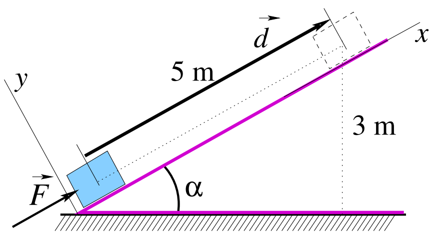
Tut #4, L1 Work and Energy ——– December 2023

Exercise 1 : You lift a book of mass m vertically and place it on a shelf at a height h. a) Express the work you've done on the book. b) What is the work done by the gravitational force on the book ? c) What is the net (total) work done on the book ? For simplicity, assume that the book is lifted without acceleration.

Solution : a) Two forces act on the book : the downward gravitation force (i.e. the book's weight) $m\vec{g}$ and the upward force $\vec{F}_{up}$ that you apply to lift the book. So, you do on the book the work $W_{\text{you}} = \vec{F}_{up} \cdot \vec{h} = F_{up} h \cos(0°) = F_{up} h$. But, since the book was lifted without acceleration, we have $\vec{F}_{up} + m\vec{g} = \vec{0}$ implying that $F_{up} = m\text{g}$. Finally $W_{\text{you}} = +m\text{g}h$. Note here that the displacement vector $\vec{h}$ is ascending like $\vec{F}_{up}$ making an angle of 0° with it, hence the (cos(0°). b) The work done on the book by the gravitational force is : $W_{\text{g}} = m\vec{g} \cdot \vec{h} = m\,\text{g}\,h \cos(180°) = -m\text{g}h$. c) The net work done on the book is $W_{\text{you}} + W_{\text{g}} = -m\text{g}h + m\text{g}h = 0$. This result is expected since the net force on the book is zero. Although the net work is zero, the person who lifts the book to a vertical height h, has to do work, as shown is a).

Exercise 2 : A block 10 kg is to be raised from the bottom to the top of an incline 5 m long and 3 m off the ground at the top (See figure i). a) Assuming no friction between the block and the incline, how much work must be done by a force $\vec{F}$ parallel to the incline pushing the block up at constant speed. b) How about the work a man would do if he were to raise the block vertically without using the incline ? c) make a comment. Take $\text{g} = 9.8\ \text{m/s}^2$.

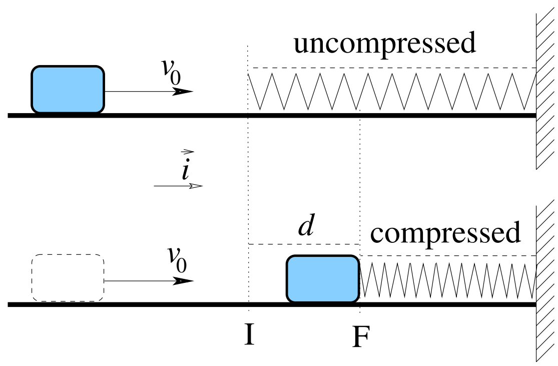


Solution : a) The forces acting on the block are $m\vec{g}$ (its weight), $\vec{F}$ (pushing force), and $\vec{N}$ (the normal force of the incline on the block), see figure ii). The work of the force $\vec{F}$ is $W_F = \vec{F} \cdot \vec{d} = Fd\cos(0^{\circ}) = Fd$. Now, we need to find F. Because the velocity of the motion is constant (i.e. not accelerated), the net force on the block must be zero. That is $\vec{F} + \vec{N} + m\vec{g} = \vec{0}$. By projecting onto x-axis, we get : $F - m\,\text{g}\sin(\alpha) = 0$ or $F = m\,\text{g}\sin(\alpha)$. Then, $W_F = m\,\text{g}\sin(\alpha) \times d$, which yields numerically $W_F = 10\,\text{kg} \times 9.8\,\text{m/s}^2 \times (3/5) = 294$ J. b) The work the man would do would be the vertical force times the vertical distance, that is, $W_{man} = \vec{F}_{vertical} \cdot \vec{d}_{vertical}$. $F_{vertical} = m\text{g}$ (because of no acceleration), and $d_{vertical} = 3$ m, so, $W_{man} = 10\,\text{kg} \times 9.8\,\text{m/s}^2 \times 3\ \text{m} = 294$ N. c) The work in b) is the same as in a). The only difference is that with the incline he need a smaller force $F = 58.8$ N to raise the block than is required without the incline ($F_{vertical} = 98$ N. On the other hand, he had to push the block a greater distance (5 m) up the incline than he had to raise the block vertically (3 m.

