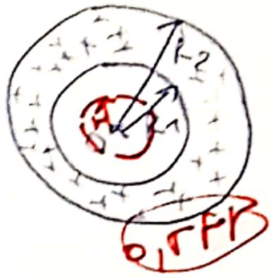
0,5pt

exo 3 6/6

1er cas $r < R_1$ 0,25pt

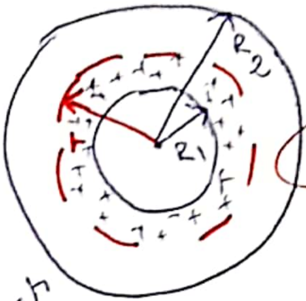
$$\phi = \oiint \vec{E}\cdot\vec{dS} = \frac{\sum q_{int}}{\varepsilon_0}$$ 0,25pt

$$\boxed{E_1 = 0}$$ 0,5pt



0,5pt

2eme cas $R_1 < r < R_2$ 0,25pt



0,5pt

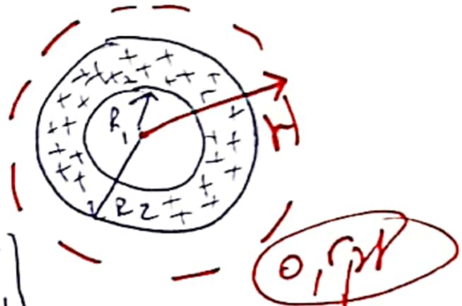
$$\phi = \oiint \vec{E}\cdot\vec{dS} = \frac{\sum q_{int}}{\varepsilon_0}$$

$$E_2 \cdot 4\pi r^2 = \frac{\iiint \rho\, dV}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\int_{R_1}^{r} 4\pi r^2 dr = \frac{4\pi\rho}{3\varepsilon_0}\left[r^3 - R_1^3\right]$$ 0,5pt 0,5pt

$$\Rightarrow \boxed{E_2 = \frac{\rho}{3\varepsilon_0}\left[r - \frac{R_1^3}{r^2}\right]} \quad \boxed{\vec{E}_2(0) = \frac{\rho}{3\varepsilon_0}\left[r - \frac{R_1^3}{r^2}\right]\vec{e}_r}$$

0,5pt

3eme cas $r > R_2$ 0,25pt



0,5pt

$$\phi = \iint \vec{E}\cdot\vec{dS} = \frac{\sum q_{int}}{\varepsilon_0}$$

$$E_3 \cdot 4\pi r^2 = \frac{\rho}{\varepsilon_0}\int_{R_1}^{R_2} 4\pi r^2 dr = \frac{\rho \cdot 4\pi}{3\varepsilon_0}\left[R_2^3 - R_1^3\right]$$ 0,5pt 0,5pt

$$E_3 = \frac{\rho}{3\varepsilon_0}\left[\frac{R_2^3 - R_1^3}{r^2}\right]$$

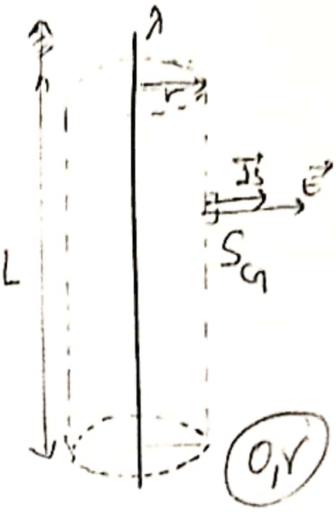
0,5pt $$\vec{E}_3(0) = \frac{\rho}{3\varepsilon_0}\left[\frac{R_2^3 - R_1^3}{r^2}\right]\vec{e}_r$$

