

**Tut#4, First year B1 — Work and Energy — December 2024**

**Exercise 1 :** The velocity of a 4- kg body varies from  $\vec{i} + 4\vec{j}$  m/s to  $-4\vec{i} + 3\vec{j} - 2\vec{k}$  m/s. Compute the change in its kinetic energy.

$$\text{Ans. : } \Delta E_k = (1/2)m(v_f^2 - v_i^2) = 0.5 \times 4 \times (29 - 17) = 24 \text{ J.}$$

**Exercise 2 :** A block of mass 100 g is attached to the end of a spring whose stiffness constant is  $k = 40$  N/m. The block slides on a horizontal surface with a coefficient of kinetic friction  $\mu_k = 0.1$ . The spring is extended by 5 cm and then released. (a) find the work done by the spring up to the point at which it is compressed by 3 cm. (b) Find the net work done on the block up to this point.

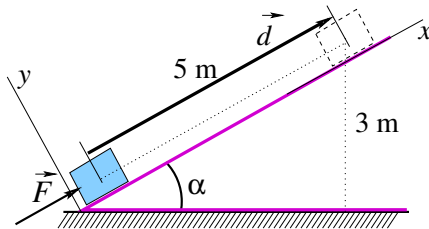
$$\text{Ans. : (a) } W_{spring} = \int_5^{-3} -kx\vec{i} \cdot d\vec{x} = -(1/2)k(x^2)]_{0.05}^{-0.03} = -0.5 \times 40 \text{ N/m} \times (0.0009 - 0.0025) = +0.032 \text{ J.}$$

(b)  $W_{f_k} = -\mu_k N \times (\text{displacement}) = -0.1 \times 0.1 \text{ kg} \times 9.8 \text{ N/kg} \times 0.08 \text{ m} = -0.0078 \text{ J}$ . The net work is therefore  $W_{net} = W_{spring} + W_{f_k} = +0.032 \text{ J} - 0.0078 \text{ J} = 0.024 \text{ J}$ .

**Exercise 3 :** You lift a book of mass  $m$  vertically and place it on a shelf at a height  $h$ . a) Express the work you've done on the book. b) What is the work done by the gravitational force on the book? c) What is the net (total) work done on the book? For simplicity, assume that the book is lifted without acceleration.

Solution : a) Two forces act on the book : the downward gravitation force (i.e. the book's weight)  $m\vec{g}$  and the upward force  $\vec{F}_{up}$  that you apply to lift the book. So, you do on the book the work  $W_{you} = \vec{F}_{up} \cdot \vec{h} = F_{up}h \cos(0^\circ) = F_{up}h$ . But, since the book was lifted without acceleration, we have  $\vec{F}_{up} + m\vec{g} = \vec{0}$  implying that  $F_{up} = mg$ . Finally  $W_{you} = +mgh$ . Note here that the displacement vector  $\vec{h}$  is ascending like  $\vec{F}_{up}$  making an angle of  $0^\circ$  with it, hence the  $\cos(0^\circ)$ . b) The work done on the book by the gravitational force is :  $W_g = m\vec{g} \cdot \vec{h} = mgh \cos(180^\circ) = -mgh$ . c) The net work done on the book is  $W_{you} + W_g = -mgh + mgh = 0$ . This result is expected since the net force on the book is zero. Although the net work is zero, the person who lifts the book to a vertical height  $h$ , has to do work, as shown in a).

**Exercise 4 :** A 10 kg block is to be raised from the bottom to the top of an incline 5 m long and 3 m off the ground at the top (See figure i). a) Assuming no friction between the block and the incline, how much work must be done by a force  $\vec{F}$  parallel to the incline pushing the block up at constant speed. b) How about the work a man would do if he were to raise, at constant speed, the block vertically without using the incline? c) make a comment. Take  $g = 9.8$  N/kg.



