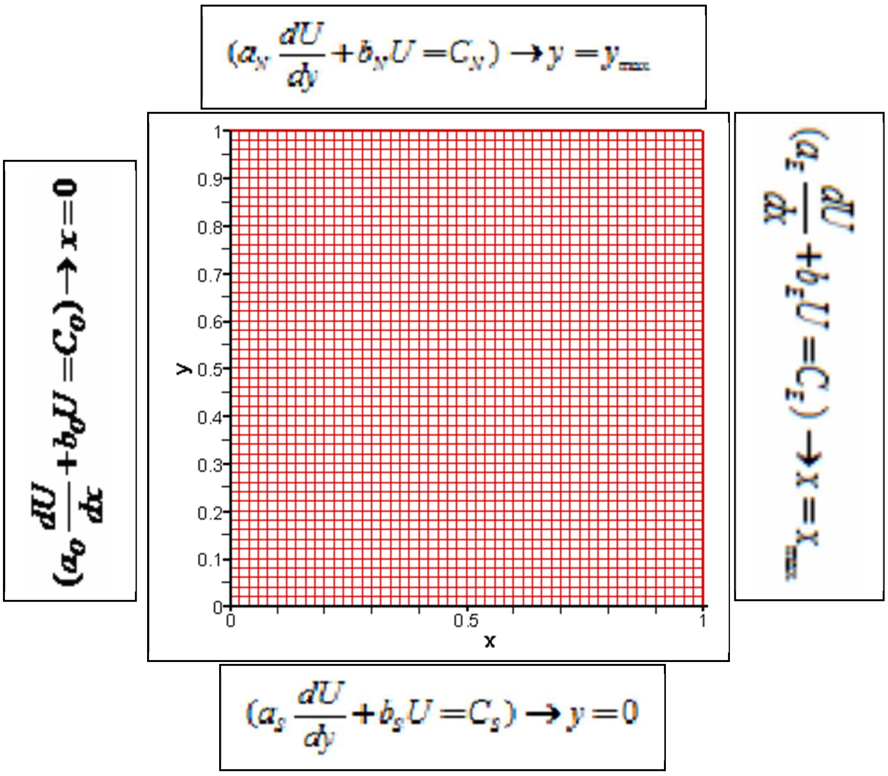
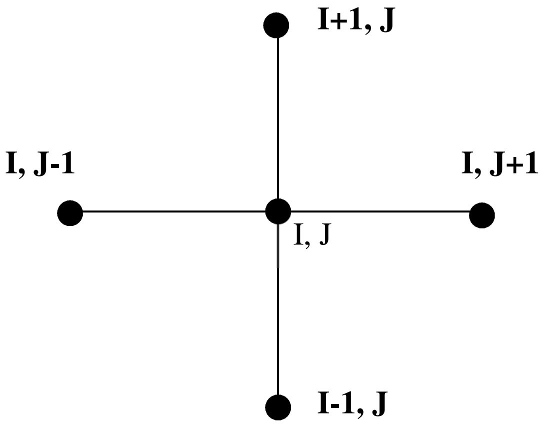
TP 4(suite) : Résoudre Équation de chaleur 2D (La méthode différence finit schéma explicite) :

$$\frac{\partial U}{\partial t}=\alpha.\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right),\quad 0\le x\le x_{\max}, 0\le y\le y_{\max}\quad ,t>0$$

- Condition initiale : $U(t=0,x,y)=g(x,y)$





$$\left(\frac{\partial^2 U}{\partial y^2}=\frac{U_{I-1,J}-2.U_{I,J}+U_{I+1,J}}{\Delta y^2}\right)^t ;\quad \left(\frac{\partial^2 U}{\partial x^2}=\frac{U_{I,J-1}-2.U_{I,J}+U_{I,J+1}}{\Delta x^2}\right)^t ;\quad \frac{\partial U}{\partial t}=\frac{U^{t+\Delta t}{}_{I,J}-U^{t}{}_{I,J}}{\Delta t}$$

