TD n° 2

exercice n° 3 (Solution)

$\sin^2 x = \frac{1-\cos 2x}{2}$

Calcul de l'énergie :

$E_{x_1} = \int_0^1 (\sin \pi t)^2 dt = \int_0^1 \sin^2 \pi t \, dt = \int_0^1 \frac{1-\cos 2\pi t}{2} dt = \boxed{\frac{1}{2}}$

$E_{x_2} = \int_0^{\frac{1}{2}} (2t)^2 dt + \int_{\frac{1}{2}}^1 (-2t+2)^2 dt = \boxed{\frac{1}{3}}$

$E_{x_3} = \int_0^1 1 \, dt = \boxed{1}$

pour déterminer la distance ~~pose~~ entre les différents signaux, il faut calculer l'énergie d'interaction :

$E_{x_1 x_2} = \int_0^{\frac{1}{2}} 2t \sin \pi t \, dt + \int_{\frac{1}{2}}^1 (-2t+2) \sin \pi t \, dt$

$= 2\underbrace{\int_0^{\frac{1}{2}} t \sin \pi t \, dt}_{I_1} - 2\underbrace{\int_{\frac{1}{2}}^1 (t \sin \pi t) dt}_{I_2} + 2\underbrace{\int_{\frac{1}{2}}^1 (\sin \pi t) dt}_{I_3}$

Calcul de I_1 : par partie

posons : $\sin \pi t \, dt = dv \Rightarrow v = -\frac{\cos \pi t}{\pi}$

$t = u \Rightarrow dt = du$.

$I_1 = \underbrace{-\frac{t \cos \pi t}{\pi} \Big|_0^{\frac{1}{2}}}_{0} + \int_0^{\frac{1}{2}} \frac{\cos \pi t}{\pi} dt = \frac{\sin \pi t}{\pi^2} \Big|_0^{\frac{1}{2}} = \frac{1}{\pi^2}$.

$I_2 = \frac{1}{\pi^2}$, $I_3 = 0$

donc $E_{x_1 x_2} = \boxed{\frac{4}{\pi^2}}$, $E_{x_2 x_3} = \int_0^{\frac{1}{2}} 2t \, dt + \int (-2t+2) dt = \boxed{\frac{1}{2}}$

$E_{x_1 x_3} = \int_0^1 \sin \pi t \, dt = \boxed{\frac{2}{\pi}}$ avec $\boxed{\pi^2 = 10}$!!!

alors $d^2(x_1, x_2) = \boxed{\frac{1}{30}}$, $d^2(x_1, x_3) = \boxed{\frac{2}{3}}$, $d^2(x_2, x_3) = \boxed{\frac{1}{3}}$

Solution TD n° 2

exercice n° 1 (Solution)

$x_1(t) = \frac{2A}{T}t - A$, $x_2(t) = -\frac{2At}{T} + A$, $x_3(t) = \begin{cases} -\frac{2A}{T}t + A & 0 \le t \le \frac{T}{2} \\ \frac{2At}{T} - A & \frac{T}{2} \le t < T \end{cases}$

$$d^2(x_1, x_2) = \frac{1}{T}\int_0^T \left[\left(\frac{2A}{T}t - A\right) - \left(-\frac{2A}{T}t + A\right)\right]^2 dt = \frac{1}{T}\int_0^T \left(\frac{4A}{T}t - 2A\right)^2 dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{16A^2}{T^2}t^2 - \frac{16A^2}{T}t + 4A^2\right) dt =$$

$$= \frac{1}{T}\left[\frac{16}{3}\frac{A^2T^3}{T^2} - \frac{16A^2T^2}{2T} + 4A^2T\right] = \boxed{\frac{4}{3}A^2}$$

$$d^2(x_1, x_3) = \frac{1}{T}\left\{\int_0^{T/2} \left[\left(\frac{2A}{T}t - A\right) - \left(-\frac{2A}{T}t + A\right)\right]^2 dt + \int_{T/2}^T \left[\left(\frac{2At}{T} - A\right) - \left(\frac{2At}{T} - A\right)\right]^2 dt\right.$$

$$d^2(x_1, x_3) = \boxed{\frac{2}{3}A^2}$$

$$d^2(x_2, x_3) = \frac{1}{T}\int_0^{T/2} \left[\left(-\frac{2At}{T} + A\right) - \left(-\frac{2At}{T} + A\right)\right]^2 dt + \int_{T/2}^T \left[\left(-\frac{2A}{T}t + A\right) - \left(\frac{2A}{T}t - A\right)\right]^2 dt$$

$$d^2(x_2, x_3) = \boxed{\frac{2}{3}A^2}$$

On remarque que : $d^2(x_1, x_3) = d^2(x_2, x_3)$.

