reference. However, if we assume that the geocentric frame of reference is a Galilean one, the laboratory frame of reference can be considered as Gallilean when the experiments involved are of short duration compared to the duration of a complete rotation, i.e. 24 hours, (and of low speeds to neglect relativistic effects). This is justified by the fact that the laboratory reference frame is, in this case, in uniform rectilinear motion relative to the geocentric reference frame.

Example 3 :
The bus shown in the figure below is making a journey from Batna 2 University to Batna 1 University. Assuming that the reference frame linked to the ground R_{sol} is Galilean, state in each of the phases of the journey whether the reference frame linked to the bus R_{bus} is Galilean or not.
Phase 1 : The bus starts from rest, accelerates on 300 m until reaching a speed of 45 km/h.

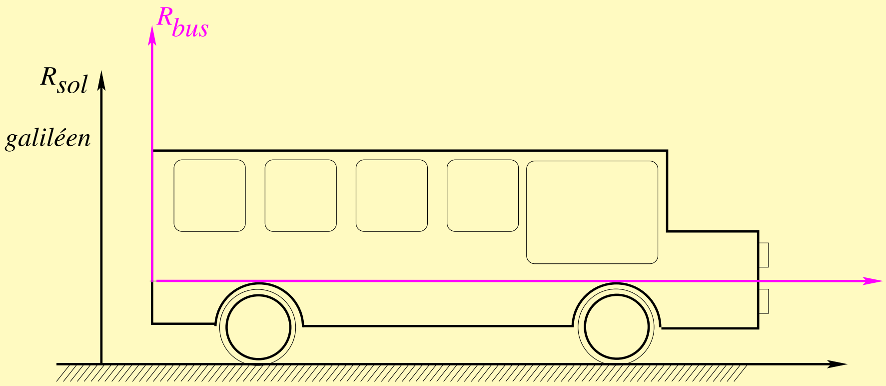


FIGURE 3.5 – Whether or not a reference frame is Galilean

Phase 2 : The bus stabilizes its speed at 45 km/h and drives in a straight line to Fesdis city.
Phase 3 : From Fesdis, while maintaining its speed, the bus takes a bend to head in the direction of Batna 1 university.
Phase 4 : At the end of the bend, the bus travels in a straight line at 45, km/h to Batna 1 university.
Phase 5 : The bus comes to a complete stop at Batna 1 university.

Solution :
Phase 1 : The bus is accelerating, so its speed is not constant relative to the ground. The reference frame R_{bus}, linked to the bus, is not Galilean.
Phase 2 : The bus (i.e. R_{bus}) moves in a straight line at constant speed ; it is a Galilean frame of reference.
Phase 3 : The speed is constant, but the motion is not straight (the bus takes a bend). The frame of reference is accelerated and, therefore, is not Galilean.
Phase 4 : The bus moves in a straight line at constant speed ; therefore R_{bus} is Galilean.
Phase 5 : The bus is at a complete standstill, so R_{bus} is Galilean.

3.5 Frictional forces

When two surfaces are in contact, a *frictional force* is exerted as soon as one of the two surfaces tends to slide over the other ; this frictional force acts to oppose sliding.

3.5.1 Experiment

1) Let's throw an object on a horizontal plane with a velocity $\vec{v}_0$ (for example, the chalkboard eraser on the desktop). The eraser slows down and stops after a few moments. We deduce that the eraser undergoes deceleration $\vec{a}$, and that a force $m\vec{a}$ (m being the mass of the eraser) opposes its movement (figure (1)).

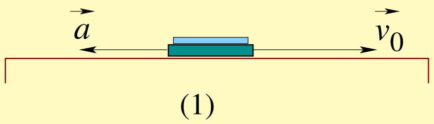


FIGURE 3.6 – Friction force on a horizontal plane

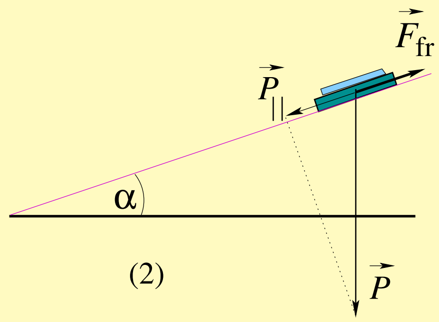


FIGURE 3.7 – Friction force on an inclined plane

This force is the result of friction between the two surfaces. The force of friction always acts to oppose the sliding of one object over another.