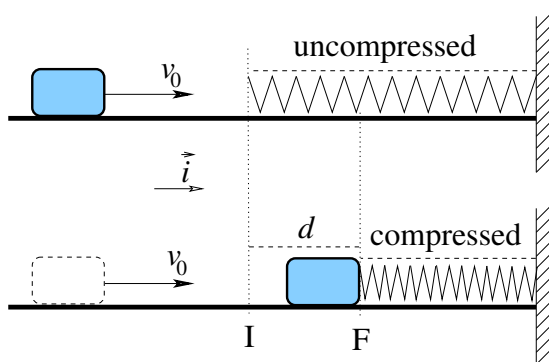
Solution : a) The forces acting on the block are  $m\vec{g}$  (its weight),  $\vec{F}$  (pushing force), and  $\vec{N}$  (the normal force of the incline on the block), see figure ii). The work of the force  $\vec{F}$  is  $W_F = \vec{F} \cdot \vec{d} = Fd \cos(0^\circ) = Fd$ . Now, we need to find  $F$ . Because the velocity of the motion is constant (i.e. not accelerated), the net force on the block must be zero. That is  $\vec{F} + \vec{N} + m\vec{g} = \vec{0}$ . By projecting onto  $x$ -axis, we get :  $F - mg \sin(\alpha) = 0$  or  $F = mg \sin(\alpha)$ . Then,  $W_F = mg \sin(\alpha) \times d$ , which yields numerically  $W_F = 10 \text{ kg} \times 9.8 \text{ m/s}^2 \times (3/5) = 294 \text{ J}$ . b) The work the man would do would be the vertical force times the vertical distance, that is,  $W_{man} =$

$\vec{F}_{vertical} \cdot \vec{d}_{vertical}$ . The vertical distance is  $d_{vertical} = 3 \text{ m}$ , and because of no acceleration  $F_{vertical} = mg$ . So,  $W_{man} = 10 \text{ kg} \times 9.8 \text{ N/kg} \times 3 \text{ m} = 294 \text{ J}$ . c) Comment : The work in b) is the same as in a). The only difference is that with the incline he need a smaller force  $F = 58.8 \text{ N}$  to raise the block than is required without the incline ( $F_{vertical} = 98 \text{ N}$ . On the other hand, he had to push the block a greater distance (5 m) up the incline than he had to raise the block vertically (3 m).

**Exercise 5 :** A man with a mass of 80 kg ascends at constant speed a staircase that is 20 m tall within 10 s. (a) Determine the power used to elevate the man. (b) If the man's body operates at 25% efficiency, how much power does he expend?

Solution : a) To ascend the staircase, the man needs to overcome his weight by exerting an upward force on himself. How can he do this? He pushes down the ground and the ground (in virtue of Newton's 3rd law) pushes him up. Because ascending speed is constant, the upward force just balances his weight and is, therefore, equal in magnitude to  $F_{up} = mg$ . So, the person must use the power  $P = F_{up} \times v = mg \times v = 80 \text{ kg} \times 9.8 \text{ N/kg} \times 20/10 \text{ m/s} = 1568 \text{ W}$ . b) An efficiency of 25% means that only 25% of the expended power  $P_{exp}$  is used to climb the staircase.  $P_{exp} \times 25/100 = 1568 \text{ W} \implies P_{exp} = 6272 \text{ W}$ . The remaining 75% is lost in the form of thermal energy (heat in the air) and chemical energy (in the body).

**Exercise 6 :** A block of mass  $m$  is moving with speed  $v_0$  on a horizontal surface toward a spring attached to a wall as shown. The spring has a force constant (spring stiffness, ثابت صلابة نابض)  $k$  and its mass is negligible. a) Find the maximum distance  $d$  the spring will be compressed if the surface is frictionless. b) Repeat the previous question if the surface is rough with a coefficient of kinetic friction  $\mu_k$ .



Solution :

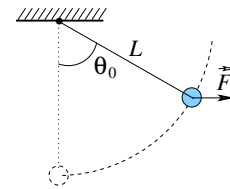
a) We can use the kinetic energy theorem :  $W_{net} = \Delta E_k$  between initial point I and final point F. At the position I, the block just touches the spring, still his speed is  $v_0$ . At F, maximum compression is reached and his speed is  $v = 0$ . During compression, three forces act on the block : Its weight  $\vec{g}$ , the normal force of the surface  $\vec{N}$  and the restoring force of the spring  $\vec{F}_s = -kx\vec{i}$ ,  $\vec{i}$  is a unit vector in the compression direction. Weight and normal force don't work because they are perpendicular to the displacement. The net work on the block is reduced to the work of the spring restoring force :  $W = \int_0^d -kx\vec{i} \cdot dx\vec{i} = -kx^2/2]_0^d$ , that is  $W = -kd^2/2$ .  $\Delta E_k = (1/2)mv_F^2 - (1/2)mv_I^2 = -(1/2)mv_0^2$ . We then have  $-(1/2)kd^2 = -(1/2)mv_0^2 \implies d = \sqrt{m/k} v_0$ .

