TD n°1 : Solution

## Vector exercises with solutions, L1 SCMI ——— October 2023

This series of questions relates to the 1st chapter. I've ticked the right answers. It's up to you to convince yourself of all the answers you've ticked. For example, in question 28, I chose answer c). I'll show you with a detailed calculation that answer c) is indeed the right answer.

**Solution of Question 28 :** $\vec{a} + \vec{b} = \vec{c} \implies (\vec{a} + \vec{b})^2 = (\vec{c})^2 \rightarrow a^2 + b^2 + 2ab\cos(\widehat{\vec{a}, \vec{b}}) = c^2 \implies 144 + 25 + 120\cos(\widehat{\vec{a}, \vec{b}}) = 169 \implies 169 + 120\cos(\widehat{\vec{a}, \vec{b}}) = 169 \implies 120\cos(\widehat{\vec{a}, \vec{b}}) = 0 \implies \cos(\widehat{\vec{a}, \vec{b}}) = 0 \implies (\widehat{\vec{a}, \vec{b}}) = \pi/2$. Answer c) is the right choice. Do the same to solve the other questions.

**Questions to be solved in tutorial classroom : 2, 5, 6, 8, 9, 12, 13, 17, 18, 20, 23, 24, 25, 29.**

**Question 1** – $P$ and $Q$ are two physical quantities with different dimensions, i.e. $[P] \neq [Q]$. Which of the following operations are possible ?

- ☐ a) $P - Q$ ;
- ☑ b) $PQ$ ;
- ☐ c) $1 - P/Q$ ;
- ☐ d) None of the above answers.

**Question 2** – $P$ et $Q$ sont deux grandeurs physiques possédant la même dimension, i.e. $[P] = [Q]$. Parmi les opérations ci-dessous, lesquelles sont possibles ?

- ☑ a) $P - Q$ ;
- ☑ b) $PQ$ ;
- ☑ c) $1 - P/Q$ ;
- ☐ d) None of the above answers.

**Question 3** – One of the health measures during the covid-19 period is to maintain a minimum distance of 1 m between individuals. Distance is a scalar quantity because

- ☑ a) to express it, we don't need to specify a direction or orientation. All you need is a number followed by a unit ;
- ☐ b) to express it, you need a number, a direction and an orientation ;
- ☐ c) a distance can be a vector ;
- ☐ d) a distance cannot be calculated from a vector.

**Question 4** – To get to a position $B$ from an initial position $A$, we need to know in what direction and at what distance $B$ is located. The displacement from $A$ to $B$ is a vector because

- ☐ a) to express it, we don't need to know the distance between $A$ and $B$ ;
- ☐ b) to express it, you just need to know the distance between $A$ and $B$ ;
- ☐ c) to express it, you don't need to know the direction ;
- ☑ d) to express it, you need to specify the direction of the vector $\overrightarrow{AB}$, in addition to the distance $AB$.

**Question 5** – Which of the following quantities are scalars ? Radius of a circle, time, force, area, mass, weight, temperature, moment of inertia, moment of a force, electrical resistance, electric field, volume, storage capacity of a hard disk, magnetic field, kinetic energy, centripetal acceleration, potential energy, pressure.

- ☐ a) Radius of a circle, force, surface area, mass, weight, moment of inertia ;
- ☐ b) weight, moment of a force, force, electric field, centripetal acceleration ;
- ☑ c) Radius of a circle, time, area, mass, moment of inertia, electrical resistance, volume, storage

capacity of a hard disk, kinetic energy, potential energy, pressure ;

☐ d) Radius of a circle, time, area, mass, temperature, moment of inertia, electrical resistance, volume, storage capacity of a hard disk, kinetic energy, centripetal acceleration, potential energy, pressure.

**Question 6** – Given two vectors $\vec{a}$ and $\vec{b}$ such that $\vec{a} = -\vec{b}$. From this equality, we deduce that the vectors $\vec{a}$ and $\vec{b}$

☑ a) have the same modulus ;

☐ b) have different moduli ;

☐ c) point in the same direction ;

☑ d) point in opposite directions.

