Exercise 1

Let $(u_n)_{n\in\mathbb{N}^*}$ be the sequence defined by :

$$u_n = \frac{n+2}{n+1}$$

1. Show that $(u_n)_{n\in\mathbb{N}^*}$ converges to 1.
2. Find an integer $n_0 \in \mathbb{N}$ such that all terms u_n of index $n \geq n_0$ are in the interval $I =]0.98; 1.2[$.

Exercise 2

Let $(u_n)_n$ a sequence of real numbers, and assume that it converges to a positive limit $l > 0$. Show that there exists some $N \in \mathbb{N}$ such that $\forall n \geq N : u_n > 0$.

Exercise 3

Study the nature of the following sequences and determine their possible limits:

1. $\sqrt{n^2+n+1} - \sqrt{n}$
2. $\dfrac{n\cos n}{n^3+1}$
3. $\dfrac{\sin n^2 + 2\cos n}{n^2}$
4. $\dfrac{a^n - b^n}{a^n + b^n}$; $a,\ b \in]0, +\infty[$
5. $(1 + \dfrac{2}{n})^n$
6. $n^3(\tan\dfrac{3}{n} - \sin\dfrac{3}{n})$

Exercise 4

Let the sequences $(x_n),\ (y_n),\ (u_n),$ and (v_n) of real numbers such that:

1. $(\forall n \in \mathbb{N},\ x_n \leq 2 \text{ et } y_n \leq 3)$ et $x_n + y_n \to 5$
2. $(\forall n \in \mathbb{N},\ 0 \leq u_n \leq 1 \text{ et } 0 \leq v_n \leq 1)$ et $u_n.v_n \to 1$

Show that the sequences defined above are convergent

Exercise 5

Let (u_n) be a sequence of real numbers

1. If $\lim u_n = +\infty$, show that $\lim E(u_n) = +\infty$.
2. If the sequence (u_n) converges, can we say that $E(u_n)$ converges?

Exercise 6

Let (u_n) be a sequence such that $u_0 = 4$ and $\forall n \in \mathbb{N}: \; u_{n+1} = 3 - \dfrac{4}{2+u_n}$.

1. Show that $u_n \geq 2$, for all $n \in \mathbb{N}$.
2. Prove that (u_n) is a monotonic sequence.
3. Study its convergence. If it converges, compute its limit.

Exercise 7

For any $n \in \mathbb{N}^*$, consider the sequence defined by: $H_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + ... + \dfrac{1}{n}$

1. Compute $H_{2n} - H_n$
2. Show that H_n is divergent.