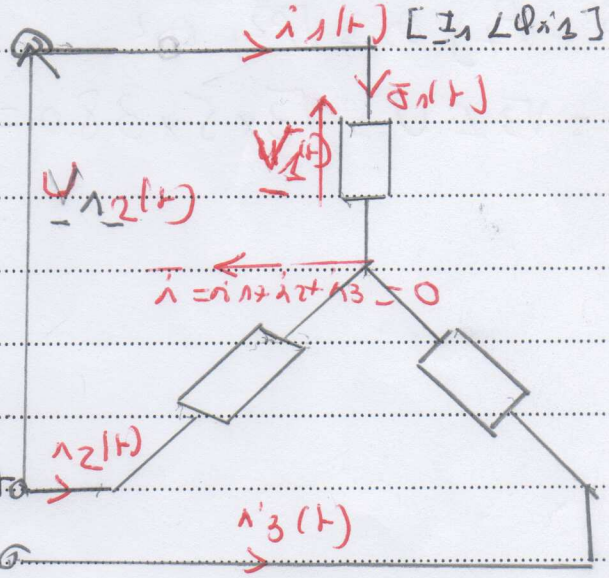
Circuits triphasés et puissances électriques — TD N°2

Exercice N°1 :

Réseau (220 / 380) V, $Z = 44\ \Omega$, $\cos\varphi = 0{,}8$ (AR)

V ↓ U = √3 V ↓ U



1°) Courant de ligne $\underline{I}$

Couplage étoile (λ) (J = I)

La tension aux bornes de $\underline{Z}$ est :

$\underline{V_1} = \underline{Z}\,\underline{I_1}$ / V_1 = Tension d'une Phase, $\underline{I_1}$ = Courant de ligne)

$$\underline{I_1} = \frac{\underline{V_1}}{\underline{Z}} = \frac{V\angle 0°}{Z\angle\varphi} \quad ; \quad \underline{I_1} = I\angle\varphi_{i1}$$

$\varphi = Arc\cos(0{,}8) = 36°$ / ($\cos\varphi = 0{,}8$ (AR) ⇒ (charge inductive).

$$\underline{I_1} = \frac{220\angle 0°}{44\angle 36°} = 5\angle -36°\ A$$

Le courant $\underline{I}$ est en arrière de −36° / à $\underline{V_1}$

2°) Expressions des $\underline{V}$ et $\underline{I}$ Instantanés

$$\begin{cases} V_1(t) = \sqrt{2}\,V\sin\omega t \\ V_2(t) = \sqrt{2}\,V\sin\left(\omega t - \frac{2\pi}{3}\right) \\ V_3(t) = \sqrt{2}\,V\sin\left(\omega t - \frac{4\pi}{3}\right) \end{cases} \qquad \begin{cases} V_1(t) = \sqrt{2}\times 220\sin\omega t \\ V_2(t) = \sqrt{2}\,220\sin(\omega t - 120°) \\ V_3(t) = \sqrt{2}\times 220\sin(\omega t - 240°) \end{cases}$$

$$\begin{cases} i_1(t) = \sqrt{2}\,I\sin(\omega t + \varphi) \\ i_2(t) = \sqrt{2}\,I\sin\left(\omega t - \frac{2\pi}{3} + \varphi\right) \\ i_3(t) = \sqrt{2}\,I\sin\left(\omega t - \frac{4\pi}{3} + \varphi\right) \end{cases} \qquad \begin{cases} i_1(t) = \sqrt{2}\,5\sin(\omega t - 36°) \\ i_2(t) = \sqrt{2}\,5\sin(\omega t - 120° - 36°) \\ i_3(t) = \sqrt{2}\,5\sin(\omega t - 240° - 36°) \end{cases}$$

3°) Diagramme de Fresnel (voir fig)

4) Puissances: P, Q, S

$P = \sqrt{3}\, U I \cos\varphi = \sqrt{3} \times 380 \times 5 \cos 36° = 2662,39$ W

$Q = \sqrt{3}\, U I \sin\varphi = \sqrt{3} \times 380 \times 5 \times \sin 36°) = 1934,34$ VAR

$S = \sqrt{3}\, U I = 3290,83$ VA

Diagramme Fresnel :

