

Exercise 1

Put the following complex numbers into algebraic form:

1. $\frac{1}{i}$

3. $\frac{i}{i+1}$

5. $\frac{1}{2+i} + \frac{1}{2-i}$

2. $\frac{2+i}{5-i}$

4. $\frac{1}{2-i}$

6. $\frac{1}{1+i} + \frac{i}{1+i}$

Exercise 2

Determine the modulus and an argument for each complex number :

1. -2

3. $2e^{-2i}$

5. $\frac{1+i}{\sqrt{3}-1}$

2. $3i$

4. $-1+i\sqrt{3}$

6. $(\sqrt{3}-i)^9$

Exercise 3

Find the points of the complex plane which satisfy the following conditions.

1. $|z| \leq 2$

2. $z + \bar{z} = 1$

3. $|z - 3 + 5i| = 2$

Exercise 4

Let $\alpha \in \mathbb{R}$. Express $\cos 5\alpha$ as a function of $\cos \alpha$, then $\sin 5\alpha$ as a function of $\sin \alpha$, give the value of $\cos \frac{\pi}{10}$.

Exercise 5

Find the square roots of the following complex numbers:

1. $5+i$

2. $6-8i$

3. $4\sqrt{3}+i$

Exercise 6

Solve the following equation in \mathbb{C} :

1. $z^2 + (2 - 2i)z = 3i + 1$

2. $z^3 = \frac{1+i}{\sqrt{2}}$

3. $z^6 = 27i$

Exercise 7

1. For all $x \in \mathbb{R}$, compute the following sums using the exponential form of a complex number:

$$A = \cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x + \cos 6x + \cos 7x$$

$$B = \sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x + \sin 6x + \sin 7x$$

2. by using Euler formulas, linearize: $\cos x^3, \sin x^3, \cos x^3 \sin x^4$.