2023/2024	**Tutorial=4**	Department: Common Core in Mathematics
Analysis 1	Limits and Continuous Functions	and Computer Science Batna 2-University.

Exercise 1

Find the domain of definition of the following functions:

1. $f_1(x) = \dfrac{1}{\sqrt{x} + \sqrt{1-x}}$
2. $f_2(x) = \ln(\ln x)$
3. $f_3(x) = \dfrac{1}{E(x) - 2}$
4. $f_4(x) = \begin{cases} 1/(1-x) & \text{si } x \geq 0 \\ 1 & \text{si.} x < 0 \end{cases}$
5. $f_5(x) = \begin{cases} 1/(3-x) & \text{si } x \geq 0 \\ x^4 - x & \text{si.} x < -2 \end{cases}$
6. $f_6(x) = \begin{cases} 1/x(2-x) & \text{si } x \geq 3 \\ 1 & \text{si.} x < 0 \end{cases}$

Exercise 2

Solve the following equations in $\mathbb{R}$

1. $\ln(x-1) + \ln(2x-1) = 0$
2. $2^{3x} - 3^{x+2} = 3^{x+1} - 2^{3x+2}$
3. $(\sqrt{x})^x = x^{\sqrt{x}}$

Exercise 3

Prove that:

1. $\lim_{x \to 1} 3x + 1 = 4$
2. $\lim_{x \to +\infty} x^2 + x - 2 = +\infty$
3. $\lim_{x \to 0^-} \dfrac{1}{x} = -\infty$

Exercise 4

Determine the following limits when x converges to 0.

1. $\dfrac{\sqrt{1+x} - \left(1 + \frac{x}{2}\right)}{x^2}$
2. $\dfrac{\sqrt{2x^2 + 5x + 9} - 3}{x}$
3. $x + 1 + \dfrac{|x|}{x}$
4. $\dfrac{\sqrt{1+x} - 1}{\sqrt[3]{1+x} - 1}$

Exercise 5

Determine the following limits when x tends to $+\infty$

1. $\dfrac{2x^2 + 3x - 1}{3x^2 + 1}$
2. $x - \sqrt{x^2 - x}$
3. $\dfrac{\sqrt{x}}{\sqrt{x + \sqrt{x}}}$
4. $\dfrac{2\ln(x) - \ln(3x^2 - 2)}{x \sin(\frac{1}{x})}$

Exercise 6

Let be the numerical function defined by:

$$f(x) = \begin{cases} 0, & si \ x \in]-\infty, 2] \\ a - \dfrac{b}{x}, & si \ x \in]2, 4] \\ 1, & si \ x \in]4, +\infty[\end{cases}$$

Determine the real parameters a and b so that the function f is continuous on $\mathbb{R}$. Then draw the graph of f.

Exercise 7

Which of the following given functions $f_i : \mathbb{R}^* \to \mathbb{R}$ can be extended to become a continuous function at 0.

1. $f_1(x) = \begin{cases} x\sin(\frac{1}{x}), & si\ x > 0 \\ \frac{\sin(x)}{x}, & si\ x < 0 \end{cases}$

2. $f_2(x) = \begin{cases} \frac{1-\cos(x)}{x^2}, & si\ x > 0 \\ x\sin(\frac{1}{x}), & si\ x < 0 \end{cases}$

3. $f_2(x) = \begin{cases} x\cos(\frac{1}{x}), & si\ x > 0 \\ \frac{1-\cos(x)}{x^2}, & si\ x < 0 \end{cases}$

Exercise 8

Let $f : [0,2] \to \mathbb{R}$ be a continuous function such that: $f(0) = f(2)$. Show that there exists at least one element α of $[0,1]$ for which we have: $f(\alpha) = f(\alpha+1)$.

Exercise 9

Let $f, g : [0,1] \to \mathbb{R}$ be two continuous functions satisfying :

$$\begin{cases} f(0) = g(1) = 0 \\ \text{and} \\ f(1) = g(0) = 1 \end{cases}$$

Show that for every $\alpha \geq 0$, we can associate an element $x_\alpha \in [0,1]$: $f(x_\alpha) = \alpha g(x_\alpha)$.

Exercise 10

Show that the function $f : [0, \frac{1}{4}] \to [0, \frac{1}{4}]$ defined by :

$$f(x) = \frac{1}{x^2+4}$$

is $\frac{1}{32}$-contraction.

Recall

A function f is said to be contracting on an interval I, if there exists a real $0 < k < 1$ such that: for all real x and y of the interval I we have :

$$|f(x) - f(y)| \leq k|x-y|$$