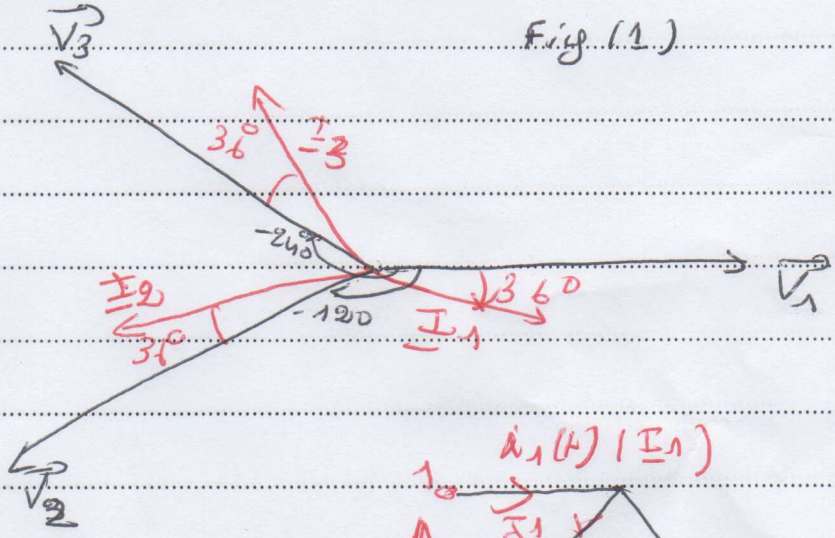
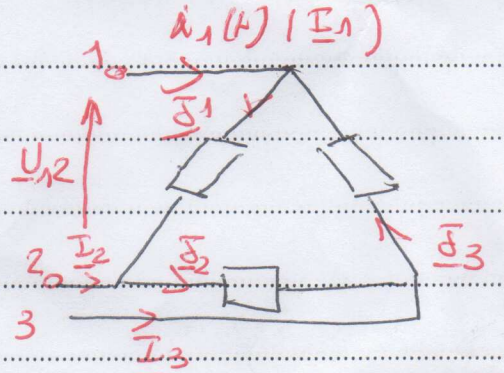


Fig. (1)



Exercice N°2:

Réseau (220 / 380) V 50 Hz

$Z = 4\Omega$; $\cos\varphi = (0,8$ AV)

1) $I = \sqrt{3}\, J$

$\underline{U}_{12} = \underline{Z}\, \underline{J}_1 \Rightarrow \underline{J}_1 = \dfrac{\underline{U}_{12}}{\underline{Z}} = \dfrac{U \angle 4}{Z \angle 4}$; $\varphi = \text{Arc}\cos(0,8) = 36°$ (charge capacitive)

$J_1 = \dfrac{380\ \angle 30°}{4\ \angle -36°} = 8,64\ \angle 66° \Rightarrow I = \sqrt{3} \times 8,64$

$I = 14,96$ A = courant de ligne.

Le courant $\underline{J}_1$ est en avance de 36° / à $\underline{U}_{12}$ (charge capacitive)

