

Ex 4

$$\begin{aligned} \cdot I_1 &= \int_0^1 \int_0^1 \int_0^1 xyz \, dx \, dy \, dz = \left( \int_0^1 x \, dx \right) \left( \int_0^1 y \, dy \right) \left( \int_0^1 z \, dz \right) = \\ & \left( \frac{x^2}{2} \Big|_0^1 \right) \cdot \left( \frac{y^2}{2} \Big|_0^1 \right) \cdot \left( \frac{z^2}{2} \Big|_0^1 \right) = \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) = \frac{1}{8} \end{aligned}$$

(11)

$$\begin{aligned} \cdot I_2 &= \int_0^1 \int_2^3 \int_0^{2x} 3x \, dz \, dx \, dy = \\ & \int_0^1 \left[ \int_2^3 \left[ \int_0^{2x} 3x \, dz \right] dx \right] dy = \\ & \int_0^1 \left[ \int_2^3 \left[ 3x z \Big|_0^{2x} \right] dx \right] dy = \\ & \int_0^1 \left[ \int_2^3 3x^2 \, dx \right] dy = \int_0^1 \left( \frac{3x^3}{3} \Big|_2^3 \right) dy = \\ & \int_0^1 13 \, dy = 13 y \Big|_0^1 = 13 \end{aligned}$$

$$\begin{aligned} \cdot I_3 &= \int_0^1 \int_0^2 \int_0^{2-y} xy^2 \, dy \, dx \, dz \\ I_3 &= \int_0^1 \int_0^2 \int_0^{2-y} xy^2 \, dz \, dy \, dx = \end{aligned}$$

$$\begin{aligned} & \int_0^1 \int_0^2 \left[ xy \frac{z^2}{2} \Big|_0^{2-y} \right] dy \, dx = \\ & \int_0^1 \int_0^2 \left[ \frac{xy(2-y)^2}{2} \right] dy \, dx = \int_0^1 \int_0^2 \left( \frac{2x^2y}{2} + \frac{xy^3}{2} + x^2y^2 \right) dy \, dx = \\ & \int_0^1 \left[ \frac{2x^3y^2}{4} + \frac{x^2y^4}{5} + \frac{2xy^3}{3} \right]_0^2 dx = \int_0^1 \left( \frac{2x^5}{4} + \frac{2x^5}{3} + \frac{2x^5}{8} \right) dx = \\ & \int_0^1 \frac{17}{24} x^5 \, dx = \frac{17}{24} \int_0^1 x^5 \, dx = \frac{17}{168} \Big|_0^1 = \frac{17}{168} \end{aligned}$$

(12)