2025/2026	**Exercise Sheet 2**	Department: Common Core in Mathematics
Analysis 2	Definite integral	and Computer Science Batna 2-University.

Darboux and Riemann sums.

Exercise 1

Let $f(x) = x^2 - 3x$ et $P = \{x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 3, x_4 = 4\}$

1. Compute: $U(P, f)$, $L(P, f)$.
2. $S(f, P)$ with $\alpha_k = x_k$

Exercise 2

1. Using Darboux sums, show that the function $f(x) = x^2$ is Riemann-integrable on the interval $[0, a], \quad a > 0$.
2. Interpret each of the following sequences as a Riemann sum (after any transformation, specify the function and integration interval),

$$1. \sum_{k=1}^{k=n} \frac{1}{n+k}, \qquad 2. \sum_{k=1}^{k=n} \frac{n}{n^2+k^2}$$

Definite integrals and antiderivatives.

Exercise 3

Compute the following definite integrals

1. $\displaystyle\int_0^5 f_i(x)dx,\ f_1(x) = |x-2|,\ f(x)_2 = \begin{cases} x^2 & : \ x < 2 \\ 3x - 2 & : \ x \geq 2 \end{cases}$
2. $\displaystyle\int_1^3 |x^2 - 3x + 2|\mathrm{dx}$
3. $\displaystyle\int_1^3 2x\sqrt{x}dx$
4. $\displaystyle\int_0^1 x^2 \arctan(x)dx$
5. $\displaystyle\int_0^{\pi/4} \frac{\sin(2x)}{2+\cos(x)}dx$

Exercise 4

Compute $\displaystyle\int_0^a \sqrt{a^2 - x^2}dx, \quad a > 0$. Deduce the area of a circle with radius r.

Exercise 5

Let

$$I = \int_1^2 \frac{dx}{2 + \sqrt{x(4-x)}}$$

1. Show that $I = \displaystyle\int_1^{\sqrt{3}} \frac{4z dz}{(1+z^2)(1+z)^2}$
2. Deduce that $I = \dfrac{\pi}{6} - 2 + \sqrt{3}$