Module TS
3ème année licence telecom.

TD n° 3

exercice n° 1 : Developpez en Série de Fourier trigonométrique les signaux suivants:

$$g(t) = \begin{cases} 1 & 0 \leq t \leq \frac{T_0}{2} \\ 0 & \frac{T_0}{2} \leq t \leq T_0 \end{cases}, \quad h(t) = \begin{cases} -t & -\frac{T_0}{2} \leq t \leq 0 \\ t & 0 < t \leq \frac{T_0}{2} \end{cases}$$

$\boxed{T_0 = 2}$ $\boxed{T_0 = 2\pi}$

exercice n° 2 : ① Developpez en Série de Fourier exponentielle les signaux (voir exercice n° 1)

② Tracez le spectre d'amplitude et de phase de chaque signal.

③ Determiner la puissance totale de chaque signal

exercice n° 3 soit $x(t) = e^{-at} u(t)$, $a > 0$.

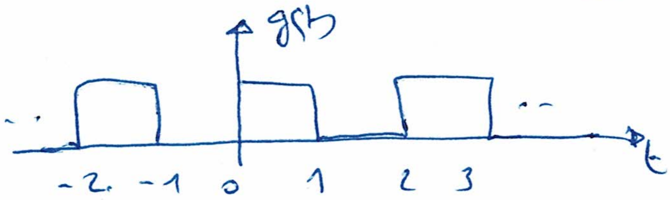
- calculer $X(f)$.
- tracez les spectres d'amplitude et de phase de $x(t)$.

exercice n° 4 : Calculer la transformée de Fourier du signal $x(t) = e^{-|t|}$

- tracez son spectre d'amplitude ~~et de phase~~.

TD n° 3 (Corrigé)

exercice n° 1 (Solution)



$$g(t) = \begin{cases} 1 & 0 \le t < \frac{T_0}{2} \\ 0 & 0 \le t \le T_0 \end{cases}$$

$T_0 = 2$

$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} = \pi$

$f_0 = \frac{1}{T_0} = \boxed{\frac{1}{2}} = 1$

$$a_0 = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t)\,dt = \frac{2}{2} \int_{-1}^{1} dt = \boxed{1}$$

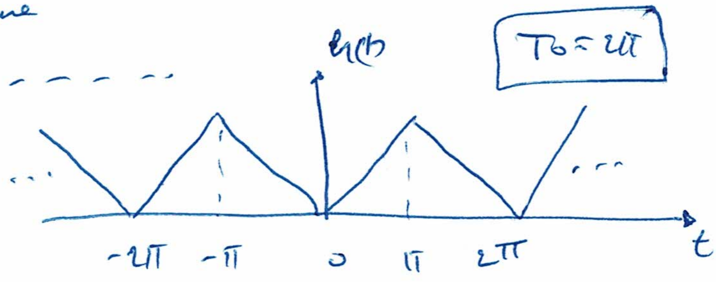
$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \cos 2\pi n f_0 t \, dt = \frac{2}{2} \int_{-1}^{0} 0 \cos 2\pi n f_0 t \, dt + \frac{2}{2} \int_{0}^{1} \cos 2\pi n f_0 t$$

$$a_n = -\frac{\sin n\pi t}{n\pi t} \Big|_0^1 = 0$$

$$b_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) \sin 2\pi n f_0 t \, dt = \int_0^1 \sin 2\pi n f_0 t \, dt = -\frac{\cos n\pi t}{n\pi} \Big|_0^1 =$$

$$b_n = \frac{-\cos n\pi + 1}{n\pi} = \frac{1 - (-1)^n}{n\pi} = \begin{cases} 0 & n : \text{paire} \\ \frac{2}{n\pi} & n : \text{impaire} \end{cases}$$

alors : $g(t) = \frac{1}{2} + \sum_{\substack{n=1 \\ n \text{ impaire}}}^{\infty} \frac{2}{n\pi} \sin n\pi t$



$\boxed{T_0 = 2\pi}$

$$h(t) = \begin{cases} -t & -\frac{T_0}{2} \le t < 0 \\ t & 0 < t \le \frac{T_0}{2} \end{cases}$$

$\boxed{f_0 = \frac{1}{T_0} = \frac{1}{2\pi}}$

le signal $h(t)$ est paire $h(-t) = h(t) \Rightarrow \underline{b_n = 0}$

Calcul de a_0 et a_n :

$$a_0 = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} h(t)\,dt = 2\left(\frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} h(t)\,dt \right) = \frac{4}{2\pi} \int_0^{\pi} t\,dt = \frac{t^2}{2} \cdot \frac{4}{2\pi} \Big|_0^{\pi} = \boxed{\pi}$$

$$a_n = \frac{2}{T_0} \int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} h(t) \cos 2\pi n f_0 t \, dt = 2\left(\frac{2}{2\pi} \int_0^{\pi} t \cos nt \, dt \right.$$

.../...

