Tutorial series 3

Exercise 1:

a) In each of the following examples, given three boolean variables A, B, and C, use a truth table to demonstrate the equality between two logical expressions.

1) $\overline{A}+\overline{B} = \overline{A.B}$
2) $A+\overline{A}.B = A+B$
3) $(A+B).(\overline{A}.\overline{B}) = 0$
4) $A.C + A.\overline{C} + B.C + B.\overline{C} = A+B$

b) Represent the following two boolean expressions using a truth table:

$S_1 = A + B.C$ $\quad$ $S_2 = (A+B).(A+C)$

What do you notice ? What can we conclude?

Exercise 2:

Simplify the expressions S_1, S_2, S_3, and S_4 using the properties of Boolean algebra.

1) $S_1= \overline{A}.B + A.\overline{B} + A.B$
2) $S_2= B + A.B + B.C + C$
3) $S_3= (B + \overline{B}.A).C + A.B.\overline{C}$
4) $S_4= (A+C).(A+\overline{B}).(A+\overline{C})$

Exercise 3 :

1) Provide the first canonical form of the following functions using the truth table:

a. $F_1(A,B,C) = (\bar{A} + \overline{B}).(B + \bar{C})$
b. $F_2(A,B,C,D) = A.\bar{C}.D + \bar{A}.B.\bar{D}$
c. $F_3(A, B, C) = \sum (1,2,7)$
d. $F_4(A,B,C,D) = \sum(0,3,6,9,10,13)$

2) Provide the second canonical form of the following functions using the truth table :

a. $H_1(A,B,C) = \bar{A}.B + \bar{B}.\bar{C} + A.B.\bar{C}$
b. $H_2(A,B,C,D) = A.B.C + \bar{A}.\bar{B}.C + B.\bar{D} + A.D$
c. $H_3(A, B, C) = \prod (1,2,4,5,6)$
d. $H_4(A, B, C, D) = \prod (4,7,12,14,15)$

3) Consider the following truth tables:

A	B	C	D	F (A,B,C,D)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

A	B	C	H (A,B,C)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

a. Provide the first canonical form of F and the second canonical form of H.
b. Deduce the simplification of each function from the truth table.
c. Demonstrate the correctness of the obtained simplification using algebraic properties

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Exercise 4 :

1. Write in the first canonical form the functions defined by the following propositions:

- **F(A,B,C)= 1** if and only if exactly two of the variables A, B, C are set to 1
- **F(A,B,C)= 1** if and only if the variables A, B, C are all set to 1
- **F(A,B,C)= A+B.C**

2. Write in the second canonical form the functions defined by the following propositions:

- **F(A,B,C)= 0** if and only if exactly one of the variables A, B, C is set to 1
- **F(A,B,C)= 0** if and only if at least two of the variables A, B, C are set to 0
- **F(A,B,C)= C.(B+A+C)**

Exercise 5 :

Using the Morgan's theorems:

1. Verify that the complements of functions F1 and F2 :

- $F_1 = A\,.B + \bar{C}$
- $F_2 = (A + B\,.\;C)\,.\,(\,\bar{A}\,.B + C)$

Are :

- $\overline{F_1} = (\bar{A} + \bar{B})\,.\,C$
- $\overline{F_2} = \bar{C} + \bar{A}\,.\;\bar{B}$

2. Express the complement of the following function:

- $F_3 = \bar{A}\,.\;\overline{C}\,.\overline{D} + A\,.D + \bar{A}\,.B.C + \bar{A}\,.\bar{B}.\bar{C}$

Exercise 6 :

Simplify the following functions algebraically:

1. $F_1(X,Y,Z) = X.Y.\bar{Z} + \bar{X}.Y.Z + \bar{X}.\bar{Y}.\bar{Z} + \bar{X}.\bar{Y}.Z + X.Y.Z + \bar{X}.Y.\bar{Z}$
2. $F_2(A,B,C) = A.\bar{B}.\bar{C} + \bar{A}.\bar{B}.C + \bar{A}.\bar{B}.\bar{C} + \bar{A}.B.\bar{C}$
3. $F_3(A,B,C) = \bar{A}.\bar{B}.C + \bar{A}.B.\bar{C} + A.\bar{B}.C + A.B.\bar{C} + A.B.C + \bar{A}.B.C$
4. $F_4(A,B,C,D) = \sum(2,3,4,5,6,7,8,9,10,11,12,13,14,15)$

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Exercise 7:

Simplify the following functions using the Karnaugh method, then provide the logic diagram for each simplified function:

1. $F_1(A,B) = A.\bar{B} + \bar{A}.\bar{B} + A.B$
2. $F_2(A,B,C) = \bar{A}.\bar{B}.\bar{C} + A.\bar{B}.\bar{C} + \bar{A}.B.\bar{C}$
3. $F_3(A,B,C) = \sum(1,2,6)$
4. $F_4(A,B,C) = \sum(0,1,2,3,4,5)$
5. $F_5(A,B,C) = A.\bar{B}.\bar{C} + A.\bar{B}.C + \bar{A}.\bar{B}.C + A.B.\bar{C}$
6. $F_6(A,B,C,D) = \bar{A}.\bar{B}.\bar{C}.\bar{D} + A.\bar{B}.\bar{C}.\bar{D} + A.\bar{B}.\bar{C}.D + \bar{A}.B.C.D + A.\bar{B}.C.D$
7. $F_7(A,B,C,D) = \sum(0,1,3,5,6,7,9,13)$
8. $F_8(A,B,C,D) = \sum(3,4,5,7,9,13,14,15)$

Exercise 8 :

For each Karnaugh map below, provide:

1. The associated logical function.
2. The simplification of the logical function found.

Exercise 9 :

We consider a logic circuit with 3 switches as inputs: Switch1, Switch2, and Switch3, and a single bulb **B** as output.

The bulb B lights up (red or green) depending on the state of the variables S_1, S_2, and S_3 (where S_1, S_2, and S_3 represent switches: Switch1, Switch2, and Switch3, respectively)

The following situations allow the bulb **B** to light up (**B** =1: green color):

1. Switch 1 alone or with either of the other two switches (Switch 2, Switch 3) can turn on bulb **B**.
2. Both switches (Switch 2, Switch 3) together can turn on bulb **B**.

Required work:

1. Create the truth table.
2. Simplify the output equation using the Karnaugh map method.
3. Draw the Logic Diagram.

F1

C \ AB	00	01	11	10
0		1		1
1	1	1		1

F2

C \ AB	00	01	11	10
0	1	1	1	
1		1	1	1

F3

CD \ AB	00	01	11	10
00	1	1		1
01	1	1	1	1
11	1	1	1	1
10	1			1

F4

CD \ AB	00	01	11	10
00	1			
01	1		1	1
11	1		1	1
10	1			1

F5

CD \ AB	00	01	11	10
00	1		1	1
01				
11				
10	1	1	1	1

Exercise 10 :

- Four keys $\mathbf{C_1}$, $\mathbf{C_2}$, $\mathbf{C_3}$, and $\mathbf{C_4}$ control the opening of a safe.

-The combinations for opening the safe are:

1. Key C1 or key C2 with both keys C3 and C4.
2. Key C2 and keys C3 and C4.
3. Both keys (C3 and C4) with key C1 or key C2.

-Create the Logic Diagram for this system.

Note: The output function is **1** if the safe is open and **0** otherwise.