Tutorial #2 : Solution —— October 2023 (3 sessions)

Exercise 1 : From the graph below (Figure 1) showing the velocity v of a particle in rectilinear motion as a function of time t, find : a) the time or times at which the velocity cancels ; b) at what instant, if any, the particle reverses the direction of its motion ; c) the average acceleration between $t = 1\,\text{s}$ and $t = 4\,\text{s}$; d) the instantaneous acceleration at $t = 3\,\text{s}$, $t = 3.5\,\text{s}$ and at $t = 5\,\text{s}$.

Exercise 2 : Find from the graph $v(t)$ below (Figure 2), a) the average velocity between $t = 0\,s$ and $t = 6\,s$. b) the average speed for the same time interval. The average speed is defined as the covered distance divided by the time taken to cover it.

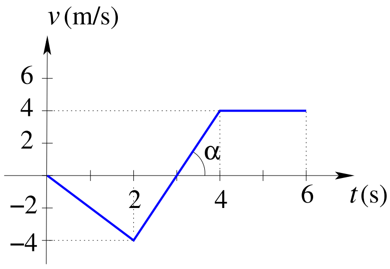


Figure 1

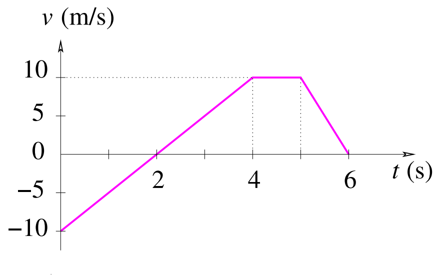


Figure 2

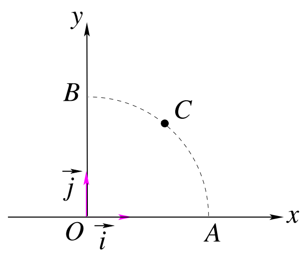


Figure 3

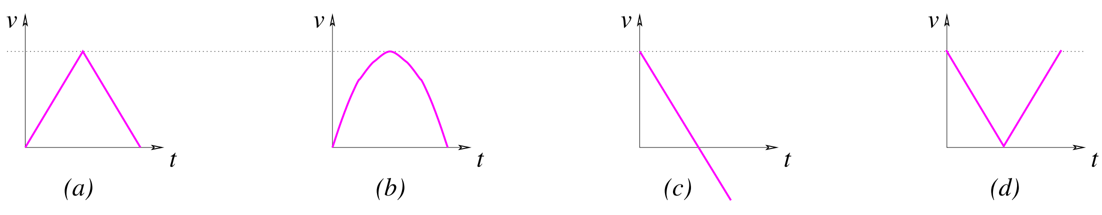
Exercise 3 : A particle covers a quarter circle, of radius $r = 5\,\text{m}$, in 5 seconds (see Figure 3 above). Initially, the particle is in B. a) What is the covered distance ? b) What is the covered displacement ? c) What is the average speed ? d) What is the average velocity ? e) Draw the velocity at point C. f) Sketch the acceleration at point C.

Solution : a) The covered distance is equal to the length of the arc $\overset{\frown}{\mathrm{BA}}$, that is : $r \times \pi/4 = 5 \times 3.14/4 = 3.93\,\text{m}$. b) The covered displacement is juste equal to $\overrightarrow{BA}$, that is : $\overrightarrow{BA} = 5\,\vec{i} - 5\,\vec{j}$. c) The average speed is distance over time, i.e. : $3.93\,\text{m}/5\,\text{s} = 0.76\,\text{m/s}$. The average velocity is displacement over time, i.e. : $\vec{v}_{av} = \overrightarrow{BA}/\text{time} = (5\,\vec{i} - 5\,\vec{j})/5 = (\vec{i} - \vec{j})\,\text{m/s}$. e) $\vec{v}_C$ is tangent to the arc in the direction of motion. f) $\vec{a}_C$ is directed to the interior of concavity.

Exercise 4 : A stone is dropped at time $t_0 = 0$, without initial velocity, into a well $490\,\text{m}$ deep.
1) Calculate the speed at which the stone reaches the bottom of the well. 2) Knowing that the speed of sound in air is $340,\text{m/s}$, how long after dropping the stone will the sound of impact be heard at the bottom of the well ? Take $\text{g} = 9.8\,\text{m/s}^2$.