Suite Corrigé TD n°3

$$a_n = \frac{2}{\pi}\int_0^{\pi} t \cos nt \, dt .$$ on intègre par partie

on pose $t = u \Rightarrow du = dt$ et $dv = \cos nt \, dt \Rightarrow v = \frac{\sin nt}{n}$

$$a_n = \frac{2}{\pi}\left[\frac{t \sin nt}{n}\Big|_0^{\pi} - \int_0^{\pi} \frac{\sin nt}{n} dt = \frac{2}{\pi}\left[0 + \frac{\cos nt}{n^2}\Big|_0^{\pi} \right.\right.$$

$$a_n = \frac{2}{\pi n^2}\left[\cos n\pi - 1\right] = \frac{2}{n^2 \pi}\left[(-1)^n - 1\right] = \begin{cases} 0 & n : \text{paire} \\ \frac{-4}{n^2 \pi} & n : \text{impaire} \end{cases}$$

alors $$h(t) = \frac{\pi}{2} + \sum_{\substack{n=1 \\ n : \text{impaire}}}^{\infty} \left(\frac{-4}{n^2 \pi}\right) \cos nt$$

exercice n°2 (Solution)

(1) Developpement en Série exponentielles des signaux.

pour $g(t)$: on a : $$g(t) = \sum_{n=-\infty}^{\infty} G_n e^{j2\pi n f_0 t} . \quad T_0 = 2$$

$$G_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} g(t) e^{-j2\pi n f_0 t} dt = \frac{1}{2}\int_0^{1} 1 . e^{-j2\pi n f_0 t} dt$$

$$G_n = \frac{1}{2}\left(\frac{1}{-j2\pi n f_0} e^{-j2\pi n f_0 t}\right|_0^1 = -\frac{1}{4j\pi n f_0}\left(e^{-j2\pi n f_0} - 1\right) =$$

$$G_n = \frac{-1}{2j\pi n}\left[\cos 2\pi n f_0 - j \sin 2\pi n f_0 - 1\right] = \frac{-1}{2j\pi n}\left[(-1)^n - 1\right]$$

$$G_n = \frac{1}{2j\pi n}\left[1 - (-1)^n\right] = \begin{cases} \frac{1}{j\pi n} & n : \text{impaire} \\ 0 & n : \text{paire} \end{cases}$$

alors : $$g(t) = \sum_{\substack{n=-\infty \\ n : \text{impaire}}}^{\infty} \left(\frac{1}{j\pi n}\right) e^{j2\pi n f_0 t}$$

.../...

$$h(t) = \sum_{n=-\infty}^{\infty} H_n e^{jn\omega_0 t}.$$

$$H_n = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} h(t)\, e^{-jn\omega_0 t}\, dt$$, le signal est paire $\boxed{T_0 = 2\pi}$

alors :
$$H_n = \frac{1}{2\pi}\Big[\underbrace{\int_{-\frac{T_0}{2}}^{0} -t\, e^{-jn\omega_0 t}\, dt}_{I_1} + \int_0^{\frac{T_0}{2}} t\, e^{-jn\omega_0 t}\, dt$$

dans l'intégrale I_1, on pose : $t = -t \Rightarrow dt = -dt$ et

$$\int_{-\frac{T_0}{2}}^{0} dt \Longrightarrow \int_0^{\frac{T_0}{2}} dt$$

donc :
$$H_n = \frac{1}{2\pi}\left[-\int_{-\frac{T_0}{2}}^{0} t\, e^{jn\omega_0 t}\, dt + \int_0^{\frac{T_0}{2}} t\, e^{-jn\omega_0 t}\, dt\right]$$

$$H_n = \frac{1}{2\pi}\left[\int_0^{\pi} t\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) dt\right] = \frac{1}{\pi}\int_0^{\pi} t \cos n\omega_0 t\, dt$$