b) Now the net work is equal to the work done by spring plus the work done by the friction :  $W_{net} = W_{spring} + W_{friction}$ . With the presence of friction, the maximum compression will be  $d'$  such that  $W_{spring} = -(1/2)kd'^2$  (computation is same as in question a)) and (since the normal force is  $N = mg$ )  $W_{friction} = -\mu_k m g \vec{i} \cdot d' \vec{i} = -\mu_k m g d'$ . Since the kinetic energy variation  $\Delta E_k$  is the same as in a), we have  $-(1/2)kd'^2 + (-\mu_k m g d') = -(1/2)mv_0^2$ . We therefore have to solve for  $d'$  the quadratic equation :  $kd'^2 + 2\mu_k m g d' - mv_0^2 = 0$  (recall that the 2 solutions for  $ax^2 + bx + c = 0$  are  $x = (-b \pm \sqrt{b^2 - 4ac})/2a$ , this leads to

$$d' = -\frac{\mu_k m g}{k} + \sqrt{\left(\frac{\mu_k m g}{k}\right)^2 + \left(\frac{mv_0}{k}\right)^2},$$

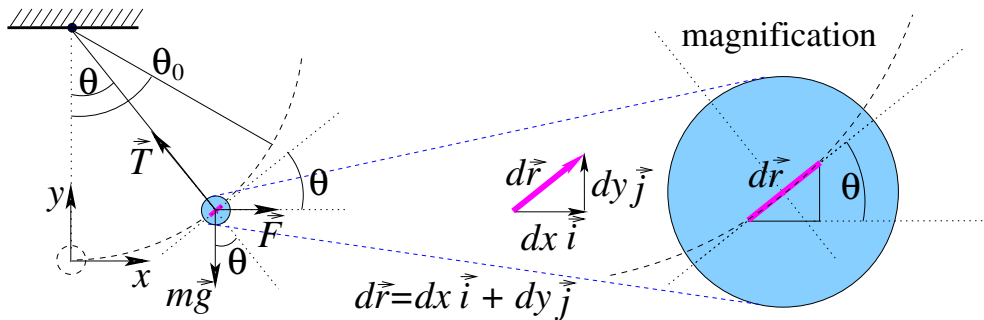
where the negative solution is rejected because  $d'$  is a distance, so  $d' \geq 0$ .

**Exercise 7 :** A horizontal force  $\vec{F}$  very slowly lifts the bob (كتلة أو ثقل) of a simple pendulum, of length  $L$ , from a vertical position to a point at which the string makes an angle  $\theta_0$  to the vertical. The magnitude of  $\vec{F}$  is varied so that the bob is essentially in equilibrium at all times. What is the work done by the force on the bob?



Solution :

a) The work done on the bob by the force  $\vec{F}$  is given by :  $W_F = \int_{\theta_0}^{\theta} \vec{F} \cdot d\vec{r}$ . To calculate the integral, we need to know how  $\vec{F}$  varies through the displacement. Since the acceleration is zero, the sum of all the forces acting on the bob is zero. So, on x-axis :  $F - T \sin \theta = 0$ , and on y-axis :  $T \cos \theta - mg = 0$ .



By eliminating  $T$  between the two equations, we get :  $F = mg \tan \theta$ . Then, by writing  $\vec{F} = mg \tan \theta \vec{i}$  and  $d\vec{r} = dx \vec{i} + dy \vec{j}$  we have  $W_F = \int_{\theta_0}^{\theta} mg \tan \theta \vec{i} \cdot (dx \vec{i} + dy \vec{j}) = mg \tan \theta dx$ . To finish calculation, we notice that (see magnified figure)  $\tan \theta = dy/dx$  or  $\tan \theta x = y$ , the integral becomes :  $W_F = \int_{\text{displacement}} mg dy$ . With respect to our  $xy$  frame, we have  $y = 0$  when  $\theta = 0$ , and  $y = y_0 = L - L \cos \theta_0$  when  $\theta = \theta_0$ ; so,  $W_F = \int_0^{y_0} mg dy = mgy_0 = mgL(1 - \cos \theta_0)$ . Calculate the work again if the applied force  $\vec{F}$  is constantly tangential to the arc.