Exercice 3 (06,1) 1^{er} cas : $r < R_1$

En utilisant le théorème de Gauss :

$$\Phi = \oiint \vec{E}\cdot\vec{dS} = \frac{\sum Q_{int}}{\varepsilon_0} \quad (0,1)$$

$$\Rightarrow \quad E \cdot S_G = \frac{\sum Q_{int}}{\varepsilon_0}$$

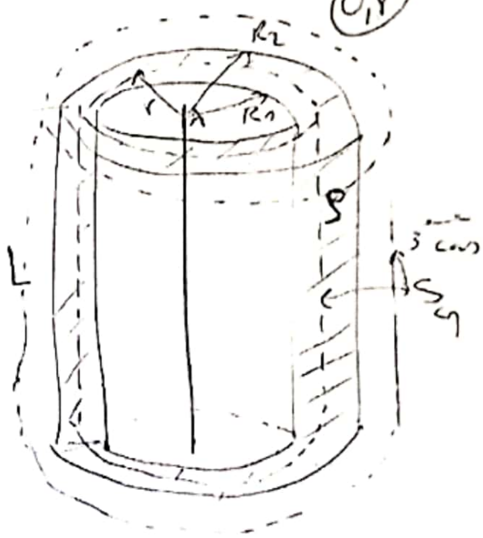


où : S_G est la surface latérale du cylindre de rayon r et de hauteur L

~~et $\sum Q_{in}$~~ $S_G = 2\pi r L$. (0,1)

$$\text{et } \sum Q_{int} = \int_{(L)} \lambda \cdot dl = \lambda \cdot L \cdot \quad (0,1)$$

$$\Rightarrow E_1 \cdot 2\pi r L = \frac{\lambda L}{\varepsilon_0} \Rightarrow \boxed{E_1 = \frac{\lambda}{2\pi\varepsilon_0 r}} \quad (1)$$



2ème cas : pour $R_1 \leq r < R_2$

$$E_2 \cdot S_G = \frac{\sum Q_{int}}{\varepsilon_0}$$

$$\sum Q_{int} = \iiint \rho \, dV + \int_{(L)} \lambda \cdot dl \quad (0,2)$$

$$V = \pi r^2 L \rightarrow dV = 2\pi r L \, dr$$

$$\sum Q_{int} = \rho \int_{R_1}^{r} 2\pi r L \, dr + \lambda L \quad (0,2)$$

$$= \rho\pi(r^2 - R_1^2) L + \lambda L$$

$$\Rightarrow E_2 = \frac{\rho\pi(r^2 - R_1^2) L + \lambda L}{2\pi r \varepsilon_0 L} \Rightarrow \boxed{E_2 = \frac{\rho(r^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}} \quad (1)$$

3ème cas : $r > R_2$:

$$\sum Q_{int} = \rho \int_{R_1}^{R_2} 2\pi r L \, dr + \lambda L = \rho\pi (R_2^2 - R_1^2) L + \lambda L \quad (0,1)$$

$$\Rightarrow E_3 = \frac{\rho\pi(R_2^2 - R_1^2) L + \lambda L}{2\pi r \varepsilon_0 L} \Rightarrow \boxed{E_3 = \frac{\rho(R_2^2 - R_1^2)}{2\varepsilon_0 r} + \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{r}} \quad (1)$$

Exercice 3: (06 pts)

1. *Expression de $\vec{E}_M(r)$:*

Par raison de symétrie du problème, il ne subsiste que la composante suivant $\vec{e}_r$:

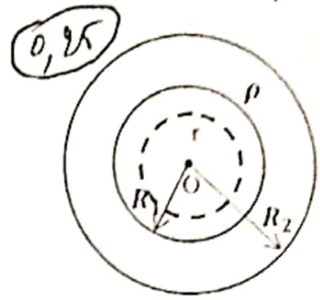
$\vec{E}_M(r) = E_M(r)\,.\vec{e}_r$ 0,50

La Surface de Gauss est une sphère S(O, r).

