# Chapter 6: Simplification of Boolean Functions

## 1. Simplification of Boolean Functions

The goal of simplifying logical functions is to:

- Reduce the number of terms in a function, which leads to a reduction in the number of logic gates,
- Obtain a smaller, faster, and less expensive circuit.

### 1.1. Algebraic Simplification

The principle is to apply the rules of Boolean algebra to eliminate variables or terms,

**Rule 1:** Apply the following rules:

| | |
|---|---|
| $A.B + A.\overline{B} = A$ | $(A + B)(A + \overline{B}) = A$ |
| $A + A B = A$ | $A(A + B) = A$ |
| $A + \overline{A} B = A + B$ | $A(\overline{A} + B) = A B$ |

Example :

1. $F(A,B,C) = A.\overline{B} + B.\overline{C} + B.C = A.\overline{B} + B.(\overline{C}+C) = A + B$
2. $F(A,B,C,D) = A.B.C + A.B.\overline{C} + A.\overline{B}.C.D = A.B.(C+\overline{C}) + A.\overline{B}.C.D$

$$= A.B + A.\overline{B}.C.D$$
$$= A(B + \overline{B}(C.D))$$
$$= A(B + C.D)$$
$$= A.B + A.C.D$$

**Rule 2:** Adding an Existing Term:

Example :

1. $A\,B\,C + \overline{A}\,B\,C + A\,\overline{B}\,C + A\,B\,\overline{C} =$

   $A\,B\,C + \overline{A}\,B\,C + A\,B\,C + A\,\overline{B}\,C + A\,B\,C + A\,B\,\overline{C} =$

   $B\,C \quad + \quad A\,C \quad + \quad A\,B$

**Rule 3:** Simplify the canonical form with the minimum number of terms:

Example : Let’s simplify the following function:

1. $F\,(A,B,C) = \sum (2, 3, 4, 5, 6, 7) = A + B$

## 1.2. Karnaugh Map

Looking at the algebraic simplification method, it is noticeable that it becomes quite difficult if **the number of variables increases**. Karnaugh’s method is a **rapid** simplification technique.

### 1.2.1. Adjacency Principle

Let's examine the following expression: : $A\,.\,B + A\,.\,\overline{B}$

Both terms have the same variables. The only difference is the state of variable B, which changes.

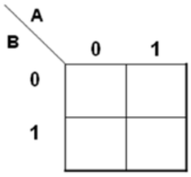
| **These terms are adjacent** | **These terms are not adjacent** |
|---|---|
| $A.B + \overline{A}.B = B$ | $A.B + \overline{A}.\overline{B}$ |
| $A.B.C + A.\overline{B}.C = A.C$ | $A.B.C + A.\overline{B}.\overline{C}$ |
| $A.B.C.D + A.B.\overline{C}.D = A.B.D$ | $A.B.C.D + \overline{A}.B.\overline{C}.D$ |

### 1.2.2. Method Principle

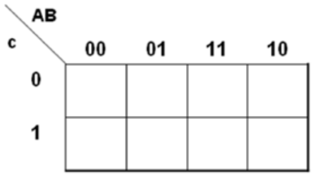
- It is a table with $2^n$ cells, where 'n' is the number of variables.
- The method can be applied to logical functions with 2, 3, 4, 5, and 6 variables.
- Rows and columns are numbered in Gray code.
- Each minterm (TT) corresponds to a cell with a value of 1.
- Each maxterm (TT) corresponds to a cell with a value of 0.
- The $0_s$ can be omitted to simplify the representation.

### 1.2.3. Simplification

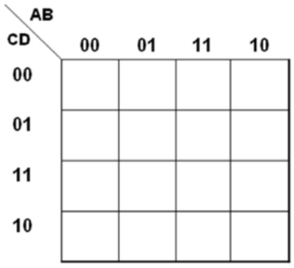
- Identify adjacent cells containing a 1 and group them in sets of $2^n$ (32, 16, 8, 4, 2, 1).
- Two cells are adjacent when they are positioned next to each other horizontally and vertically.
- Cells at the ends of the same row or column are adjacent (cylindrical form).
- The 4 corner cells are considered adjacent.
- The size of a group must be a power of 2.
- With the Karnaugh method, the goal is to **minimize the number of groups** (minimize the number of terms) that contain **the maximum number of $1_s$** (minimize the number of variables).
- Eliminate variables that change states within a group.
- The final function is the sum of terms where the variable states do not change within a group.
- One or more cells can be common to several groups.
- Stop when there is no 1 outside the groups.




TWO-VARIABLE ARRAY




THREE-VARIABLE ARRAY




FOUR-VARIABLE ARRAY

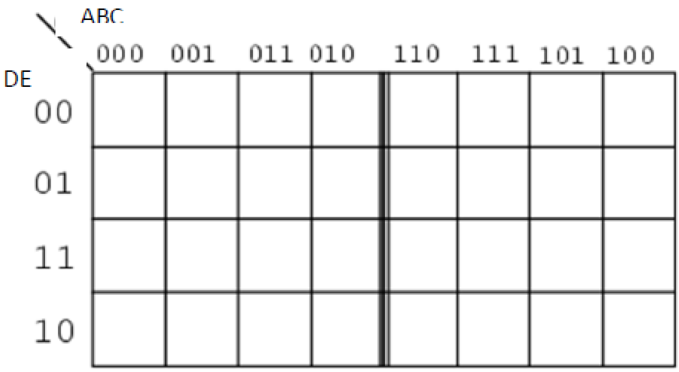
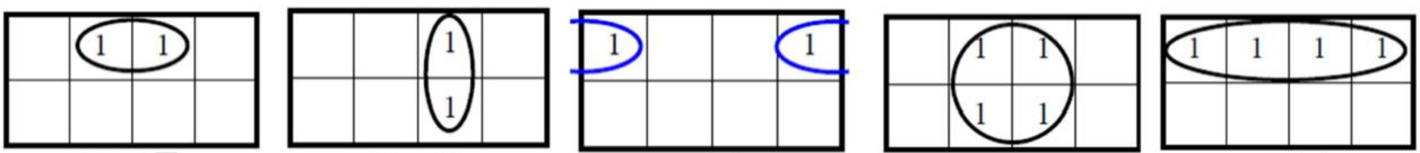