- **On remplace dans les équation de chaleur :**

$$\frac{U^{t+\Delta t}{}_{I,J}-U^{t}{}_{I,J}}{\Delta t}=\alpha.\left(\frac{U^t_{I,J-1}-2.U^t_{I,J}+U^t_{I,J+1}}{\Delta x^2}+\frac{U^t_{I-1,J}-2.U^t_{I,J}+U^t_{I+1,J}}{\Delta y^2}\right)$$ on note : $Rx=\frac{\alpha.\Delta t}{\Delta x^2}$; $Ry=\frac{\alpha.\Delta t}{\Delta y^2}$

$$U^{t+\Delta t}_{I,J}=R_x*\left(U^t_{I,J-1}+U^t_{I,J+1}\right)-\left(2.\left(R_x+R_y\right)-1\right).U^t_{I,J}+R_y*\left(U^t_{I-1,J}+U^t_{I+1,J}\right)$$

Pour $K=1,Nt$ **% la boucle de temps**

$t=t+\Delta t$

Pour $I=2,Ny-1$; $J=2,Nx-1$

$$U^{t+\Delta t}_{I,J}=R_x*\left(U^t_{I,J-1}+U^t_{I,J+1}\right)-\left(2.\left(R_x+R_y\right)-1\right).U^t_{I,J}+R_y*\left(U^t_{I-1,J}+U^t_{I+1,J}\right)$$

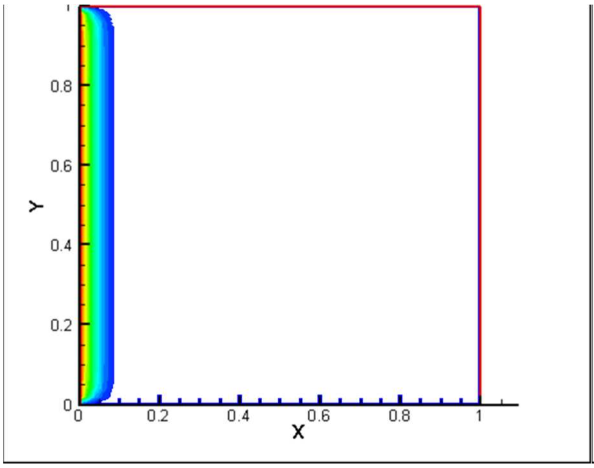
Pour $I=1$; $J=1,Nx$ $\rightarrow U^{t+\Delta t}_{I,1}=\left(C_S-\frac{a_S}{\Delta y}.U^{t+\Delta t}_{I,2}\right)/\left(C_S-\frac{a_S}{\Delta y}\right)$

Pour $I=Ny$; $J=1,Nx$ $\rightarrow U^{t+\Delta t}_{I,NJ}=\left(C_N+\frac{a_N}{\Delta y}.U^{t+\Delta t}_{I,NJ-1}\right)/\left(C_N+\frac{a_N}{\Delta y}\right)$

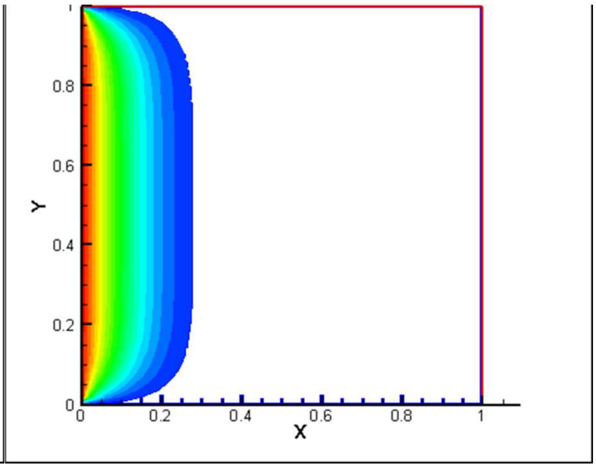
Pour $I=1,Ny$; $J=1$ $\rightarrow U^{t+\Delta t}_{1,J}=\left(C_O-\frac{a_O}{\Delta x}.U^{t+\Delta t}_{2,J}\right)/\left(C_O-\frac{a_O}{\Delta x}\right)$

