University of Batna 2
Faculty of Mathematics and Computer Science
Department of Mathematics
1st year of Mathematics Degree

Tutorial Series 01: Mathematical Algebra 1 – Year 2024/2025

Exercise 01

Using truth tables, show that:

$$\overline{(P \wedge Q)} \Leftrightarrow \overline{P} \vee \overline{Q}$$

$$\overline{(P \vee Q)} \Leftrightarrow \overline{P} \wedge \overline{Q}$$

$$((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$$

Exercise 02

Show that: Write the following propositions in another form:

$$(\overline{P} \wedge Q) \Rightarrow R$$

$$\overline{(P \vee \overline{Q})} \wedge (S \Rightarrow T)$$

$$\overline{(P \wedge Q)} \wedge (P \vee Q)$$

Exercise 03

Let a, b, c be real numbers. Write the negation of the following propositions:

1. $a < 1$ and $b > 2$
2. $a + b = 4$ or $a - b > 6$
3. $a \geq -2$ or $b \leq 5$

Exercise 04

Find which of the following propositions are true:

1. 24 is a multiple of 8 and 3 divides 19.
2. 24 is a multiple of 8 or 2 divides 19.
3. $\exists x \in \mathbb{R},\ (x+1=0 \text{ and } x+2=0)$
4. $(\exists x \in \mathbb{R},\ x+1=0)$ and $(\exists x \in \mathbb{R},\ x+2=0)$
5. $\forall x \in \mathbb{R},\ (x+1 \neq 0 \text{ or } x+2 \neq 0)$
6. $\exists x \in \mathbb{R}^*, \forall y \in \mathbb{R}^*, \forall z \in \mathbb{R}^*,\ z - xy = 0$
7. $\forall y \in \mathbb{R}^*, \forall z \in \mathbb{R}^*,\ y + z = 0$
8. $\forall y \in \mathbb{R}^*, \forall z \in \mathbb{R}^*, \exists x \in \mathbb{R}^*,\ z - xy = 0$
9. $\exists a \in \mathbb{R}, \forall \varepsilon > 0, |a| < \varepsilon$
10. $\forall \varepsilon > 0, \exists a \in \mathbb{R},\ |a-1| < \varepsilon$

Exercise 05

Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Deny (negate) the following propositions:

1. $\forall x \in \mathbb{R},\ f(x) \neq 0$
2. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R},\ f(x) = y$
3. $\forall x \in \mathbb{R},\ f(x) > 0 \implies x \leq 0$
4. $\forall M > 0, \exists A > 0, \forall x \geq A,\ f(x) > M$

Exercise 06

1. Show that if $x, y \in \mathbb{Q}$ then $x + y \in \mathbb{Q}$.
2. Show that for all $x \in \mathbb{R}$,
$$|x-1| \leq x^2 - x + 1.$$
3. Let $a, b \geq 0$. Show that if
$$\frac{a}{1+b} = \frac{b}{1+a}$$
then $a = b$.
4. Let $a, b \in \mathbb{R}_+$. Show that if $a \leq b$ then
$$a \leq \frac{a+b}{2} \leq b \quad \text{and} \quad a \leq \sqrt{ab} \leq b.$$
5. Show that for all $n \in \mathbb{N}$, $n(n+1)$ is divisible by 2.
(Hint: Consider two cases: n even and n odd.)