Le Flux: $\Phi = \oiint_{S_G} \vec{E}_{1M}(r)\,.\overrightarrow{dS} = \oiint_{S_G} E_{1M}(r).\vec{e}_r.dS.\vec{n} = E_{1M}(r)\,.4\pi\,.r^2$
0,50 0,50

1er Cas : $r < R_1$

1er Cas : $r < R_1$ 0,25

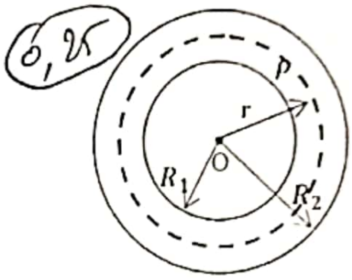


$$\Phi = \oiint_{S_G} \vec{E}_{1M}(r)\,.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0} \quad 1.00 \qquad Q_{int} = 0 \quad 0,25$$

$$\Rightarrow \; E_{1M}(r) = 0 \Rightarrow \vec{E}_{1M}(r) = \vec{0} \quad 0,25$$

2er Cas : $R_1 < r < R_2$

2er Cas : $R_1 < r < R_2$ 0,25



$$\Phi = \oiint_{S_G} \vec{E}_{2M}(r)\,.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$$

0,25

$$Q_{int} = \iiint \rho\, dv = \int_{R_1}^{r} \rho\,.4\pi r^2 dr = \rho.\frac{4}{3}\pi\,[r^3]_{R_1}^{r}$$

$$= \rho.\frac{4}{3}\pi.(r^3 - R_1^3) \quad 0,25$$

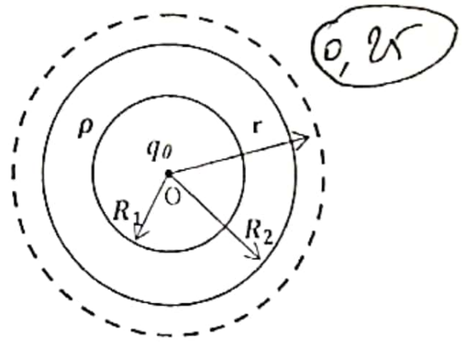
$$\Rightarrow \; E_{2M}(r).4\pi\,.r^2 = \rho.\frac{4}{3\varepsilon_0}\pi.(r^3 - R_1^3)$$

$$E_{2M}(r) = = .\frac{\rho}{3\varepsilon_0}.\left(r - \frac{R_1^3}{r^2}\right) \qquad \vec{E}_{2M}(r) = = .\frac{\rho}{3\varepsilon_0}.\left(r - \frac{R_1^3}{r^2}\right)\vec{e}_r$$

0,25 0,25

3er Cas : $r > R_2$ et 2°

0,25



3er Cas : $r \geq R_2$

$$\Phi = \oiint_{S_G} \vec{E}_{2M}(r)\,.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$$

0,25 0,25

$$Q_{int} = \iiint \rho\, dv = \int_{R_1}^{R_2} \rho\,.4\pi r^2 dr = \rho.\frac{4}{3}\pi\,[r^3]_{R_1}^{R_2} = \rho.\frac{4}{3}\pi.(R_2^3 - R_1^3)$$

0,25 0,25

$$E_{3M}(r).4\pi\,.r^2 = \rho.\frac{4}{3\varepsilon_0}\pi.(R_2^3 - R_1^3) \qquad E_{3M}(r) = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0}\frac{1}{r^2} \qquad \vec{E}_{3M}(r) = \frac{\rho(R_2^3 - R_1^3)}{3\varepsilon_0}\frac{1}{r^2}\vec{e}_r$$

2 *Valeur de $\vec{E}_M(r)$ en présence q_0:*

On est dans le 3ème cas du 1°, avec une charge supplémentaire $q_0 = -\rho\frac{4}{3}\pi(R_2^3 - R_1^3)$ placée en O

$$Q_{int} = Q_{sphère} + q_0 = \rho.\frac{4}{3}\pi.(R_2^3 - R_1^3) + \left(-\rho\frac{4}{3}\pi(R_2^3 - R_1^3)\right) = 0. \quad 0,50$$

$$\Rightarrow \vec{E}_M(r) = \vec{0} \quad 0,25$$