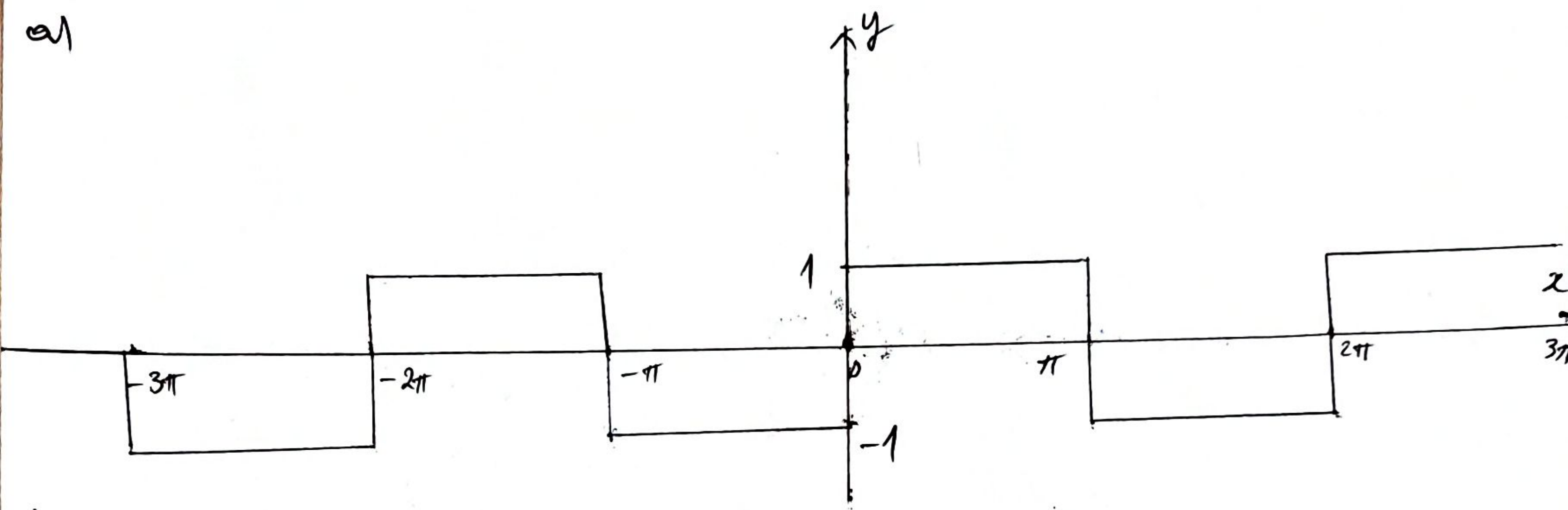
Master I : MMTH _ Module: Outils Numériques _ 15/12/22

## Solution de l'Examen Final

Présentation 2 pts

### Exercice N°1

a)




b)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$ 0,25 pt

où

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$ 0,25 pt

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \qquad n = 1, 2, 3, \dots,$$ 0,25 pt

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$ 0,25 pt

$f(x)$ est impaire donc

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = 0$$ 0,5 pt

$f(x)\cos nx$ est impaire donc

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = 0$$ 0,5 pt

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -\sin nx\,dx + \int_{0}^{\pi} \sin nx\,dx\right\}$$ 0,5 pt

$$= \frac{1}{\pi}\left\{\left[\frac{\cos nx}{n}\right]_{-\pi}^{0} + \left[-\frac{\cos nx}{n}\right]_{0}^{\pi}\right\}$$ 0,5 pt

$$= \frac{1}{\pi}\left\{\left(\frac{1}{n} - \frac{\cos n\pi}{n}\right) + \left(-\frac{\cos n\pi}{n} + \frac{1}{n}\right)\right\}$$ 0,25 pt

$$= \frac{1}{\pi}\left\{\frac{2}{n} - 2\frac{\cos n\pi}{n}\right\} = \begin{cases} 0 & \text{si } n \text{ est pair} \\ \dfrac{4}{\pi n} & \text{si } n \text{ est impair} \end{cases}$$ 0,5 pt

Donc

$$f(x) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin(2n-1)x \qquad 0{,}25\,pt$$

1 pt

Exercice N°2

a) $\mathcal{L}(e^{5t}) = F(s-5)$, où $F(s) = \underbrace{\mathcal{L}(1)}_{1pt} = \frac{1}{s}$ , $s>0$

Donc $\mathcal{L}(e^{5t}) = \frac{1}{s-5}$ 1pt

b) $\mathcal{L}(-\frac{1}{2}+2t) = \underbrace{-\frac{1}{2}\mathcal{L}(1) + 2\mathcal{L}(t)}_{1pt} = \underbrace{-\frac{1}{2s}}_{1pt} + \underbrace{\frac{2}{s^2}}_{1pt}$ , $s>0$

Exercice N°3

a) $f(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{A(s+1)+Bs}{s(s+1)} = \frac{(A+B)s+A}{s(s+1)}$ 1pt

$\Leftrightarrow \begin{cases} A+B=0 \\ A=1 \end{cases} \Leftrightarrow \begin{cases} B=-A=-1 \\ \text{et } A=1 \end{cases}$ 1pt $\Leftrightarrow \begin{cases} A=1 \\ B=-1 \end{cases}$ 1pt

Donc $\frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$

b) $\mathcal{L}^{-1}(f(s)) = \mathcal{L}^{-1}\left(\frac{1}{s(s+1)}\right) = \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \mathcal{L}^{-1}\left(\frac{1}{s+1}\right)$ 1pt

$= \underbrace{1}_{1pt} - \underbrace{e^{-t}}_{1pt}$