2) Eraser placed on a plane inclined by α with respect to the horizontal (figure (2)). As long as α is not too large (below a certain critical value α_{max}), the eraser remains in equilibrium. This means that there is a force $\vec{F}_{\text{fr}}$ that compensates for $\vec{P}_{\|}$, $\vec{P}_{\|}$ being the plane-parallel component of the eraser's weight. The force $\vec{F}_{\text{fr}}$ is the force of friction, meaning that an object can be subject to a frictional force even when it's not moving.

3.5.2 Static friction coefficient

Let's continue the experiment with the eraser placed on the horizontal plane and pulled with a horizontal force $\vec{F}$.
At (a), $\vec{F} = 0$ and only the forces $\vec{P}$ (eraser weight) and $\vec{N}$ (plane normal reaction) act on the eraser.

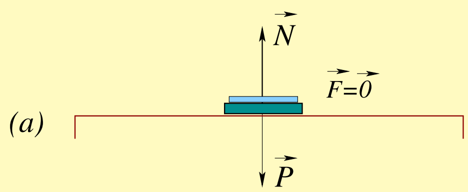


FIGURE 3.8 – Vertical equilibrium

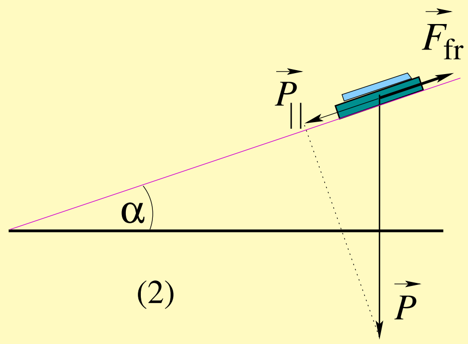


FIGURE 3.9 – Friction force $\vec{F}_{\text{fr}}$.

The equilibrium of the eraser is expressed by : $\vec{N} + \vec{P} = \vec{0}$. This latter equation remains true in the rest of the experiment.

In (b), $\vec{F}$ is different from 0 but not sufficient to set the eraser in motion, the eraser remains stationary. We deduce that there is a force $\vec{F_s}$ which compensates $\vec{F}$: $\vec{F_s} + \vec{F} = \vec{0}$. The force F_s is the *static friction force*. We gradually increase $\vec{F}$ (figure (c)) and there's still no movement. We deduce that F_s increases accordingly to compensate for F.

Remarque : When an object is placed on a support (table, floor, etc.), the latter exerts a contact force on the object, called *reaction*. This reaction force R generally has two components : a normal component N which prevents the object from sliding back into the support, and a tangential component which opposes the sliding of the object on the support. It's this tangential component that makes up the frictional force $F_{friction}$. So we have : $\vec{R} = \vec{N} + \vec{F}_{\text{friction}}$. In the absence of friction, the reaction force $\vec{R}$ is reduced to the normal component $\vec{N}$, hereinafter referred to as the normal force.

We continue to gradually increase $\vec{F}$ until equilibrium is broken, i.e. until the eraser starts to move. The value of the force $\vec{F}$ is then $\vec{F_r}$. To find the 'limit' equilibrium - in other words, to find the maximum value

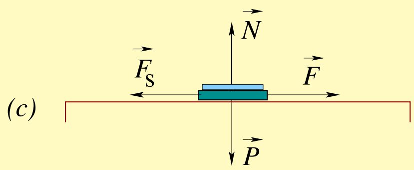


FIGURE 3.10 – If $\vec{F}$ is increased, then $\vec{F_s}$ increases accordingly to balance $\vec{F}$.

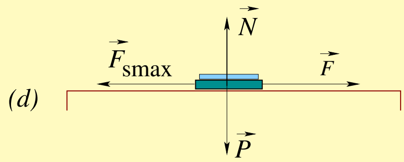


FIGURE 3.11 – Finding the limit equilibrium.

that $\vec{F}$ must not exceed to avoid breaking equilibrium - we take a value of $\vec{F}$ slightly lower than $\vec{F_r}$ and repeat the experiment, increasing $\vec{F}$ by very small amounts until equilibrium is broken again. At this point, friction has reached its maximum value $\vec{F}_{\text{smax}}$ (figure (d)). It means that if $\vec{F}$ increases further and exceeds $\vec{F}_{\text{smax}}$, equilibrium will be broken.
If we cut the eraser in half and stack the pieces on top of each other (figure (e)), $\vec{F}_{\text{smax}}$ remains unchanged. We deduce that $\vec{F}_{\text{smax}}$ does not depend on the extent of the surface in contact.