Exercise 3 : A man with a mass of 80 kg ascends at constant speed a staircase that is 20 m tall within 10 s. (a) Determine the power required to elevate the person. (b) If the man's body operates at 25% efficiency, calculate the expended power.

Solution : a) To ascend the staircase, the man needs to overcome his weight by exerting an upward force on himself. How can he do this ? He pushes down the ground and the ground (in virtue of Newton's 3rd law) pushes him up. Because ascending speed is constant, the upward force just balances his weight and is, therefore, equal in magnitude to $F_{up} = m\,g$. So, the person must deliver the power $P = F_{up} \times v = mg \times v = 80 \times 9.8 \times 20/10 = 1568$ W. b) An efficiency of 25% means that only 25% of the power expended P_{exp} is used to climb the stairs. $P_{exp} = 1568 \times 100/25 = 6272$ W. The remaining 75% is lost in the form of thermal energy (heat in the air) and chemical energy (in the body).

Exercise 4 : A block of mass m is moving with speed v_0 on a horizontal surface toward a spring attached to a wall as shown. The spring has a force constant (spring stiffness, ثابت صلابة نابض) k and its mass is negligible. a) Find the maximum distance d the spring will be compressed if the surface is frictionless. b) Repeat the previous question if the surface is rough with a coefficient of kinetic friction μ_k.



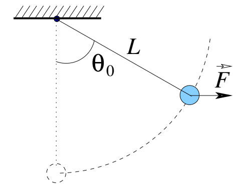
Solution :

a) We can use the kinetic energy theorem : $W_{net} = \Delta E_k$ between initial point I and final point F. At the position I, the block just touches the spring, still his speed is v_0. At F, maximum compression is reached and his speed is $v = 0$. During compression, three forces act on the block : Its weight $\vec{g}$, the normal force of the surface $\vec{N}$ and the restoring force of the spring $\vec{F_s} = -kx\vec{i}$, $\vec{i}$ is a unit vector in the compression direction. Weight and normal force don't work because they are perpendicular to the displacement. The net work on the block is reduced to the work of the spring restoring force : $W = \int_0^d -kx\vec{i}dx\vec{i} = -kx^2/2$, $\Delta E_k = (1/2)m0^2 - (1/2)mv_0^2 = -(1/2)mv_0^2$. We then have $-(1/2)kd^2 = -(1/2)mv_0^2 \implies d = \sqrt{m/k}\,v_0$.
b) Now the net work is equal to the work done by spring plus the work done by the friction : $W_{net} = W_{spring} + W_{friction} =$. With the presence of friction, the maximum compression will be d' such that $W_{spring} = -(1/2)kd'^2$ (calculation is the same as in question a)) and (since the normal force is $N = m\,g$) $W_{friction} = -\mu_k m\,g\vec{i} \cdot d'\vec{i} = -\mu_k m\,gd'$. Since the kinetic energy variation ΔE_k is the same as in a), we have $(-(1/2)kd'^2) + (-\mu_k m\,gd') = -(1/2)mv_0^2$. We therefore have to solve for d' the quadratic equation : $kd'^2 + 2\mu_k m\,gd' - mv_0^2 = 0$ (recall that the 2 solutions for $ax^2 + bx + c = 0$ are $x = (-b \pm \sqrt{b^2 - 4ac})/2a$, this leads to

$$d' = -\frac{\mu_k m\,g}{k} + \sqrt{\left(\frac{\mu_k m\,g}{k}\right)^2 + \left(\frac{mv_0}{k}\right)^2},$$

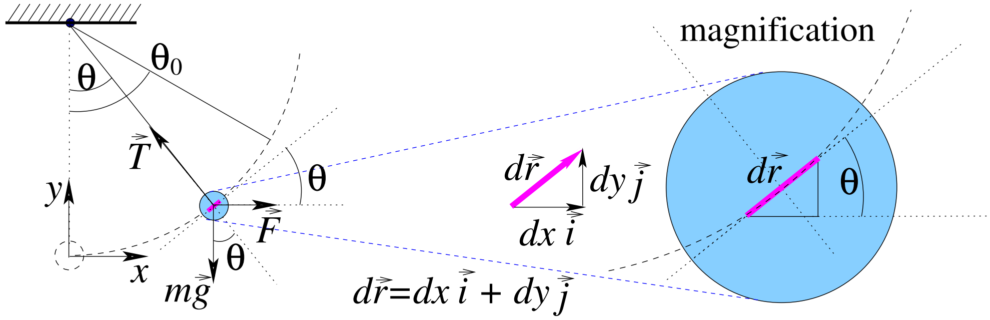
where the negative solution is rejected because d' is a distance, so $d' \geq 0$.

