

University of Batna 2

Faculty of Mathematics and Computer Science

Department of Mathematics

1st year of mathematics degree



جامعة باتنة 2
كلية الرياضيات والاعلام الآلي
قسم الرياضيات
السنة الأولى ليسانس رياضيات

Tutorial series 2

Mathematical Analysis 1

Year 2024/2025

Exercise 01

Put the following complex numbers into algebraic form

- $z_1 = (1 + 2i) - 2(3 - 2i)$
- $z_2 = (2 + 3i)(4 - 3i)$
- $z_3 = \frac{3+6i}{3-4i}$
- $z_4 = \frac{2+3i}{1-i} + \frac{2-3i}{1+i}$
- $z_5 = (1 + i)^7 + (1 - i)^7$

Exercise 02

1. Solve the following equations, with unknown $z \in \mathbb{C}$:
 - $z + 2i = iz - 3$
 - $2\bar{z} + 5i = z + 4$
2. Solve the following systems of unknowns the complex numbers z_1 and z_2 :
$$\begin{cases} 2iz_1 - 3z_2 = -8 + 5i \\ 5z_1 + iz_2 = 6 + 7i \end{cases}$$

Exercise 03

- Find the square roots of the following numbers:
 $8 + 6i, 3 - 4i, i, -2i.$
- Solve the following equations;
 - $z^2 - 10z + 29 = 0$
 - $z^2 - (3 - 4i)z - 1 - 5i = 0$
 - $z^2 - 2(\cos \theta)z + 1 = 0, \theta \in \mathbb{R}$
 - $z^2 - (i + 2a)z + ia + a^2 = 0, a \in \mathbb{R}$
 - $z^2 - (1 + a)(1 + i)z + (1 + a^2)i = 0$

Exercise 04

- 1) Write the following numbers in trigonometric and exponential form.

$$z_1 = 1 + i\sqrt{3}, z_2 = -1 + i, z_3 = z_1 z_2, z_4 = \frac{z_1}{z_2},$$

$$z_5 = \frac{1+i\tan(\theta)}{1-i\tan(\theta)}, z_6 = 1 + \cos(\theta) + i\sin(\theta), \theta \in]0, \pi[.$$

- 2) Linearize $(\cos \theta)^3$ and $(\sin \theta)^3$

Exercise 05

Let $u = 1 + i, v = \sqrt{3} + i$

- 1) Write u and v in trigonometric and exponential form
- 2) Write $z_1 = \frac{u}{v}$ and $z_2 = uv$ in trigonometric form
- 3) Write $\frac{u}{v}$ and uv in algebraic form
- 4) Deduce the values of $\cos \frac{\pi}{12}, \sin \frac{\pi}{12}, \cos \frac{5\pi}{12}$ and $\sin \frac{5\pi}{12}$
- 5) Find the values of the naturel n so that z_1^n is real

Exercise 06

Let $z = \sqrt{2 + \sqrt{3}} + i\sqrt{2 - \sqrt{3}}$

- 1) Calculate z^2
- 2) Write z^2 in trigonometric form
- 3) Deduce the trigonometric form of z
- 4) Deduce $\cos \frac{\pi}{12}, \sin \frac{\pi}{12}$
- 5) Calculate z^{2028}

Exercise 07

- 1) Solve the following equation $Z^4 = -4$ and write the solutions in trigonometric and algebraic form
- 2) Give the solutions of $(z + 1)^4 + 4(z - 1)^4 = 0$ In algebraic form.

Exercise 08

Let $j = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

- 1) Solve in \mathbb{C} the equation $Z^3 = 1$
- 2) Write the solutions in trigonometric and algebraic form
- 3) Show that $\bar{j} = j^2$ and $j^{-1} = j^2$
- 4) Show that $1 + j + j^2 = 0$
- 5) Calculate $\frac{1}{1+j}$
- 6) Calculate j^n for all $n \in \mathbb{N}$

Exercise 09

Let $\beta \in \mathbb{C}$ such that $\beta^7 = 1, \beta \neq 1$ show that:

$$\frac{\beta}{1 + \beta^2} + \frac{\beta^2}{1 + \beta^4} + \frac{\beta^3}{1 + \beta^6} = -2$$