2) Tensions et courant instantanés

$u_{12}(t) = \sqrt{2} \times 380 \sin(\omega t + 30°)$

$u_{23}(t) = \sqrt{2} \times 380 \sin(\omega t - 120 + 30°)$

$u_{31}(t) = \sqrt{2} \times 380 \sin(\omega t - 240 + 30°)$

$j_1(t) = \sqrt{2} \times 8,64 \sin(\omega t + 66°)$

$j_2(t) = \sqrt{2} \times 8,64 \sin(\omega t - 120 + 66°)$

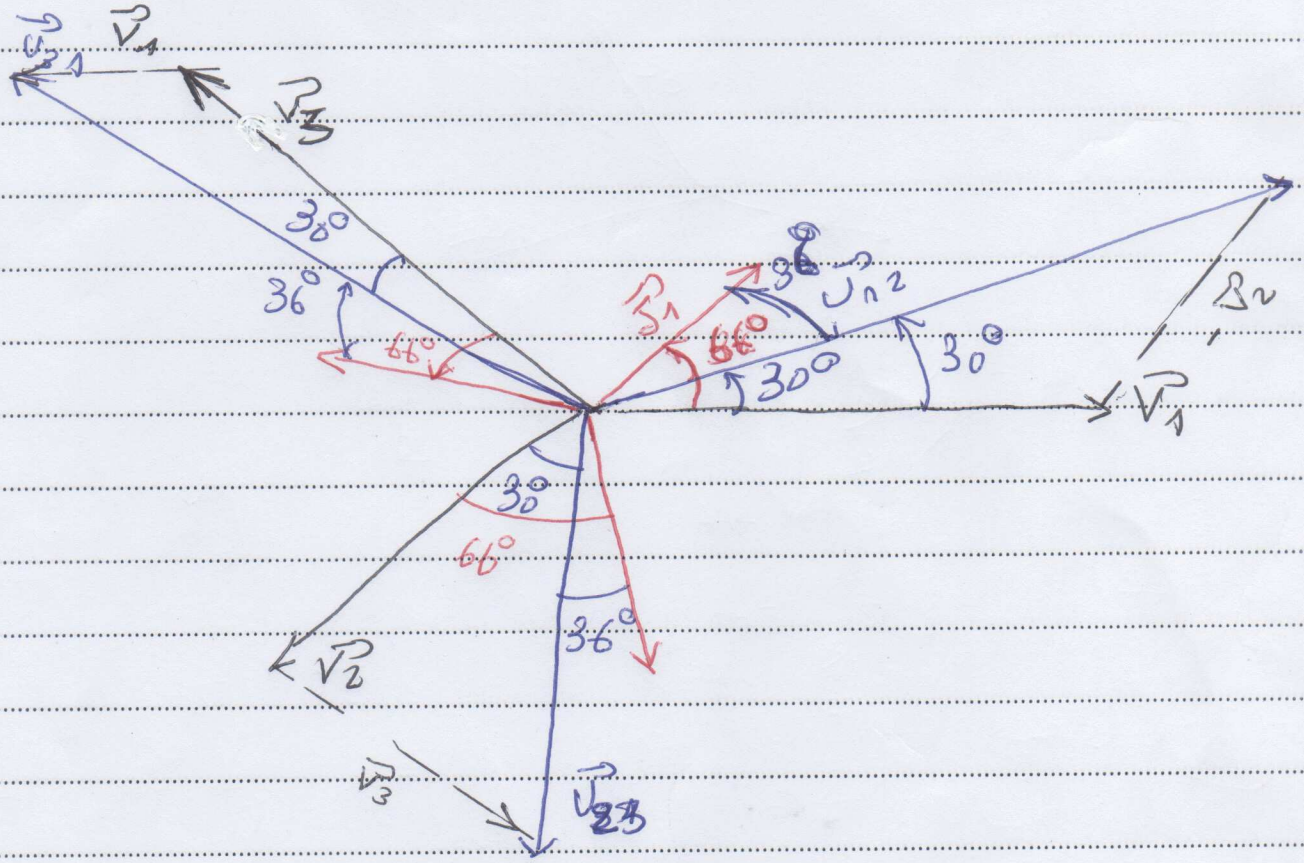
$j_3(t) = \sqrt{2} \times 8,64 \sin(\omega t - 240 + 66°)$

Puissances

$P = \sqrt{3}\, U I \cos\varphi = \sqrt{3} \times 14,96 \times 380 \times 0,8 = 7877,09$ W

$Q = \sqrt{3}\, U I \sin\varphi = \sqrt{3} \times 14,96 \times 380 \times \sin(-36°) = -5787,54$ VAR

$S = \sqrt{3}\, U I = \sqrt{3} \times 380 \times 14,96 = 9846,36$ VA



L3

Exercice N°3 : 1ère Méthode :

Réseau (220/380) V – 50 Hz ; Couplage (2) ; $\underline{Z} = R - jX$

$W_1 = 782$ W ; $W_2 = 1980$ W ; (charge capacitive)

a) Facteur de puissance :

$$tg\varphi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{782 - 1980}{782 + 1980} = -0,75$$

$\varphi = \text{Arc tg}\,\varphi = \text{Arc}(-0,75) = -36°$

$F_p = \cos\varphi = \cos(-36°) = 0,8$

b) Courant de ligne :

$W_1 + W_2 = \sqrt{3}\,U I \cos\varphi \Rightarrow I = \dfrac{W_1 + W_2}{\sqrt{3} \times U \times \cos\varphi}$

$$I = \frac{782 + 1980}{\sqrt{3} \times 380 \times 0,8} \simeq 5,25\ A$$

c) Impédance d'une phase

$$\underline{Z}_s = \frac{\underline{V}_1}{\underline{I}_1} = Z \angle \varphi \quad ; \quad Z = \frac{V}{I} = \frac{220}{5,25} = 41,9\ \Omega$$

$\underline{Z} = 41,9 \angle -36°\ \Omega = (33,9 - j\,24,62)\ \Omega$

d) Capacité d'une phase

$X = \dfrac{1}{C\omega}$; $\omega = 2\pi f = 2\pi \times 50 = 314$ rd/s

$$C = \frac{1}{X \cdot \omega} = \frac{1}{24,62 \times 314} \simeq 129\ \mu F$$

e) Le déphasage courant / tension :