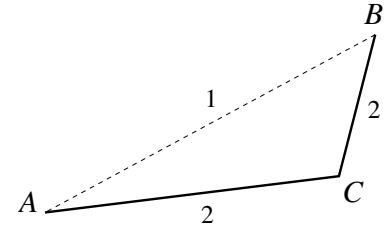
**Exercise 8 :** A 2 kg block slides on a horizontal surface a distance  $d = 3$  m before coming to rest. Knowing that its initial velocity is  $v_i = 4$  m/s, what is the coefficient of sliding (kinetic) friction on the surface? Hint : Total energy variation  $E_m^f - E_m^i = \text{work of frictional force}$ .

Solution :

a)  $\Delta E_m = W(\text{frictional force}) \implies E_m^f - E_m^i = \vec{F}_k \cdot \vec{d}$ , or  $(E_k^f + E_p^f) - (E_k^i + E_p^i) = -F_k \times d$ . We choose  $E_p = 0$  at surface level, which implies  $E_p^f = E_p^i = 0$ . Then  $(0 + 0) - (0 + mv_i^2/2) = -F_k d \implies mv_i^2/2 = F_k d$ .

If  $\mu_k$  is the kinetic friction coefficient, we know that  $F_k = \mu_k N = \mu_k m g$ , so,  $mv_i^2/2 = \mu_k m g d \implies \mu_k = v_i^2/(2gd) = 4^2/(2 \times 9.8 \times 3) = 0.27$ .

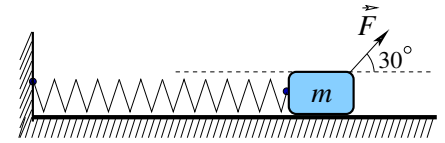
**Exercise 9 :** Compute the work done by the force of kinetic friction in sliding a crate (صندوق) of mass  $m$  along a horizontal surface from position  $A$  to position  $B$  using two different paths : 1) going straight from  $A$  to  $B$ , and 2) going from  $A$  to  $B$  via  $C$ , as depicted in figure opposite. Deduce that friction is a non-conservative force. Assume that the coefficient of kinetic friction between the crate and the ground is  $\mu_k$ .



**Solution :**  $W^{path1} = \vec{F}_k \cdot \vec{AB} = -F_k AB = -\mu_k m g AB$ ,  $W^{path2} = \vec{F}_k \cdot (\vec{AC} + \vec{CB}) = -F_k AC - F_k CB = -\mu_k m g (AC + CB)$ . Since  $AB$  is not equal to  $AC + CB$ , we get  $W^{path1} \neq W^{path2}$ . So, we deduce that friction is a non-conservative force.

**Exercise 10 :** A force  $F = 36$  N acts at  $30^\circ$  above the horizontal on a mass  $m = 3$  kg attached to a spring as shown in the adjacent figure. The spring stiffness constant is  $k = 40$  N/m.

The coefficient of kinetic friction between the block and the horizontal plane is  $\mu_k = 0.15$ . Initially, the system was at rest and the spring not extended. If the block moves a distance  $d = 30$  cm, find the work done (a) by the force  $F$ ; (b) by the spring; (c) by the friction. (d) Compute the final speed of the block. Take  $g = 10$  N/kg.



**Ans. :** (a)  $W_a = F \cos(36.8^\circ) \times d = 36 \times 0.8 \times 0.3 = 8.64$  J. (b)  $W_b = \int_0^d -kx\vec{i} \cdot dx\vec{i} = -(1/2)kx^2]_0^d = -(1/2)kd^2 = -0.5 \times 40 \times 0.3^2 = -1.8$  J. (c)  $W_c = -\mu_k N \times d = -\mu_k(mg - F \sin 36.8^\circ) \times d = -0.15 \times (30 - 36 \times 0.6) \times 0.3 = -0.378$  J. (d) Using Work-Energy theorem  $\Delta E_k = \sum W_{ext}$  we get  $(1/2) \times 3 \times v_f^2 = 8.64 - 1.8 - 0.378 \implies v_f = \sqrt{6.46} = 2.54$  m/s.