$$H_n = \frac{1}{\pi}\int_0^{\pi} t \cos nt\, dt = \frac{1}{\pi}\left[\frac{t \sin nt}{n}\Big|_0^{\pi} + \frac{1}{n^2}\cos nt\Big|_0^{\pi}\right] = \frac{1}{n^2\pi}\left[(-1)^n - 1\right]$$

$$H_n = \begin{cases} \dfrac{-2}{n^2\pi} & n : \text{impaire} \\ 0 & n : \text{paire} \end{cases}$$

alors :
$$h(t) = \sum_{\substack{n=-\infty \\ n : \text{impaire}}}^{\infty} \left(\frac{-2}{n^2\pi}\right) e^{jn\omega_0 t}.$$

② tracé des spectres :

pour $g(t)$.

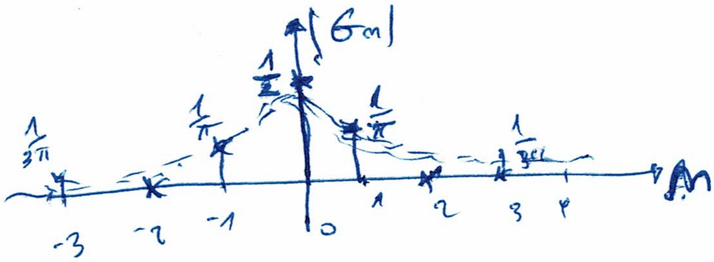
…/…

Spectre de g(t)

- Spectre d'amplitude : $|G_n| = \frac{1}{j\pi n}$ n : impaire

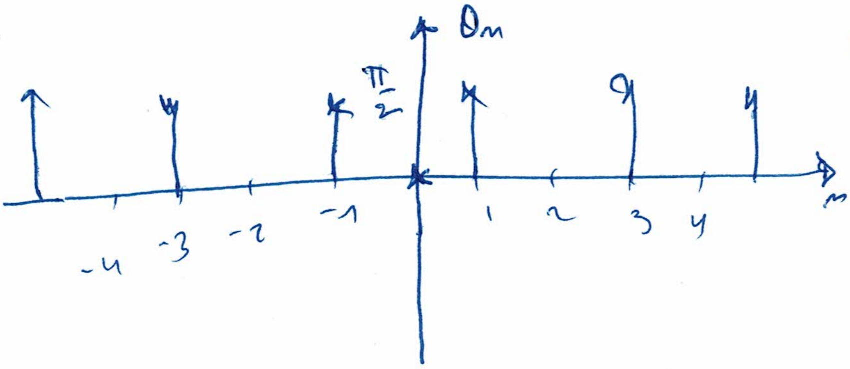
le spectre est bilatéral

$$G_0 = \frac{1}{2}\int_0^1 g(t)\,dt = \frac{1}{2}\Big[t\Big]_0^1 = \frac{1}{2}$$



Spectre de phase : $tg\,\theta = \frac{y}{x} = \frac{-j/\pi n}{0} = -\infty \Rightarrow \theta_n = -\frac{\pi}{2}$

pour $n = 0 \Rightarrow \theta_0 = 0$



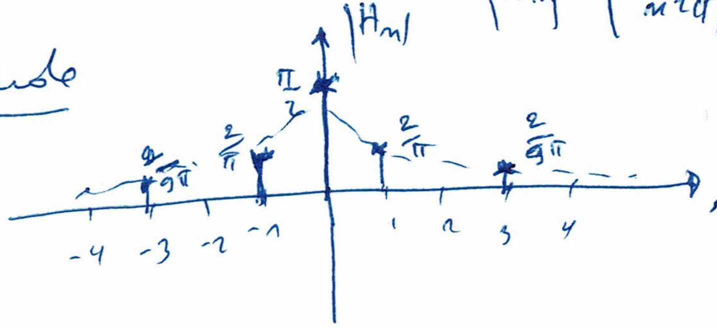
~~Hn~~

Spectre de h(t)