Pour $I=1,Ny$; $J=Nx$ $\rightarrow U^{t+\Delta t}_{NI,J}=\left(C_E+\frac{a_E}{\Delta x}.U^{t+\Delta t}_{NI-1,J}\right)/\left(C_E+\frac{a_E}{\Delta x}\right)$

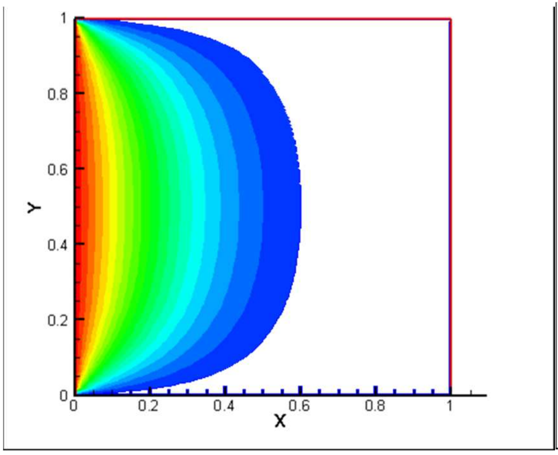
Fin la boucle de temps



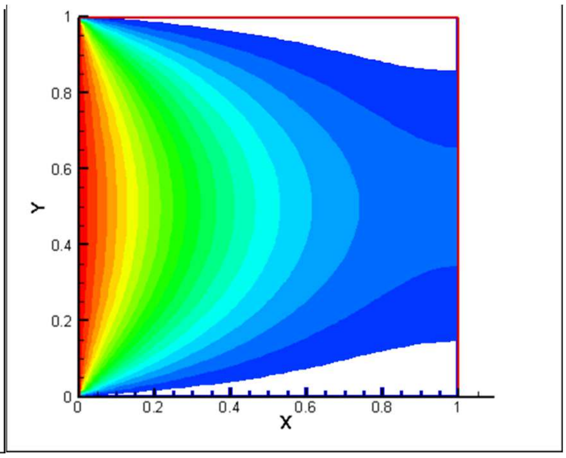
$Nt = 10$



$Nt = 100$



$Nt = 500$



Nt=1000

```
%  Remarque I--> y  ; J--->x; U(I,J)=U(y,x)
clc  ;    clear all
nt=1000
ny=51 ;ymax=1      ;      dy=ymax/(ny-1) ;
nx=51 ;xmax=1      ;      dx=xmax/(nx-1) ;
ao=0     ;  bo=1      ;  co=400 ;
ae=1     ;  be=0      ;  ce=0 ;
as=0     ;  bs=1      ;  cs=300 ;
an=0     ;  bn=1      ;   cn=300 ;
dt=0.1  ; d=0.001 ;    rx=dt*d/(dx^2)  ;  ry=dt*d/(dy^2) ;
t=0 ;    u(1:ny,1:nx)=300 ;    %   valeur intial
for k=1:nt-1      %------- debut boucle de temps
for i=2:ny-1
for j=2:nx-1
u1(i,j)=ry*(u(i-1,j)+u(i+1,j))+rx*(u(i,j-1)+u(i,j+1)) - ...
                        (2*(rx+ry)-1)*u(i,j);
end
end
% ======== condition aux limites =================
        u1(1,2:nx-1) =(cs-(as*u1(2, 2:nx-1)/dy))   /(bs-(as/dy));
        u1(ny, 2:nx-1)=(cn+(an*u1(ny-1, 2:nx-1)/dy))/(bn+(an/dy));
         u1(:,1)=(co-(ao*u1(:,2)/dx))    /(bo-(ao/dx));
        u1(:,nx)=(ce+(ae*u1(:,nx-1)/dx))/(be+(ae/dx));
  u(:,:)=u1(:,:) ;  t=t+dt ;
end %-----------------------  fin  boucle de temps
 [x,y]=meshgrid(0:dx:xmax,0:dy:ymax);
surf(x,y,u1,'lineStyle','none','faceColor','interp');
```