Exercise 5 : A horizontal force $\vec{F}$ very slowly lifts the bob (كتلة أو ثقل) of a simple pendulum, of length L, from a vertical position to a point at which the string makes an angle θ_0 to the vertical. The magnitude of $\vec{F}$ is varied so that the bob is essentially in equilibrium at all times. What is the work done by the force on the bob ?



Solution :

a) The work done on the bob by the force $\vec{F}$ is given by : $W_F = \int_{\theta_0}^{\theta} \vec{F} \cdot d\vec{r}$. To calculate the integral, we need to know how $\vec{F}$ varies through the displacement. Since the acceleration is zero, the sum of all the forces acting on the bob is zero. So, on x-axis : $F - T \sin\theta = 0$, and on y-axis : $T\cos\theta - m\,g = 0$.



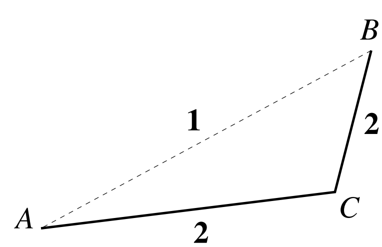
By eliminating T between the two equations, we get : $F = m\,g\tan\theta$. Then, by writing $\vec{F} = m\,g\tan\theta\vec{i}$ and $d\vec{r} = dx\vec{i} + dy\vec{j}$ we have $W_F = \int_{\theta_0}^{\theta} m\,g\tan\theta\vec{i}\cdot(dx\vec{i}+dy\vec{j}) = m\,g\tan\theta\,dx$. To finish calculation, we notice that (see magnified figure) $\tan\theta = dy/dx$ or $\tan\theta\, x = y$, the integral becomes : $W_F = \int_{\text{displcement}} m\,g\,dy$. With respect to our xy frame, we have $y = 0$ when $\theta = 0$, and $y = y_0 = L - L\cos\theta_0) =$ when $\theta = \theta_0$; so, $W_F = \int_0^{y_0} m\,g\,y_0 = m\,gL(1-\cos\theta_0)$. Calculate the work again if the applied force $\vec{F}$ is constantly tangential to the arc.

Exercise 6 : A block of mass 2 kg slides, on a horizontal surface, a distance $d = 3$ m before coming to rest. Knowing that its initial velocity is $v_i = 4$ m/ s, what is the coefficient of sliding (kinetic) friction on the surface ? *Hint : Total energy variation $E_m^f - E_m^i$ = work of frictional force.*

Solution :

a) $\Delta E_m = W(\text{frictional force}) \implies E_m^f - E_m^i = \vec{F}_k\cdot\vec{d}$, or $(E_k^f + E_p^f) - (E_k^i + E_p^i) = -F_k \times d$. We choose $E_p = 0$ at surface level, which implies $E_p^i = E_p^f = 0$. Then $(0+0)-(0+mv_i^2/2) = -F_k d \implies mv_i^2/2 = F_k d$. If mu_k is the kinetic friction coefficient, we know that $F_k = \mu_k N = \mu_k m\,g$, so, $mv_i^2/2 = \mu_k m\,gd \implies \mu_k = v_i^2/(2\,gd) = 4^2/(2\times 9.8\times 3) = 0.27$.

Exercise 7 : Compute the work done by the force of kinetic friction in sliding a crate (صندوق) of mass m along a horizontal surface from position A to position B using two different paths : 1) going straight from A to B, and 2) going from A to B via C, as depicted in figure opposite. Deduce that friction is a non-conservative force. Take $AB = d$, $AC = d_1$, $CB = d_2$, and assume that the coefficient of kinetic friction between the crate and the ground is μ_k.



Solution : $W^{path1} = \vec{F}_k\cdot\overrightarrow{AB} = -F_k AB = -\mu_k m\,gd$, $W^{path2} = \vec{F}_k\cdot(\overrightarrow{AC}+\overrightarrow{CB}) = -F_k AC - F_k CB = -\mu_k m\,g(d_1+d_2)$. Since $W^{path1} \neq W^{path2}$, we deduce that friction is a non-conservative force.