**Question 7** – Which of the following vectors are null vectors ?

☐ a) velocity of a body in uniform motion on a circle ;

☐ b) velocity of a body in uniform rectilinear motion ;

☑ c) vector position of the origin of a coordinate system ;

☑ d) displacement vector of a stationary object.

**Question 8** – How must two non-zero vectors be arranged so that their sum gives a resultant of minimum length ?

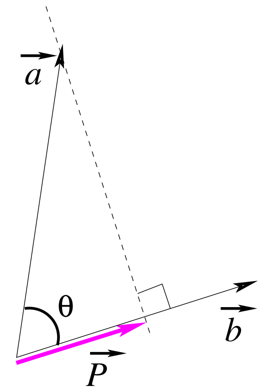
☐ a) perpendicularly ;

☐ b) parallel and in the same direction ;

☑ c) parallel but in opposite directions ;

☐ d) at 45° from each other.

**Question 9** – In the figure below, the vector $\vec{P}$ represents the orthogonal projection of $\vec{a}$ onto $\vec{b}$. If $a$ and $b$ denote the moduli of $\vec{a}$ and $\vec{b}$ respectively, $\vec{P}$ can be expressed by



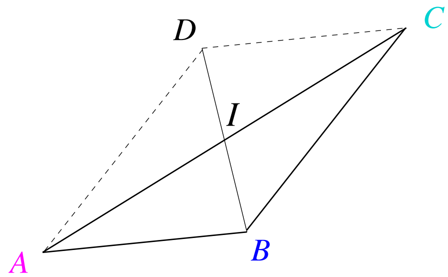

☑ a) $\vec{P} = a\cos\theta\,\frac{\vec{b}}{b}$ ;

☐ b) $\vec{P} = b\cos\theta\,\frac{\vec{b}}{a}$ ;

☐ c) $\vec{P} = (\vec{a}\cdot\vec{b})\,\frac{\vec{b}}{a}$ ;

☑ d) $\vec{P} = \left(\vec{a}\cdot\frac{\vec{b}}{b}\right)\frac{\vec{b}}{b}$.

**Question 10** – The points $A$ and $C$ being given, for any point $B$, we have




☑ a) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ (head-to-tail method or triangle method : the 2 vectors are arranged so that the head of the 1st vector coincides with the tail of the 2nd vector. The 3rd side which completes a triangle is the resultant vector of the sum.)

☑ b) $\overrightarrow{AB} - \overrightarrow{AC} = \overrightarrow{CB}$ (end of the 2nd vector to end of the 1st vector)

☑ c) $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$ ;

☑ d) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AA} = \vec{0}$.

**Question 11** – On the figure in the exercise 10, we also have

☑ a) $\overrightarrow{AB} = \overrightarrow{DC}$ and $\overrightarrow{AD} = \overrightarrow{BC}$ ;

☑ b) $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$ ;

☑ c) $\overrightarrow{BA} + \overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} = \overrightarrow{BD}$ ;

☑ d) $\overrightarrow{IA} + \overrightarrow{IC} = \vec{0}$ and $\overrightarrow{IB} + \overrightarrow{ID} = \vec{0}$.

**Question 12** – Given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ such that $\vec{c}$ is perpendicular to the plane $(\vec{a}, \vec{b})$. Which of the following relationships are possible ?

☑ a) $\vec{a} \times \vec{b} = \vec{c}$;

☑ b) $\vec{a} \times \vec{b} = -\vec{c}$;

☐ c) $\vec{a} + \vec{b} = \vec{c}$;

☐ d) $\vec{a} + \vec{b} = -\vec{c}$ .