$\arg(\underline{I}_1) = \arg(\underline{V}_1) - \arg(\underline{Z}) = -\arg(\underline{Z}) = -(-36)°$

$\arg(\underline{I}_1) = 36° \Rightarrow \underline{I}_1 = 5,25 \angle 36°$

Le courant est en avance de 36° / à la tension.

f) Puissances P, Q, S

$P = W_1 + W_2 = \sqrt{3}\,U I \cos\varphi = 782 + 1980 = 2762$ W

$Q = \sqrt{3}(W_1 - W_2) = \sqrt{3}\,U I \sin\varphi = \sqrt{3}(782 - 1980) \simeq -2075$ VAR

$S = \sqrt{3}\,U I = \sqrt{P^2 + Q^2} = \sqrt{2762^2 + 2075^2} = 3454,6$ V.A

Exercice N°3 : 2ème Méthode :

Réseau (220 / 380) V – 50 Hz couplage (Δ)

V U f

$\underline{Z} = R - jX$ (Impédance capacitive)

$W_1 = 782$ W ; $W_2 = 1980$ W.

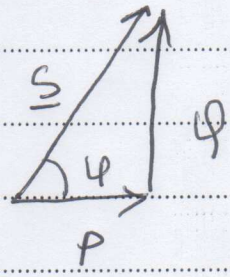
1°) Puissance P, Q, S

$P = \sqrt{3}\, U I \cos\varphi = W_1 + W_2 = 782 + 1980 = 2762$ W.

$Q = \sqrt{3}\, U I \sin\varphi = \sqrt{3}(W_1 - W_2) = \sqrt{3}(782 - 1980) = -2075$ VAR

$S = \sqrt{3}\, U I = \sqrt{P^2 + Q^2}$

$= \sqrt{2762^2 + (-2075)^2} = 3454{,}6$ VA



2°) $F_p = \cos\varphi = \frac{P}{S} = \frac{2762}{3454{,}6} \simeq 0{,}8$

3°) Courant de ligne I

$I = \frac{S}{\sqrt{3}\, U} = \frac{3454{,}6}{\sqrt{3} \times 380} = 5{,}25$ A

4°) Impédance $\underline{Z}$

$\underline{Z} = Z \angle \varphi$; $\varphi = -\text{Arc}\cos(0{,}8) = -36°$

$Z = \frac{V}{I} = \frac{220}{5{,}25} = 41{,}9\ \Omega$

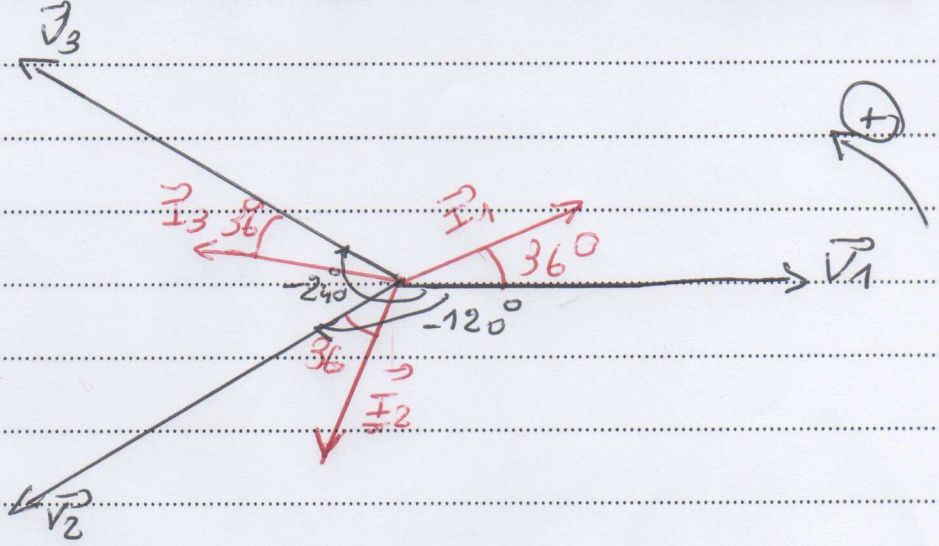
$\underline{Z} = 41{,}9 \angle -36° = (33{,}9 - j\,24{,}62)\ \Omega$

