Algebra tutorial series 1 (continued)

Exercise 1 *Consider the two vector spaces of $\mathbb{R}^3$*

$$E = \left\{ (x, y, z) \in \mathbb{R}^3 / \quad x + z = 0 \quad and \quad x + y - 2z = 0. \right\}$$

$$F = span\left\{ u, v \right\}, \quad u = (1, -2, 1), v = (2, 0, 1).$$

1. *Determine a basis and the dimension of each of the subspaces : $E, F, E \cap F$ and $E + F$.*
2. *Do we have $\mathbb{R}^3 = E \oplus F$?*
3. *Verify that $u = (2, -6, -2) \in E$ and determine the coordinate of the vector u in the basis of E.*

Exercise 2 *Consider the following polynomials in $\mathbb{R}_2[X]$:*

$$P_1 = X + 1, \quad P_2 = X^2 - 1, \quad P_3 = X^2 - 2X + 1, \quad P_4 = -X^2 + X + 6.$$

1. *Show that $B = \{P_1, P_2, P_3\}$ is a basis of $\mathbb{R}_2[X]$.*
2. *Determine the coordinates of P_4 in the base B.*

Exercise 3 *Let's consider $E = \mathbb{R}^3$.*
Prove that $F = \{(x, y, z) \in \mathbb{R}^3 / \quad x - y + z = 0\}$ and $G = \{(x, x, x) \in \mathbb{R}^3\}$ are complements in E.

Exercise 4 *Consider the vectors of $\mathbb{R}^4$: $u = (1, -2, 4, 1)$ and $v = (1, 0, 0, 2)$.*

1. *Determine $Vect(u, v)$.*
2. *Complete the set (u, v) adding two vectors of the canonical basis of $\mathbb{R}^4$ in order to have a basis of $\mathbb{R}^4$.*
3. *In $\mathbb{R}^3$, let $u = (2, 3, 5), v = (4, 6, 10)$ and $w = (-2, -3, -5)$. Find $rg(u, v, w)$.*

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