**Question 13** – Given two vectors $\vec{a}$ et $\vec{b}$. Their scalar product cancels out if

☑ a) $\vec{a}$ is zero, and $\vec{b}$ is non-zero ;

☑ b) $\vec{b}$ is zero, and $\vec{a}$ is non-zero ;

☑ c) $\vec{a}$ and $\vec{b}$ are both zero ;

☑ d) $\vec{a}$ and $\vec{b}$ are non-zero and perpendicular to each other.

**Question 14** – Given two vectors $\vec{a}$ and $\vec{b}$ making an angle $\theta$ between them (we're talking about the angle between 0° and 180°). Their vector product $\vec{a} \times \vec{b}$

☑ a) is, by definition, given by $ab \sin\theta \vec{n}$, $\vec{n}$ being a unit vector directly perpendicular to $\vec{a}$ and $\vec{b}$ ;

☑ b) can be calculated, in a direct orthonormal basis $(\vec{i}, \vec{j}, \vec{k}\,)$, using the determinant $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_i & a_j & a_k \\ b_i & b_j & b_k \end{vmatrix}$ ;

☑ c) represents, in modulus, the area of the parallelogram built on the two vectors ;

☑ d) is anti-commutative : $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$.

**Question 15** – Given two vectors $\vec{a}$ and $\vec{b}$. Their cross product is zero if

☑ a) $\vec{a}$ is zero, and $\vec{b}$ is non-zero ;

☑ b) $\vec{b}$ is zero, and $\vec{a}$ is non-zero ;

☑ c) $\vec{a}$ and $\vec{b}$ are both zero ;

☑ d) $\vec{a}$ and $\vec{b}$ are non-zero and parallel to each other.

**Question 16** – Given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. The double vector product $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

☑ a) $(\vec{b} \cdot \vec{a})\, \vec{c} - (\vec{c} \cdot \vec{a})\, \vec{b}$ ;

☐ b) $(\vec{b} \cdot \vec{a})\, \vec{c} + (\vec{c} \cdot \vec{a})\, \vec{b}$ ;

☐ c) $(\vec{b} \times \vec{a}) \cdot \vec{c} - (\vec{c} \times \vec{a}) \cdot \vec{b}$ ;

☐ d) $(\vec{b} \times \vec{a})\, \vec{c} + (\vec{c} \times \vec{a}) \cdot \vec{b}$.

**Question 17** – Given three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$. The scalar triple product (or mixed product) $\vec{a} \cdot (\vec{b} \times \vec{c})$ cancels out if

☑ a) one of the three vectors is zero ;

☐ b) one of the three vectors is perpendicular to the plane of the other two ;

☑ c) two of the three vectors are parallel ;

☑ d) all three vectors lie in the same plane.

**Question 18** – Given three non-zero, and non-collinear vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ that are not in the same plane. Then

☐ a) we can write $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, where $\alpha$ and $\beta$ are real numbers ;

☑ b) their mixed product is not 0 ;

☑ c) the three vectors are linearly independent ;

☑ d) it is impossible to express one as a combination of the other two.

**Question 19** – If $\vec{a} \times \vec{b} = \vec{c}$, which of the following statements are correct ?

☑ a) $\vec{c}$ is perpendicular to $\vec{a}$ ;

☑ b) $\vec{c}$ is perpendicular to $\vec{b}$ ;

☑ c) $\vec{c}$ is perpendicular to $\vec{a}$ and to $\vec{b}$ ;

☐ d) $\vec{c}$ is in the plane of $\vec{a}$ and $\vec{b}$.

**Question 20** – Consider three points $A(-2, 1, 5)$, $B(1, 3, 5)$ and $C(-1, 3, 1)$ in a direct orthonormal coordinate system $(O; \vec{i}, \vec{j}, \vec{k})$. Coordinates are in metres. The volume of the parallelepiped built on the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AO}$ is

☑ a) $48\,\mathrm{m}^3$ ;

☐ b) $-48\,\mathrm{m}^3$ ;

☐ c) $AB \times AC \times AO$ ;

☐ d) $\overrightarrow{AB} \times \overrightarrow{AC} \times \overrightarrow{AO}$.

