Exercice 01 (06 points) ━━━━〰〰〰

Let's consider the sequence v_n defined by :

$$\begin{cases} v_0 &= \dfrac{3}{2} \\ v_{n+1} &= \sqrt{3v_n - 2} \end{cases}$$

1 Show that : $1 \le v_n \le 2$, $\forall n \in \mathbb{N}$.

2 Establish the monotonicity of the sequence v_n and determine its limit.

Exercice 02 (10 points) ━━━━〰〰〰

1 Find the values of a and b that make the following function differentiable on $\mathbb{R}$

$$f(x) = \begin{cases} e^x + 2, & si\ x \ge 0 \\ a\cos(x) + bx + 1, & si\ x < 0 \end{cases}$$

2 Compute the derivative of the following function

$$f(t) = e^{\tan h(t)}, \quad \text{with } t > 0, h(t) = \frac{\pi}{2 + 2t}$$

Exercice 03 (04 points) ━━━━〰〰〰

Using the Mean Value Theorem on the function $f(t) = \ln(t)$, show that :

$$\forall x \in]0, +\infty[: \ln(x) \le x - 1$$

good luck