$|H_n| = \left|-\frac{2}{n^2\pi}\right|$

- Spectre d'amplitude

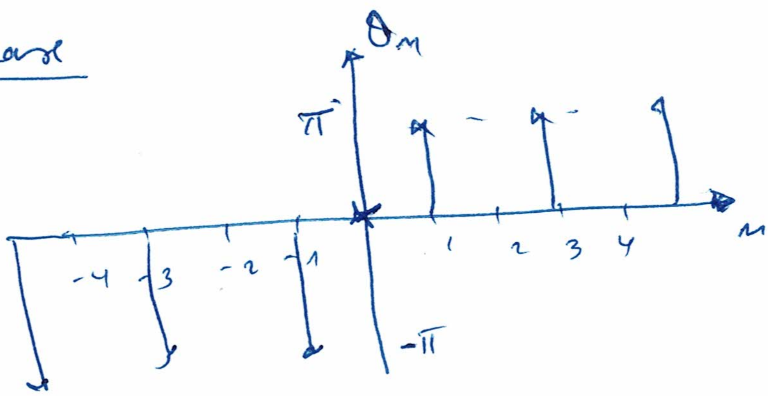
$$H_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t\,dt = \frac{\pi}{2}$$



Spectre de phase

$$tg\,\theta = \frac{y}{x} = \frac{0}{-2/n^2\pi} = 0$$

$$\theta = \begin{cases} \pi & n > 0 \\ -\pi & n < 0 \end{cases}$$



(3) La puissance totale

$$P_{g(t)\,totale} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |g(t)|^2\,dt = \frac{1}{2}\int_0^1 dt = \boxed{\frac{1}{2}}$$

$$P_{h(t)\,totale} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |h(t)|^2\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2\,dt = \boxed{\frac{\pi^2}{3}}$$

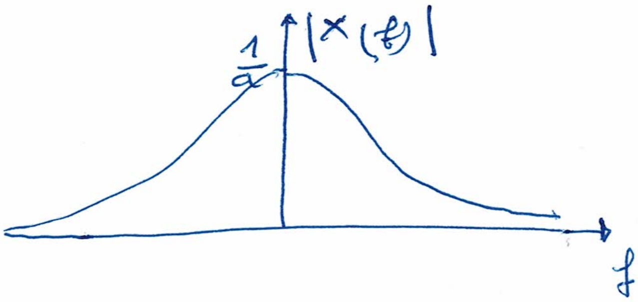
exercice 3 (Solution)

$$x(t) = e^{-at}\,u(t). \quad a > 0 \quad \text{et} \quad u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- $$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{0}^{\infty} e^{-at} e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} e^{-(a+j2\pi f)t}\, dt$$

$$X(f) = -\frac{1}{(a+j2\pi f)} e^{-(a+j2\pi f)t} \Big|_{0}^{\infty} = \boxed{\frac{1}{a+j2\pi f}} \Rightarrow |X(f)| = \frac{1}{\sqrt{a^2+(2\pi f)^2}}$$

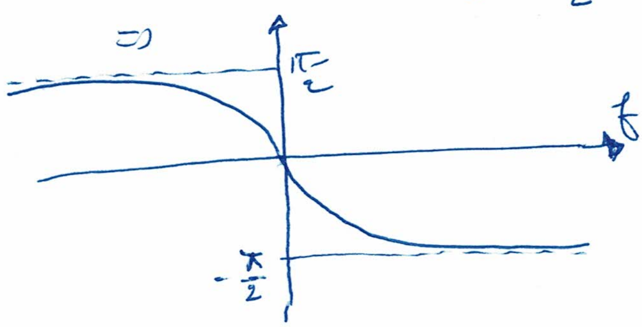
- Spectre d'amplitude



- Spectre de phase :

$$\theta(f) = -\operatorname{arctg}\left(\frac{2\pi f}{a}\right) = \pm\frac{\pi}{2}$$

$\Rightarrow$



exercice n° 4 (Solution) : $x(t) = e^{-|t|}$

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{0} e^{t} e^{-j2\pi f t}\, dt + \int_{0}^{\infty} e^{-t} e^{-j2\pi f t}\, dt$$

$$X(f) = \frac{1}{1-j2\pi f} e^{(1-j2\pi f)t} \Big|_{-\infty}^{0} + \left(-\frac{1}{1+j2\pi f} e^{-(1+j2\pi f)t}\right)\Big|_{0}^{\infty}$$

$$X(f) = \frac{1}{1-j2\pi f} + \frac{1}{1+j2\pi f} = \boxed{\frac{2}{1+4\pi^2 f^2}}$$