**Question 21** – The data from question 20 are repeated. The distance from $O$ to the plane $(ABC)$ is

☐ a) $48\,\mathrm{m}$ ;

☐ b) $-48\,\mathrm{m}$ ;

☑ c) $12/\sqrt{14}\,\mathrm{m}$ ;

☑ d) $6\sqrt{2/7}\,\mathrm{m}$.

**Question 22** – Let $\vec{i}$ , $\vec{j}$ and $\vec{k}$ be the respective unit vectors of the orthogonal axes $x, y, z$. The vector $3\vec{i} + 4\vec{j} - 5\vec{k}$ has modulus

☐ a) 2 ;

☐ b) 50 ;

☑ c) $5\sqrt{2}$ ;

☐ d) 0.

**Question 23** – The angle between the $3\vec{i} + 4\vec{j} - 5\vec{k}$ vector and the $z$ axis is

☐ a) $45°$ ;

☑ b) $135°$ ;

☐ c) $-45°$ ;

☐ d) $75°$ .

**Question 24** – Let $\vec{V} = p\vec{i} + \frac{1}{2}\vec{j} + \frac{1}{2}\vec{k}$. What is $p$ if $\vec{V}$ is a unit vector ?

☐ a) $p = \pm 1$ ;

☐ b) $p = \pm 1/2$ ;

☐ c) $p = \pm 1/4$ ;

☑ d) $p = \pm 1/\sqrt{2}$ .

**Question 25** – We give the vectors $\vec{A} = -\vec{j} + 2\vec{k}$ and $\vec{B} = \vec{i} - 2\vec{j} + \vec{k}$. Their scalar product is

☐ a) 0 ;

☐ b) 2 ;

☑ c) 4 ;

☐ d) 8 .

**Question 26** – The vector product of the vectors $\vec{A}$ and $\vec{B}$ from question 25 is

☐ a) $3\vec{i} + 2\vec{j} - \vec{k}$ ;

☑ b) $3\vec{i} + 2\vec{j} + \vec{k}$ ;

☐ c) $3\vec{i} - 2\vec{j} + \vec{k}$ ;

☐ d) $-3\vec{i} + 2\vec{j} + \vec{k}$ .

**Question 27** – Take the vectors $\vec{A}$ and $\vec{B}$ from question 25 and consider a third vector $\vec{C} = s\vec{i} + \vec{j} + t\vec{k}$ . For what values of $s$ and $t$, does the vector $\vec{C}$ lie in the same plane as $\vec{A}$ and $\vec{B}$ while being perpendicular to $\vec{B}$ ?

☐ a) $s = 0,\ t = 0$ ;

☑ b) $s = -2,\ t = 4$ ;

☐ c) $s = 2,\ t = 0$ ;

☐ d) $s = 2,\ t = -4$.

**Question 28** – Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors such that $\vec{a} + \vec{b} = \vec{c}$ and $a = 12$, $b = 5$, $c = 13$. The angle between $\vec{a}$ and $\vec{b}$ is

☐ a) 0 ;

☐ b) $\pi$ ;

☑ c) $\pi/2$ ;

☐ d) $\pi/4$ .

**Question 29** – Let $\vec{a}$ be a vector which, when added to the resultant of the vectors $2\vec{i} - 3\vec{j} + 4\vec{k}$ and $\vec{i} + 5\vec{j} + 2\vec{k}$, gives a unit vector along the $y$ axis in the $+$ direction. Then the vector $\vec{a}$ is

☑ a) $-3\vec{i} - \vec{j} - 6\vec{k}$ ;

☐ b) $3\vec{i} + \vec{j} - 6\vec{k}$ ;

☐ c) $3\vec{i} - \vec{j} + 6\vec{k}$ ;

☐ d) $3\vec{i} + \vec{j} + 6\vec{k}$.