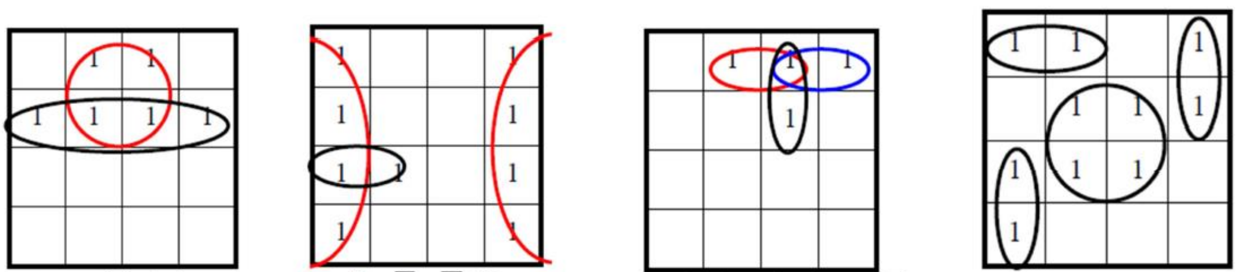

**Tableau 5 variables**

Note:

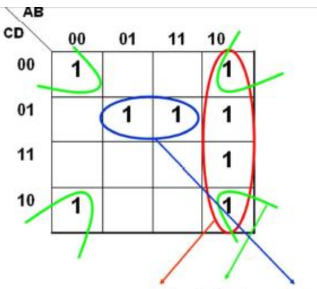
Beyond 6 variables, since the Karnaugh method is no longer applicable, the Mc-cluskey method is used.



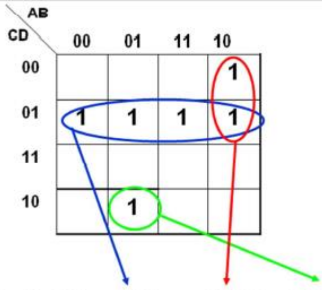
$F = B\overline{C}$ $F = AB$ $F = \overline{B}\,\overline{C}$ $F = B$ $F = \overline{C}$



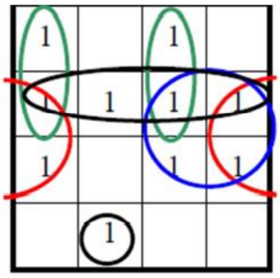
$F = B\overline{C} + \overline{C}D$ $F = \overline{B} + \overline{A}\,CD$ $= B\overline{C}\overline{D} + A\overline{C}\overline{D} + AB\,\overline{C}$ $= BD + \overline{A}\,\overline{C}\,\overline{D} + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C$



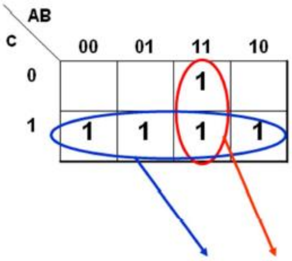
$$F(A,B,C,D) = A\overline{B} + \overline{B}\,\overline{D} + B\overline{C}D$$



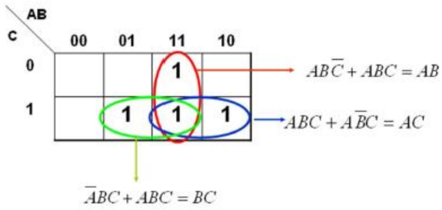
$$F(A,B,C,D) = \overline{C}.D + A.\overline{B}.\overline{C} + \overline{A}.B.C.\overline{D}$$



$$= \overline{C}D + AD + \overline{B}D + \overline{A}\,\overline{B}\,\overline{C} + AB\overline{C} + \overline{A}BC\overline{D}.$$

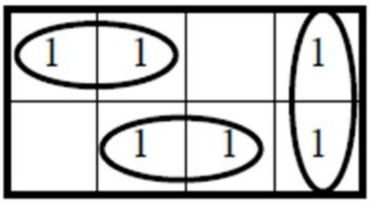


$$F(A,B,C) = C + AB$$

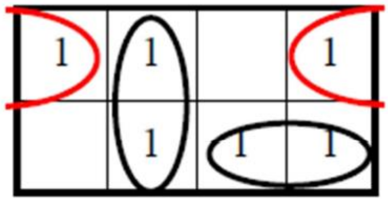



$$F(A,B,C) = AB + AC + BC$$

The simplification result may not be unique

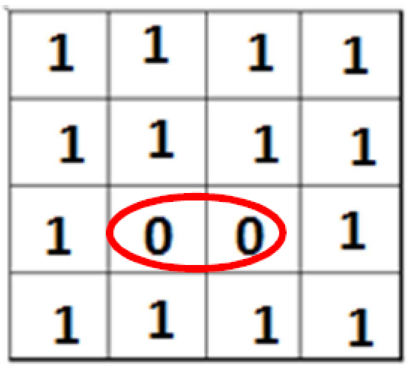


$$= \overline{A}\,\overline{C} + BC + A\overline{B}$$



$$= \overline{B}\,\overline{C} + \overline{A}\,B + AC$$

We can apply the Karnaugh method on MAXTERMS



$$= \overline{B} + \overline{C} + \overline{D}$$