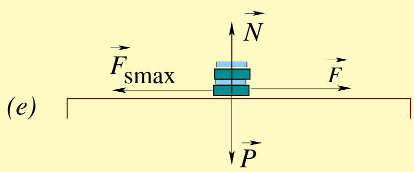


FIGURE 3.12 – $\vec{F}_{\text{smax}}$ does not depend on extent of the surface in contact

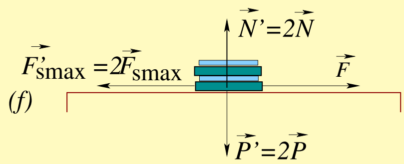


FIGURE 3.13 – Doubling the weight

Let's take two erasers and stack them one on top of the other (figure (f)). We then have $\vec{P}' = 2\vec{P}$ since the weight has doubled, and consequently, $\vec{N}' = 2\vec{N}$ and $\vec{F}'_{\text{smax}} = 2\vec{F}_{\text{smax}}$. We can see that if the weight doubles, the normal force doubles and the maximum friction force doubles. However, the ratio of the modulus of the maximum static friction force to the modulus of the normal force is constant : $F'_{\text{smax}}/N' = F_{\text{smax}}/N$. We define this ratio as the *coefficient of static friction*, denoted μ_s.

$$\mu_s = \frac{F_{\text{smax}}}{N} \tag{3.1}$$

On a donc

$$F_{\text{s}} \leq F_{\text{smax}} = \mu_s N. \tag{3.2}$$

The μ_s coefficient depends on the nature of the surfaces in contact, their cleanliness, polish, the amount of moisture present, etc. For friction between metals under ordinary conditions, μ_s varies from 0.3 to 1.0. If the surfaces are lubricated, the value of μ_s will be extremely reduced. At the hip joint in the human body, synovial liquid reduces μ_s to around 0.003. If, on the other hand, these surfaces are first cleaned and then brought into contact under vacuum, enormous adhesion forces come into play, making the value of μ_s very high.

3.5.3 Kinetic friction coefficient

Let's continue with the previous experiments.

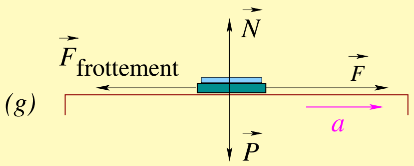


FIGURE 3.14 – F increases and becomes > à $\vec{F}_{\text{smax}}$

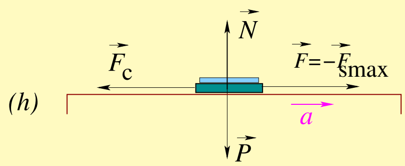


FIGURE 3.15 – F lowered to F_{smax}

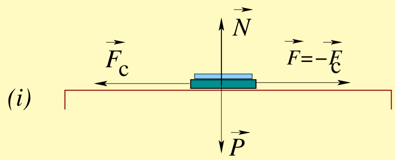
Let's increase the intensity of $\vec{F}$ beyond F_{smax}. As soon as F exceeds F_{smax}, the eraser starts to move. It accelerates if we continue to apply the force F (figure (g)).

If, while the eraser is in motion, F is lowered to F_{smax}, the eraser remains in accelerated motion (albeit with a lower acceleration $\vec{a}$ than in (g)). We deduce that the frictional force decreases during motion. During motion, friction is not static but kinetic. It's the *kinetic friction force*, $\vec{F_c}$, that comes into play. Sometimes it's also called dyamic or sliding friction force. We have :

$$F_c \leq F_{\text{smax}} \tag{3.3}$$

The intensity is further reduced to F_c. At this point, $\vec{F} + \vec{F_c} = \vec{0}$ and the eraser will have reached a speed v_0 and, by virtue of the principle of inertia, continues its movement in a straight line with this (constant) speed v_0.

Consequently, to start the movement, a force greater than F_{smax} must be applied, whereas to keep it going, a force F_c less than F_{smax} is sufficient. In other words, we know from experience that the most difficult part of moving a body (a bed for example) is first initiating the slide. It then takes much less effort to maintain it. It's easier to keep a moving object in motion than it is to move a stationary object.



In a similar way, we define the coefficient of kinetic friction as the ratio of the modulus of the kinetic friction force F_c to the modulus of the normal force N :

$$\mu_c = \frac{F_{\text{c}}}{N} \rightarrow F_{\text{c}} = \mu_c\, N \tag{3.4}$$

Puisque $F_{\text{c}} \leq F_{\text{smax}}$, alors

$$\mu_c \leq \mu_s \tag{3.5}$$