5°) Capacité ; $\omega = 2\pi f = 2\pi \times 50 = 314$ rd/s

$X = 24{,}62 = \frac{1}{C\omega} \Rightarrow C = \frac{1}{X\omega} = \frac{1}{24{,}62 \times 314}$

$C \simeq 129\ \mu F$

g) Diagramme vectoriel des courants et tensions



b) Couplage (Δ)

$$\underline{J}_1 = \frac{\underline{U}_{12}}{\underline{Z}} = J_1 \angle \varphi_{J_1} \quad ; \quad J_1 = \frac{U}{Z} = \frac{380}{41,9} \simeq 9,07\,A$$

$\text{Arg}(\underline{J}_1) = \arg(\underline{U}_{12}) - \arg(\underline{Z}) = 30 - (-36) = 66°$

$\underline{J} = 9,07 \angle 66°$

Puissance P, Q, S

$P = \sqrt{3}\,U\,I\cos\varphi \Rightarrow I = \sqrt{3} \times J = \sqrt{3} \times 9,07 \simeq 15,71\,A$

$P = \sqrt{3} \times 380 \times 15,71 \times 0,8 \simeq 8272\,W$

$Q = \sqrt{3}\,U\,I\sin\varphi = \sqrt{3} \times 380 \times 15,71 \sin(-36) \simeq -6077,70\,VAR$

$S = \sqrt{3}\,U\,I \simeq 10360\,VA$

Exercice N°4

Réseau (250 / 4,34) V – 50 Hz, $\underline{Z}_{M1} = (1,25 + j\,2,17)\,\Omega$

$I_{M_2} = 120\,A$; $\cos\varphi_2 = 0,87$ (AR)

1°) Courant $\underline{I}_{M_1}$

$$\underline{I}_{M_1} = \frac{\underline{V}_1}{\underline{Z}_{M_1}} \quad ; \quad \underline{Z}_{M_1} = Z \angle \varphi_1 \Rightarrow Z = \sqrt{1,25^2 + 2,17^2} = 2,5\,\Omega$$

$$\varphi_1 = \arg(\underline{Z}_{M_1}) = \text{arc tg}\left(\frac{2,17}{1,25}\right) \simeq 60°$$

$$\underline{Z}_{M_1} = 2,5 \angle 60°\,\Omega \Rightarrow \underline{I}_{M1} = \frac{V \angle 0°}{Z_{M_1} \angle 60°} = \frac{250}{2,5} \angle -60°$$

$\underline{I}_{M1} = 100 \angle -60°\,A = (50 - j\,86,6)\,A$

courant fourni par la source

$$\underline{I} = \underline{I}_{M1} + \underline{I}_{M2}$$

$$\underline{I}_{M2} = I_{M2} \angle \varphi_{i2} \; ; \quad I_{M2} = 120\,A \; ; \; \cos\varphi_2 = 0{,}87 \; (AR)$$

$$\varphi_2 = \arg(\underline{Z}_{M2}) = \arccos\varphi_2 = \arccos(0{,}87) = 29{,}5° \Rightarrow$$

(Impédance Inductive)

$$\varphi_{i2} = \arg(\underline{I}_{M2}) = \arg(\underline{V}_1) - \arg(\underline{Z}_{M1}) = 0 - 29{,}5° = -29{,}5°$$

$$\underline{I}_{M2} = 120 \angle -29{,}5° A = (104{,}44 - j59)\,A$$

$$\underline{I} = \underline{I}_{M1} + \underline{I}_{M2} = (50 - j86{,}6) + (104{,}44 - j59)$$

$$= (154{,}44 - j145{,}6)\,A = I \angle \varphi_i$$

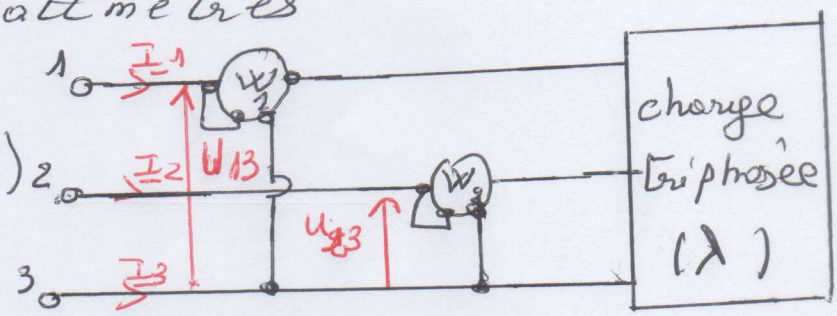
$$I = \sqrt{154{,}44^2 + (145{,}6)^2} = 212{,}25\,A \; ; \; \varphi_i = \arctg\left(\frac{-154{,}44}{154{,}44}\right)$$

$$\varphi_i = -43{,}31° \Rightarrow \underline{I} = 212{,}25 \angle -43{,}31°\,A$$