exercice n°2 (Solution)

$$x(t) = e^{-\alpha t}\cdot u(t), \quad \alpha>0 \quad \text{et} \quad u(t)=\begin{cases}1 & t\geq 0\\ 0 & t<0\end{cases}$$

1er cas $\tau>0$.

$$x(t)-y(t) = \begin{cases} e^{-\alpha t} & 0<t<\tau \\ e^{-\alpha t} - e^{-\alpha(t-\tau)} & \text{pour } t\geq\tau\end{cases}$$

alors:

$$d^2(x,y) = \int_0^\tau (e^{-\alpha t})^2\, dt + \int_\tau^\infty \left[e^{-\alpha t} - e^{-\alpha(t-\tau)}\right]^2 dt$$

$$= \int_0^\tau e^{-2\alpha t}\, dt + \int_\tau^\infty \left[e^{-\alpha t}(1-e^{\alpha\tau})\right]^2 dt$$

$$= \frac{-1}{2\alpha} e^{-2\alpha t}\Big|_0^\tau + (1-e^{\alpha\tau})\left(\frac{-e^{-2\alpha t}}{2\alpha}\right)\Big|_\tau^\infty = \boxed{\frac{1}{\alpha}(1-e^{-\alpha\tau})}$$

2ème cas $\tau<0$

$$x(t)-y(t)=\begin{cases} -e^{-\alpha(t-\tau)} & \text{pour } \tau\leq t\leq 0\\ e^{-\alpha t}-e^{-\alpha(t-\tau)} & \text{pour } t>0\end{cases}$$

alors:

$$d^2(x,y) = \int_\tau^0 (-e^{\alpha\tau} e^{-\alpha t})^2\, dt + \int_0^\infty \left[e^{-\alpha t}(1-e^{\alpha\tau})\right]^2 dt$$

$$= e^{2\alpha\tau}\cdot\left(-\frac{1}{2\alpha} e^{-2\alpha t}\Big|_\tau^0 - \frac{1}{2\alpha}(1-e^{\alpha\tau})\,(e^{-2\alpha t}\Big|_0^\infty\right.$$

$$= \frac{1}{2\alpha}\left(1-e^{2\alpha\tau}+1-2e^{\alpha\tau}+e^{2\alpha\tau}\right) = \boxed{\frac{1}{\alpha}(1-e^{\alpha\tau})}$$

donc $d^2(x,y) = \boxed{\frac{1}{\alpha}\left(1-e^{-\alpha|\tau|}\right)}$

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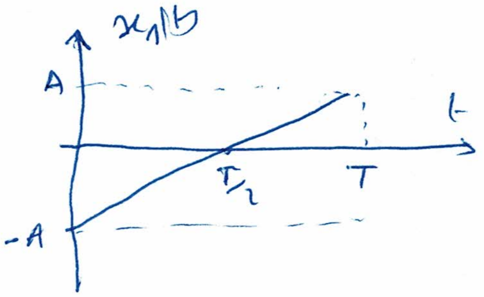
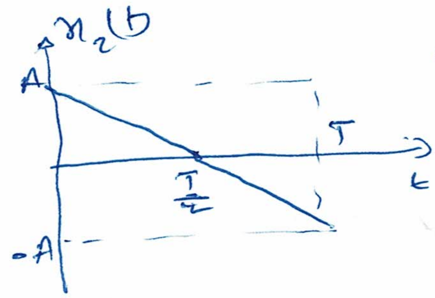
TD n° 2

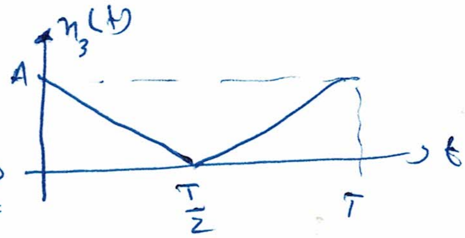
exercice n° 1 : Evaluez et comparez les distances suivantes :

$$d^2(x_i, x_j) \text{ avec } d^2(x_i, x_j) = \frac{1}{T}\int_0^T \left| x_i(t) - x_j(t) \right|^2 dt$$

pour les signaux suivants :







exercice n° 2 calculer la distance euclidienne entre les signaux suivants :

$$x(t) = e^{-at} \cdot u(t) \quad a > 0 \,, \quad u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$y(t) = x(t - \tau)$$

exercice n° 3 . calculer l'énergie des signaux suivants :

en déduire la distance euclidienne $d^2(x_i, x_j)$.

entre les signaux pris deux à deux. Sachant que :

$$d^2(x_i, x_j) = \|x_i\|^2 + \|x_j\|^2 - 2\,\mathrm{Re}\left[\langle x_i, x_j^* \rangle\right] = E_{x_i} + E_{x_j} - 2E_{x_i x_j}$$