Exercise 5 : A ball is launched from the roof of a 44 m high building with initial velocity v_0 directed at an angle θ below the horizontal. The ball lands $2\,\text{s}$ later at $32\,\text{m}$ from the base of the building. Find θ and v_0. Take $\text{g} = 9.8\,\text{m/s}^2$.

Exercice 6 : A marble is thrown vertically upwards, rises to a maximum height and then falls back to the ground. Which of the graphs below best represents the variation of its velocity as a function of time ? Justify.

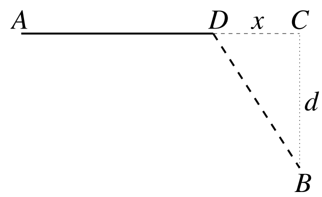


Exercise 7 : The position of a point M over time, in an orthonormal basis $(\vec{i}, \vec{j}, \vec{k})$, is given by : $\vec{r} = \vec{i} + 4t^2\vec{j} + t\vec{k}$.

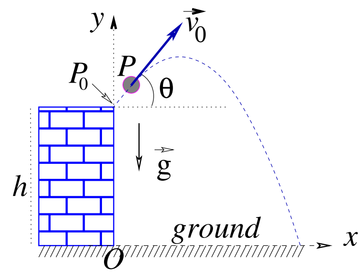
a) Express velocity and acceleration of M as a function of time. b) What is the shape of M's trajectory ?

Exercise 8 : A van is travelling in a straight line in a west-east direction at $v_1 = 90$ km/h. Suddenly, it begins a phase of constant deceleration (braking) over a distance of distance of 80 m, reducing its speed to $v_2 = 54$ km/h. a) Express v_1 and v_2 in m/s. b) Calculate the magnitude and direction of acceleration ? c) Calculate the duration of the acceleration. d) Assuming that the van continues with the same deceleration beyond the 80 m, how long and how far will it take to come to a complete stop ?

Exercise 9 : A farm tractor starts at point A on a straight road to reach a point B located in a field at a distance distance $d = CB$ from the road (see Figure on the right).
At which point D (i.e. at what distance $DC = x$) must the vehicle leave the road to complete the ADB path in the minimum time ? The paths AD and DB are supposed to be straight and travelled at constant speed by the tractor that travels half as fast in the field as on the road.



Exercise 10 : From the roof of a building of height $h = 16$m, a projectile P is launched with initial speed $v_0 = 21$m/s at 30° from the horizontal. Air resistance is ignored.
a) At any instant t, express the velocity $\vec{v}$ and the position $\overrightarrow{OP}$ of the projectile as a function of $\vec{v}_0$ and the gravity acceleration $\vec{g}$. Deduce that the projectile's motion takes place in the vertical plane containing $\vec{v}_0$ and $\vec{g}$. b) Calculate the time of flight. Take g $= 9.8$m/s^2. c) Find the horizontal amplitude R of the motion. d) Find, in terms of the projectile speed, the maximum height attained above the ground. e) Calculate the velocity when the projectile is 2 m above the building. f) Calculate the angle of impact of the projectile on the ground.



Exercise 11 : Two cars, A and B, are driving towards each other, on the same straight road, at speeds of 16 m/s and 8 m/s respectively.
When they are 45 m apart, both drivers apply the brakes. both drivers apply the brakes. The two cars are then decelerating at rates of 2 m/s^2 for A and 4 m/s^2 for B. a) When and where do the two cars collide ? b) If car A could brake more strongly, what would be the minimum rate of braking required to avoid a collision ? c) For same speeds and braking rates as in a), what is the minimum distance required between the two cars when they start braking to avoid a collision ?

Exercice 12 : Consider the circular helix of parametric equations ($x = R\cos\theta,\ y = R\sin\theta,\ z = h\theta$), where R and h are positive constants and θ is the angle between Ox and $\overrightarrow{Om}$, m being the orthogonal project of the point (x, y, z) onto the plane (xy). a) By a calculation similar to that in the previous exercise, show that the radius of curvature at any point on the helix is $\rho = R(1 + h^2/R^2)$. b) Calculate the length of the helix arc between θ and $\theta + \pi$.