University of Batna 2-Institute of Industrial Hygiene and Safety

L1-Math1
2023/2024

Tutorial n°3

Real Functions of One Real Variable

Exercice 1

Evaluate the following limits :

$$\lim_{x\to+\infty} \frac{\sqrt{1+x^2}-\sqrt{1+x}}{x^2}, \qquad \lim_{x\to-\infty} \frac{4x^2-sin(5x)}{x^2+7} \text{ (use squeeze theorem)}$$

Exercice 2

1. Show that f has a continuous extension at $x=2$, and find that extension.

$$f(x) = \frac{x^2-x-2}{x^2-4}, \qquad x \neq 2$$

2. Determine the value of a and b for which the function g is continuous at $x=0$.

$$g(x) = \begin{cases} \dfrac{\sin((a+1)x)+\ln(x+1)}{x} & \text{for } x<0 \\ b & \text{for } x=0 \\ \dfrac{\sqrt{x+x^2}-\sqrt{x}}{x\sqrt{x}} & \text{for } x>0 \end{cases}$$

Exercice 3

1. Examine the differentiability of f on $\mathbb{R}$, where

$$f(x) = \begin{cases} x^2\sin(\frac{1}{x}) & \text{for } x \neq 0 \\ 0 & \text{for } x=0 \end{cases}$$

2. Discuss the differentiability of g at $x=0$, where

$$g(x) = \ln(1+|x|)$$

Exercice 4

1. Let f be the function defined by : $\quad f(x) = 2x^2-16x+1$
   (a) Find the extremum of f on $\left[0,9\right]$
   (b) Prove that the equation $f(x)=0$ has a unique solution α on $\left[0,3\right]$.

2. Let g be the function defined by : $\quad g(x) = \begin{cases} \dfrac{1-\cos(2\pi x)}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x=0 \end{cases}$

   Show that there exist $c \in \left]-1,1\right[$ such that $g'(c)=0$