2) Lecture des deux wattmètres

$$W_1 = \vec{U}_{13}\vec{I}_1 = UI\cos(\alpha(\vec{U}_{13}, \vec{I}_1))$$

$$W_2 = U_{23} \cdot \underline{I}_2 = UI\cos(\alpha(\vec{V}_{23}, \vec{I}_2))$$



$$W_1 = UI\cos(43{,}31 - 30°) \quad (13{,}31°)$$

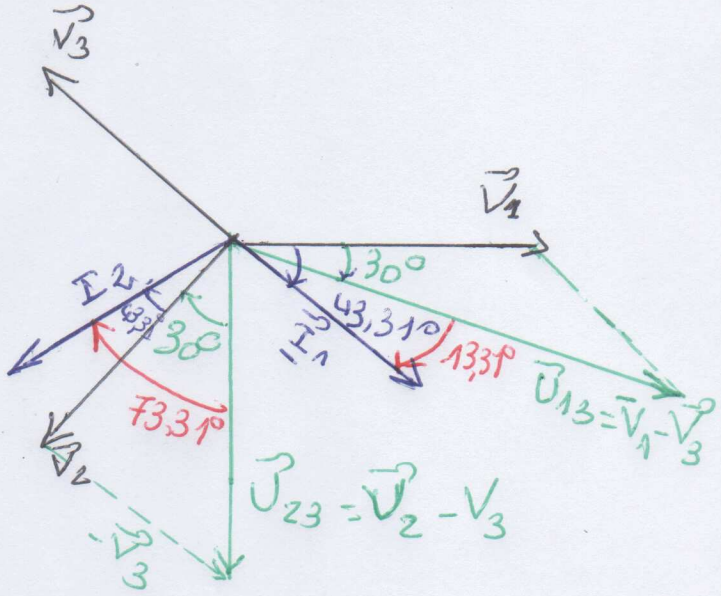
$$= 434 \times 212{,}25 \cos(43{,}31 - 30)$$

$$= 89642{,}13\,W$$

$$W_2 = UI\cos(43{,}31 + 30°) \quad (73{,}31°)$$

$$= 434 \times 212{,}25 \cos(43{,}31 + 30°)$$

$$\simeq 26455{,}2\,W$$



3°) Impedance $\underline{Z}_{M2}$

$$Z_{M2} = \frac{V_1}{\underline{I}_{M2}} = Z_{M2} \angle \varphi_2 = Z_{M2} \angle 29{,}5°\,\Omega$$

$$Z_{M2} = \frac{V_1}{I_{M2}} = \frac{250}{120} = 2{,}083\,\Omega$$

$\underline{Z}_{M2} = 2,083 \angle 29,5° \ \Omega$

4° Facteur de puissance.

$F_p = \cos\varphi = \cos(43,31°) = 0,727$ (AR)

5° Capacité "C" des condensateurs couplés en (Δ) :

$\cos\varphi = 0,727$

$P = W_1 + W_2$

$= 89642,13 + 26455,25$

$= 116097,37 \ W$

$Q_1 = \sqrt{3}(W_1 - W_2)$

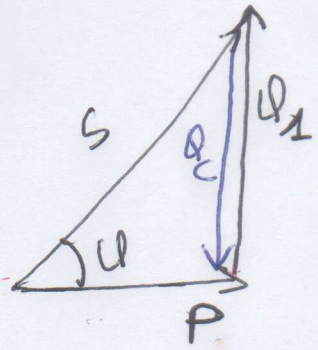
$= \sqrt{3}(89642,13 - 26455,25)$

$Q_1 = 109442,9 \ VAR$

$\cos\varphi' = 1$

$P = 116097,37 \ W$

$Q_2 = P \ tg\ \varphi' = 0$



La puissance des 3 condensateurs :

$Q_c = Q_2 - Q_1 = -109442,9 \ VAR$

$Q_c = -3V^2 C\omega \Rightarrow C = \dfrac{Q_c}{-3V^2\omega}$ / $\omega = 2\pi f = 2\pi \times 50 = 314 \ rd/s$)

$Q_c = \quad C = \dfrac{-109442,9}{-3 \times 434^2 \times 314} \simeq 617 \mu F$

6) Diagramme Fresnel