2023/2024	**Exercise Sheet 6**	Department: Common Core in Mathematics
Analysis 1	Elementary Functions	and Computer Science Batna 2-University.

Exercise 1

Show that for all real numbers x and y :

$$e^{\frac{x+y}{2}} \leq \frac{e^x + e^y}{2}$$

Exercise 2

Simplify the following expressions :

1. $\cos(\arcsin x)$
2. $\sin(\arccos x)$
3. $\tan(arcsinx)$
4. $cos(2\arctan x)$

Exercise 3

According to the values of x, find the limits of x^n when $n \to +\infty$

Exercise 4

1. Show that for all $x \in]0, \frac{\pi}{2}[$:
   a) $\sin(x) = \dfrac{\tan(x)}{\sqrt{1+\tan^2(x)}}$
   b) $\cos(x) = \dfrac{1}{\sqrt{1+\tan^2(x)}}$
2. $0 < \arctan(\frac{1}{2}) + \arctan(\frac{3}{2}) < \frac{\pi}{2}$
3. Solve $\arcsin(x) = \arctan(\frac{1}{2}) + \arctan(\frac{3}{2})$

Exercise 5

Show the following assertions:

1. $\forall x \in \mathbb{R} : argsh(x) = \ln(x + \sqrt{x^2+1})$
2. $\forall x \in]1, +\infty[: argch(x) = \ln(x + \sqrt{x^2-1})$
3. $\forall x \in]-1, 1[: argth(x) = \frac{1}{2}\ln(\frac{1+x}{1-x})$

Exercise 6

1. Compute: $\cosh(\frac{1}{2}\ln(3))$ and $\sinh(\frac{1}{2}\ln(3))$
2. Show that: $\cosh(a+b) = \cosh(a)\cosh(b) + \sinh(a)\sinh(b)$
3. Deduce the solutions of the equation: $2\cosh(x) + \sinh(x) = \sqrt{3}\cosh(5x)$