People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
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Master's Degree 1, CS Option

Practical work of Signal processing

Practical work N° 03 : Fourier series

1. Introduction

In analysis, Fourier series are a fundamental tool in the study of periodic functions. It is from this concept that the branch of mathematics known as harmonic analysis has developed. The study of a periodic function through Fourier series involves two aspects:
• analysis, which consists of determining the sequence of its Fourier coefficients,
• synthesis, which allows us to recover, in a certain sense, the function using the sequence of its coefficients.

Beyond the problem of decomposition, the theory of Fourier series establishes a correspondence between the periodic function and its Fourier coefficients. As a result, Fourier analysis can be seen as a new way of describing periodic functions. Operations such as differentiation can be expressed simply in terms of Fourier coefficients. The construction of a periodic function that solves a functional equation can be reduced to constructing the corresponding Fourier coefficients. The most classic way to represent a time signal is to plot the graph of $x(t)$. Another method is to represent the coefficients of the decomposition as a function of frequencies $f_n=nf$. This yields a **spectral representation**. The spectrum of a periodic signal is discrete since the coefficients are defined only for particular frequencies $(f_n=nf)$. For more details, see the course.

2. Objective of the Lab

The objective of this lab is to establish the correspondence between the amplitude spectrum of a periodic signal and its Fourier series (trigonometric and complex) expansion.

3. Manipulations

3.1. Manipulation 1

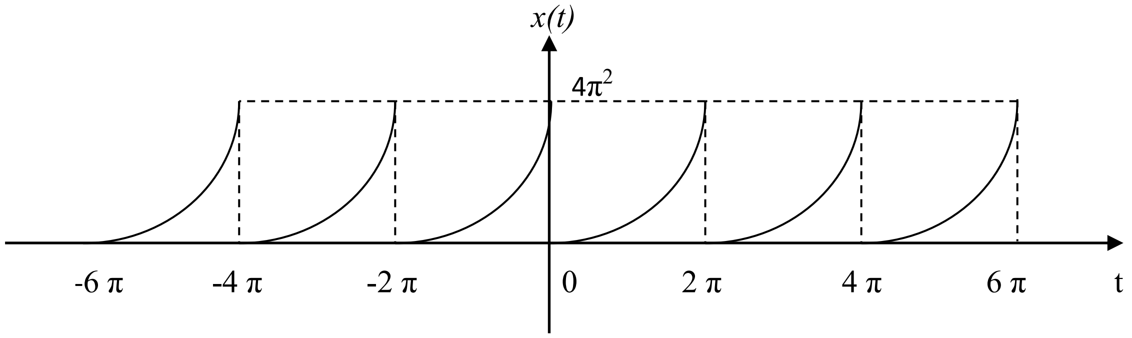
A pulse train with a duty cycle of $\frac{1}{2}$ can be decomposed as a sum of sinusoids with frequencies $nf_0 = n/T_0$, where n is odd, as follows:

$$x(t) = 4/\pi*(\sin(2\pi f_0 t) + 1/3\ \sin(2\pi 3 f_0 t) + 1/5\ \sin(2\pi 5 f_0 t) + \ldots)$$

1. Suppose T_0= 1 second. Create a vector of 1000 points representing 10 seconds. Using a for loop, generate a signal by taking into account10000 harmonics.
2. Plot the curve of the signal $x(t)$ in the time domain and verify theoretically that the Fourier series decomposition corresponds to what is expected.
3. Find the mathematical expression for this signal between $\frac{-T_0}{2}$ and $\frac{+T_0}{2}$.
4. Plot the frequency spectrum for 5 harmonics for the trigonometric form.

3.2. Manipulation 2

1. Develop theoretically $x(t) = t^2$ for $0 < t < 2\pi$ into a Fourier series (trigonometric form) if the period is T_0= 2π.



2. Draw the curve of the signal in the time domain.
3. Then, we will look to plot the shape of $x(t)$ when the sum is taken over the first 1, 2, 3... terms. Observe that after a few terms, the shape is quite close to the desired signal.
4. Write a program that allows obtaining the frequency spectrum of this signal for 5 harmonics in complex form with:

$$|C_n| = |C_{-n}| = \frac{\sqrt{a_n^2+b_n^2}}{2} \qquad \text{for n=1..5}$$

With : $C_0=a_0$

$$\text{Arg}\{C_n\} == -arctg\left(\frac{b_n}{a_n}\right) \quad \text{and} \quad \text{Arg}\{C_{-n}\} == arctg\left(\frac{b_n}{a_n}\right)$$